# E308: Persistent currents of a particle in a ring

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**The problem:**

A particle with mass M and charge e is placed on a one dimensional ring of length L. the flux through the ring is $\Phi$. The particle is in the state ground of the ring.

(1) Find the currrent I($\Phi$)= - $\frac{dE}{d\Phi}$ that flows in the ring.
(2) Draw a schematic grafh of the current as a function of the flux.
(3) Explain (qualitivly) what is the effect on the result when a scatter is present.

**The solution:**

(1) The Hamiltonian is :

$$\hat{H} = \frac{1}{2M}(\hat{p} - \frac{e}{c}A)^2$$

The flux through the ring (Aharonov-Bohm geometry):

$$\Phi = AL$$

We get a partricle in the interval 0<x<L with periodical boundary conditions,so $\hat{H}$ is:

$$\hat{H} = \frac{1}{2M}(\hat{p} - \frac{e\Phi}{cL})^2$$

The eigenstates of the hamiltonian are the momentum states $|\mathrm{k}\rangle$ and The eigenvalues of the energy of a particle in a closed ring are:

$$E(n) = \frac{1}{2M}(\frac{2\pi\hbar}{L}n - \frac{e\Phi}{2\pi})^2 = \frac{1}{2M}(\frac{2\pi\hbar}{L})^2(n - \frac{e\Phi}{2\pi})^2$$

The current which is created by an electron is:

$$I(\Phi, n) = -\frac{dE(n)}{d\Phi} = -\frac{d}{d\Phi}[\frac{1}{2M}(\frac{2\pi\hbar}{L})^2(n - \frac{e\Phi}{2\pi})^2]$$

$$I(\Phi, n) = \frac{2\pi\hbar}{ML^2}\frac{e}{c}n - \frac{e^2\Phi}{ML^2c^2}$$

$$n = 0, \pm 1, \pm 2....$$

The unit $\Phi_0 = \frac{2\pi\hbar}{e}c$ is called "fluxon". The current in the ground state is a periodic function with the period $\Phi_0$, its graph has a saw tooth shape.n=0 is the ground state to the interval:

$$-\frac{\Phi_0}{2} < \Phi < \frac{\Phi_0}{2} \qquad With: \qquad I(\Phi) = -\frac{e^2}{ML^2c^2}\Phi$$

And the plot repeats itself to the intervals:

$$-\frac{\Phi_0}{2} + \Phi_0 n < \Phi < \frac{\Phi_0}{2} + \Phi_0 n \qquad With: \qquad I(\Phi, n) = \frac{2\pi\hbar}{ML^2}\frac{e}{c}n - \frac{e^2}{ML^2c^2}\Phi$$

(2) The grafh of the current as a function of $\Phi$ in the ground state will be a saw tooth.

(3) If u$\rightarrow \infty$ the energy levels will not be dependent on the flux,similar to a particle in a infinite potential well, so there will be no current I=0.For u$\rightarrow$0 the energy levels as a function of the flux will be parabolic, in the ground state the current will behave as a saw tooth grafh, as u increase the amplitude of the current (periodic behaviour) as a function of $\Phi$ will decrease and the edges of the saw tooth will be more round.