Exe.251 Influance of bump in a potential well on time delay

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The problem:

Consider a particle scattered on the potential well $0 < x < R$, as a result of the time it spens inside the well, the particle suffers a phase shift. Given a shielded potential well, $V(x) = u\delta(x-R)$ assume that the floor of the well V can receive different values from $-V_0$ to V_0.

(1) Calculate the time delay resulting from the phase shift.
(2) Draw a chart of the time delay vs. V (the potential well's floor).
(3) Analyze the chart's different regions and explain the classical limit.

The solution:

(1)The wave function of the particle outside the well: $\psi_(x) = exp(-ikx) - exp(i2\delta)exp(ikx)$. inside the well $\psi = \sqrt{\frac{2}{R}}sin(\alpha x)$; were $\alpha = \sqrt{2(E-V)}$ and $k = \sqrt{2E}$.
We will use the notation $\hbar = m = 1$.

The right matching for shielded well at the point R:

$$\frac{\psi'}{\psi} - \frac{\psi'}{\psi} = 2u$$

We can easily derive the phase shift:

$$\delta = arccot\left(\frac{2u + \alpha\cot(\alpha x)}{k^2 + (2u + \alpha\cot(\alpha x))^2}\right) - kR$$

Deriving the following: (eather by yourself or with the help of the Mathematica)

$$\tau = \frac{d(2\delta)}{dE} = \frac{d(2\delta)}{dk}\frac{dk}{dE}$$

notice that α is also a function of E.

(2) To see the chart and the equation of the time delay please open the file

(3)Analysis:

(a) On the right side of the chart we can see that τ receives values under zero, classicaly it means that the particle did not spend time in the well at all, it was reflacted by the potential. The limit is $\tau = \frac{-2R}{v}$.

(b) The point where the chart have no continuousness is neturally the point where $V = E$.

(c) On the left, one can observe the resonances each time the particle's energy meets one of the known energies in a potential well $E = \frac{1}{2m}(\frac{\pi \cdot n}{R})^2$. Classicaly, the floor is lower than the energy of the particle - consequently, the velocity get larger and the particle moves out of the well quicker.
(d) In order to receive the classical limit; $\frac{E}{V} >> 1$ and $kR >> 1$. the classical limit results from:
The particle's velosity $v = \sqrt{2(E-V)}$ The particle's time delay $\tau = \frac{R}{\sqrt{2(E-V)}} - \frac{R}{\sqrt{2E}}$.

Note: using Mathematica it is hard to get rid of the oscilations on the left side of the chart, but for large E one can see that their amplitude is dumpting.