E268: Scattering of a particle of 4 states

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The problem:

A particle can be painted in 4 different colors (marked as 1,2,3,4).
When a particle is scattered, it might change its color.
If the particle was in "color 1" then after scattering it might get "color 2" or stay 1.
A particle colored 2 can be painted 1 or stay 2.
A particle colored 3 can't be 2 after scattering.

(1) Can a particle colored 3 be painted 1?

(2) If a particle is colored 4, which colors can we say it won't be painted?

Instruction: Use the properties of the scattering matrix.

The solution:

The scattering matrix according to the question is:

$$\mathcal{S} = \begin{pmatrix} \alpha & \gamma & 0 & 0 \\ \beta & \delta & 0 & 0 \\ \epsilon & 0 & e & f \\ a & b & c & d \end{pmatrix}$$

With ϵ, a, b, c, d, e, f are unkown.

We know that a scattering matrix is unitary, which means $S \dagger S = I$,
so calculating the main diagonal of the matrix multiplication we get:
a) $|\alpha|^2 + |\beta|^2 + |\epsilon|^2 + |a|^2 = 1$
b) $|\gamma|^2 + |\delta|^2 + |b|^2 = 1$

We also know that the sum of scattering probabilities of each particle is 1:
c) $|\alpha|^2 + |\gamma|^2 = 1$
d) $|\beta|^2 + |\delta|^2 = 1$

If we sum a) and b), and use c) and d) we get:
e) $|\epsilon|^2 + |a|^2 + |b|^2 = 0$
The only solution for this equation is $a = b = \epsilon = 0$

Hence we can see that:

(1) ϵ=0, which means that a "color 3" particle can not be scattered into a "color 1" particle.

(2) a=0, b=0 which means that a "color 4" particle can not be scattered into a "color 1" nor 2 particle.