Ex2670: Inelastic scattering on a two level atom

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The problem:

Consider a scattering problem that is described by the Hamiltonian $H = \frac{p^2}{2m} - \frac{h}{2}\sigma_z + u\sigma_x\delta(x)$. The target is prepared in the lower level. The kinetic energy of the incident particle is E_v.

(1) Assuming $E_v > h$, express the "free wave" solutions in the scattering channels in terms of reflection and transmission amplitudes.
(2) Write down the matching equation for $\psi_\sigma(x)$ and its derivative at $x = 0$, where σ is the spin index.
(3) Find the total transmission T as a function of v.
(4) Repeat (1) and (3) for the case $E_v < h$.

The solution:

(1) Looking at the energies of the system for the $x \neq 0$ space, $H|\psi >= E|\psi >$ will yield the energies in the form of $E = E_v \pm \frac{h}{2}$. It will be more conviniant to look upon a shifted Hamiltonian: $\widetilde{H} = H + \frac{h}{2} \cdot \hat{I}$, with the energies $E_0 = E_v,\ E_1 = E_v + h$, for $x \neq 0$. The according momentums will be:

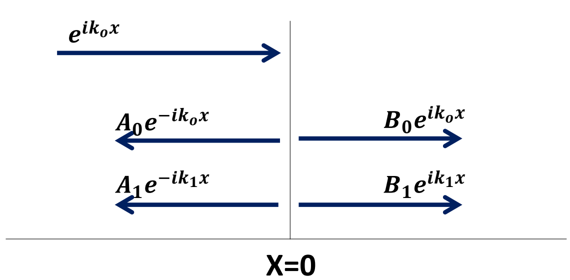
$$k_0 = \sqrt{2mE_0},\ k_1 = \sqrt{2m\left(E_0 - h\right)}$$

While $E_n, k_n,\ n = \{0, 1\}$ denotes the n site's energy and momenutm accordingly (n=0 being the lower level). The "free wave" solution will be represented by:

$$\Psi = \begin{pmatrix} \psi_0 \\ \psi_1 \end{pmatrix}$$

Assuming the incident wave propagates from the left side of the barrier to the right:

$$\psi_0 = \begin{cases} e^{ik_0x} + A_0e^{-ik_0x} & ,\ x < 0 \\ B_0e^{ik_0x} & ,\ x > 0 \end{cases}, \qquad \psi_1 = \begin{cases} A_1e^{-ik_1x} & ,\ x < 0 \\ B_1e^{ik_1x} & ,\ x > 0 \end{cases}$$



(2) The boundry conditions on $\Psi(x=0)$:

$$\psi_\sigma\left(0^{+}\right)=\psi_\sigma\left(0^{-}\right)$$

$$\Psi'\left(0^{+}\right)-\Psi'\left(0^{-}\right)=-2mu\sigma_x\Psi\left(0\right)$$

(3) The boundry conditions of $\Psi(x)$ in x=0 yields:

$$1+A_0=B_0,\ A_1=B_1$$

$$\left(\begin{array}{c} ik_0B_0-(ik_0-ik_0A_0) \\ iB_1k_1-(-iA_1k_1) \end{array}\right)=-2mu\left(\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array}\right)\left(\begin{array}{c} B_0 \\ B_1 \end{array}\right)$$

Solving for B_0, B_1:

$$B_0=\frac{k_0k_1}{k_0k_1+(mu)^2}\ ,\qquad B_1=\frac{imuk_0}{k_0k_1+(mu)^2}$$

Noting that current conservation:

$$1=\frac{1}{v_0}\sum_{n=0}^{1}|A_n|^2v_n+|B_n|^2v_n=R+T$$

We finally idnetify the transmission coefficient T as a function of v $\left(v=\frac{k_0}{m},\ k_1=\sqrt{k_0^2-2mh}\right)$ for the case $E_v>h$:

$$T=|B_0|^2+|B_1|^2\cdot\frac{k_1}{k_0}=\frac{k_0k_1}{k_0k_1+(mu)^2}$$

(4) For $E_0=E_v<h$, k_1 is an imaginary number. Now, defining: $k_1=i\sqrt{2m\left(h-E_v\right)}=iq_1$, if the particle will scater to $E=E_1$ state it will be bound, so now $T=|B_0|^2$, and all the wave probability will be at the state $E=E_0$, and now:

$$\psi_0=\begin{cases} e^{ik_0x}+A_0e^{-ik_0x} & ,\ x<0 \\ B_0e^{ik_0x} & ,\ x>0 \end{cases}\ ,\qquad \psi_1=\begin{cases} A_1e^{q_1x} & ,\ x<0 \\ B_1e^{-q_1x} & ,\ x>0 \end{cases}$$

From the previous boundry equations: $B_0=\frac{ik_0q_1}{(mu)^2+ik_0q_1}$.

And finally, the total transmission T as a function of v $\left(v=\frac{k_0}{m},\ k_1=\sqrt{k_0^2-2mh}\right)$ for the case $E_v<h$:

$$T=|B_0|^2=\frac{(k_0q_1)^2}{(mu)^4+(k_0q_1)^2}$$