Exercise 256: scattering over a step

Submitted by: ahmed suleman

for given potential:

$$V(x) = \begin{cases} 0 & \text{for } x < 0 \\ V_0 & \text{for } x > 0 \end{cases}$$

There is a particle with $E > V_0$, left of the potential .

a. finde the transmission and reflectin coefficient.

b. find the S matrix.

solution:

The stationary solution , of schrödinger equation are :

$$\Psi_L(x) = \frac{1}{\sqrt{v_1}}(A_L e^{iK_1x} - B_L e^{-iK_1x}) \qquad K_1^2 = \frac{2mE}{\hbar^2} \quad v_1 = \frac{K_1}{m}$$

$$\Psi_R(x) = \frac{1}{\sqrt{v_2}}(A_R e^{-iK_2x} - B_R e^{iK_2x}) \qquad K_2^2 = \frac{2m(E-v_0)}{\hbar^2} \quad v_2 = \frac{K_2}{m}$$

Note:
In the class, doron write thise equation in genarly in spherical coordinate,(function of "r").
In thise problem you must be attention that we do a 1D problem then,
for left side of the potential $r \equiv -x$ and for right side of the potential $r \equiv x$.

The matching condition at $x = 0$,the continuty of $\Psi(x)$ and $\Psi'(x)$
Yields:

$I.\ \ \Psi_L(0) = \Psi_R(0)$

$II.\ \ \Psi'_L(0) = \Psi'_R(0)$

So:

1. $\dfrac{A_L}{\sqrt{v_1}} - \dfrac{B_L}{\sqrt{v_1}} = \dfrac{A_R}{\sqrt{v_2}} - \dfrac{B_R}{\sqrt{v_2}}$

2. $\dfrac{K_1 A_L}{\sqrt{v_1}} - \dfrac{K_1 B_L}{\sqrt{v_1}} = -\dfrac{K_2 A_R}{\sqrt{v_2}} - \dfrac{K_2 B_R}{\sqrt{v_2}}$

we obtain:

$$1. \quad B_R = \frac{K_1 - K_2}{K_1 + K_2} A_R - \frac{2\sqrt{K_1 K_2}}{K_1 + K_2} A_L$$

$$2. \quad B_L = \frac{K_2 - K_1}{K_1 + K_2} A_L - \frac{2\sqrt{K_1 K_2}}{K_1 + K_2} A_R$$

a. consider incident particle from $x = -\infty \longleftrightarrow A_R = 0$

then:

$$B_R = -\frac{2\sqrt{K_1 K_2}}{K_1 + K_2} A_L$$

$$B_L = B_L = \frac{K_2 - K_1}{K_1 + K_2} A_L$$

The transmission coefficient is:

$$T = |\frac{B_R}{A_L}|^2 = \frac{4K_1 K_2}{(K_1 + K_2)^2}$$

the reflectin coefficient is :

$$R = |\frac{B_L}{A_L}|^2 = (\frac{K_2 - K_1}{K_1 + K_2})^2 = 1 - \frac{4K_1 K_2}{(K_1 + K_2)^2}$$

we can see that $\quad T + R = 1$

b. The S matrix.

$$\begin{pmatrix} B_R \\ B_L \end{pmatrix} = S \begin{pmatrix} A_R \\ A_L \end{pmatrix} \longleftrightarrow S_{matrix} = \begin{pmatrix} \frac{K_1 - K_2}{K_1 + K_2} & -\frac{2\sqrt{K_1 K_2}}{K_1 + K_2} \\ -\frac{2\sqrt{K_1 K_2}}{K_1 + K_2} & \frac{K_2 - K_1}{K_1 + K_2} \end{pmatrix}$$