E557: A particle in an accelerated box

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The problem:

A particle is placed in an accelerated box.

(1) Write the transformation from the laboratory system to the box system.
(2) Write the Hamiltonian in the box system.
(3) distinguish between a partial transformation and a full one.

The solution:

(1) The partial s transformation from the laboratory system to the accelerated box is:

$$S = e^{\frac{iat^2\hat{p}}{2}} \tag{1}$$

(2) in the laboratory system the hemiltonian is:

$$H \ = \ \frac{p^2}{2m} + V(x - \frac{1}{2}at^2) \tag{2}$$

transform of the hemiltonian to the accelerated box system

$$\tilde{H} = SHS^{-1} + i\frac{\partial S}{\partial t}S^{-1} \tag{3}$$

$$SHS^{-1} = \frac{p^2}{2m} + V(x) \tag{4}$$

$$i\frac{\partial S}{\partial t}S^{-1} = i(iat)e^{\frac{iat}{2}^2\hat{p}}e^{-\frac{iat}{2}^2\hat{p}} = -atp \tag{5}$$

$$\tilde{H} = \frac{p^2}{2m} + V(x) - atp = \frac{1}{2m}(p^2 - 2matp) + V(x) = \tag{6}$$

$$\frac{1}{2m}(p^2 - 2matp + (mat)^2) - \frac{(mat)^2}{2m} + V(x) = \frac{1}{2m}(p - mat)^2 + V(x) - \frac{1}{2}ma^2t^2 \tag{7}$$

$$\tilde{A}(x) = mat \tag{8}$$

$$\tilde{V}(x) = V(x) - \frac{1}{2}ma^2t^2 \tag{9}$$

using maxwell equations in one dimension :

$$\tilde{\epsilon}(x) = -\frac{d}{dx}\tilde{V}(x) - \frac{d\tilde{A}(x)}{dt} = -\frac{d}{dx}V(x) - ma = \epsilon - ma \tag{10}$$

(3) We have got rid of the moving walls, but the price was non-zero vector potential. Next, we would like to gauge away the vector potential, so as to have only scalar potential. that can be done with:

$$\Lambda(x) = mat\hat{x}$$

We see that the guage needed is in fact boost transformation:

$$S = e^{-imat\hat{x}} \tag{11}$$

The Hemiltonian is:

$$\tilde{H} \;=\; \frac{1}{2m}(p - mat)^2 + V(x) - \frac{1}{2}ma^2t^2 \tag{12}$$

"Guaging"' it:

$$\tilde{\tilde{H}} = S\tilde{H}S^{-1} + i\frac{\partial S}{\partial t}S^{-1} \tag{13}$$

$$S\tilde{H}S^{-1} = \tilde{H}(S\hat{x}S^{-1}, S\hat{p}S^{-1}) = \tilde{\tilde{H}}(\hat{x}, \hat{p} + mat) \tag{14}$$

$$i\frac{\partial S}{\partial t}S^{-1} = max \tag{15}$$

$$\tilde{\tilde{H}} \;=\; \frac{p^2}{2m} - \frac{m}{2}a^2t^2 + V(x) + max \tag{16}$$

at the accelerated box:

$$\tilde{\tilde{A}}(x) = 0 \tag{17}$$

$$\tilde{\tilde{V}}(x) = +V(x) + max - \frac{m}{2}a^2t^2 \tag{18}$$

using maxwell equations in one dimension:

$$\tilde{\tilde{\epsilon}}(x) = -\frac{d}{dx}\tilde{V}(x) - \frac{d\tilde{A}(x)}{dt} = -\frac{d}{dx}V(x) - ma = \epsilon - ma \tag{19}$$

we see that the boost is a gauge, with no effect over the electric field