EX146: The neutrino problem

Submitted by: Ilya Osherov

The problem:

A neutrino particle of an electron type is produced as a result of interaction with an electron. We'll denote its state as $|\nu_e\rangle$. A neutrino of a different type is produced as a result of interaction with a muion, we'll denote it's state as $|\nu_\mu\rangle$. These are different types of particles. Some sensors are able to detect only particles of one of these types. Energy eigenstates of these particles are given as (*)

$$|\nu_1\rangle = cos\theta|\nu_\mu\rangle + sin\theta|\nu_e\rangle$$

$$H|\nu_1\rangle = E_1|\nu_1\rangle$$

$$|\nu_2\rangle = -sin\theta|\nu_\mu\rangle + cos\theta|\nu_e\rangle$$

$$H|\nu_2\rangle = E_2|\nu_2\rangle$$

If at a time t=0, a neutrino of an electron type was produced, what is the probobility that at a time t the sensor will detect a nutrino of the same type?

The solution:

By solving (*) with respact to $|\nu_e\rangle$ we get

$$|\nu_e\rangle = sin\theta|\nu_1\rangle + cos\theta|\nu_2\rangle$$

Lets denote the time depandant state of a particle as $|\psi(t)\rangle$. Now we can write

$$|\psi(t=0)\rangle = |\nu_e\rangle = \sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$$

And by adding the time depandance we get

$$|\psi(t)\rangle = \sin\theta e^{-iE_1t}|\nu_1> + \cos\theta e^{-iE_2t}|\nu_2\rangle$$

Lets find the probobility of detecting an electron type nutrino

$$Prob(\nu_e, t) = |\langle\nu_e|\psi(t)\rangle|^2$$

$$= |(\sin\theta\langle\nu_1| + \cos\theta\langle\nu_2|)(e^{-iE_1t}\sin\theta|\nu_1> + e^{-iE_2t}\cos\theta|\nu_2>)|^2$$

And after some calculating we get

$$Prob(\nu_e, t) = \sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta\cos(E_2 - E_1)t$$

We can easily see that the maximum value of the $Prob(\nu_e, t)$ is 1. Because the maximum value of $\cos(E_2 - E_1)t$ is 1, in which case we get

$$Prob(\nu_e, t) = \sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta = (\sin^2\theta + \cos^2\theta)^2 = 1$$