# Proton Spin State

**Submitted by: Ido Ziv**

**The problem:**
The spin state of a system that includes three quarks is described in the standard presentation- $2 \otimes 2 \otimes 2$. The proton is a particle made up of three quarks (quarks are particles of Dirak, ie fermions with a half spin).For the purpose of the question use the state called "polarized proton" in a form: $|\mathrm{p}\uparrow\rangle = |\mathrm{uud}\rangle \oplus |\mathrm{spinstate}\rangle$. The charge q of quarks u and d is respectively 2/3 and 1/3 of the charge of the proton, and the mass of quark is 1/3 of the mass of the proton. Due to symmetry considerations-the spin state of the proton is a spin $\frac{1}{2}$ state with even symmetry to replace u. Tip: It is convenient to perform the presentation in the following order:$(2 \otimes 2) \otimes 2$. When a Dirak particle is placed in a magnetic field in the direction of the Z axis its energy is: $E = -g\frac{q}{2M}S_z B = -\mu \mathrm{B}$

(1) What is the spin state of the proton - Write it as a superposition of states at the standard base.

(2) Express the magnetic moment of the $\mu_p$ proton as a linear combination of $\mu_u$ and $\mu_d$.

**The solution:**

(1) From the lecture notes p.83 [15.5], we know that:

$$(2l+1) \otimes (2s+1) = (2 \mid s+l \mid +1) \oplus ... \oplus (2 \mid l-s \mid +1)$$

in our case we get:

$$(2 \otimes 2) \otimes 2 = (3 \oplus 1) \otimes 2 = 4 \oplus 2 \oplus 2$$

the upper stat is:

$$\mid \frac{3}{2}, \frac{3}{2}\rangle = \mid\uparrow\uparrow\uparrow\rangle$$

after using j-:

$$\mid \frac{3}{2}, \frac{1}{2}\rangle = \frac{1}{\sqrt{3}}(\mid\uparrow\uparrow\downarrow\rangle + \mid\uparrow\downarrow\uparrow\rangle + \mid\downarrow\uparrow\uparrow\rangle)$$

we are looking for the orthogonal states for this state and we get:

$$\mid \frac{1}{2}, \frac{1}{2}\rangle_1 = \frac{1}{\sqrt{6}}(-2\mid\uparrow\uparrow\downarrow\rangle + \mid\uparrow\downarrow\uparrow\rangle + \mid\downarrow\uparrow\uparrow\rangle)$$

$$\mid \frac{1}{2}, \frac{1}{2}\rangle_2 = \frac{1}{\sqrt{2}}(\mid\uparrow\downarrow\uparrow\rangle - \mid\downarrow\uparrow\uparrow\rangle)$$

we can see that the proton state as even symmetry to replace is:

$$\mid \frac{1}{2}, \frac{1}{2}\rangle = \frac{1}{\sqrt{6}}(-2\mid\uparrow\uparrow\downarrow\rangle + \mid\uparrow\downarrow\uparrow\rangle + \mid\downarrow\uparrow\uparrow\rangle)$$

(2) the energy of the proton is:

$$\langle E_p \rangle = (P(|\uparrow\uparrow\downarrow\rangle)E\ |\uparrow\uparrow\downarrow\rangle + P(|\uparrow\downarrow\uparrow\rangle)E\ |\uparrow\downarrow\uparrow\rangle + P(|\downarrow\uparrow\uparrow\rangle)E\ |\downarrow\uparrow\uparrow\rangle)$$

the probability of the states are:

$$P(|\uparrow\uparrow\downarrow\rangle) = \frac{4}{6}$$

$$P(|\uparrow\downarrow\uparrow\rangle) = P(|\downarrow\uparrow\uparrow\rangle) = \frac{1}{6}$$

we know that:

$$E = -\mu B$$

so:

$$E_(u, \uparrow) = -\mu_u B$$

$$E_(u, \downarrow) = \mu_u B$$

$$E_(d, \uparrow) = -\mu_d B$$

$$E_(d, \downarrow) = \mu_d B$$

when E of of the different states are:

$$E\ |\uparrow\uparrow\downarrow\rangle = E_(u, \uparrow) + E_(u, \uparrow) + E_(d, \downarrow) = (-2\mu_u + \mu_d)B$$

$$E\ |\uparrow\downarrow\uparrow\rangle = E\ |\downarrow\uparrow\uparrow\rangle = E_(u, \uparrow) + E_(u, \downarrow) + E_(d, \downarrow) = -\mu_d B$$

we know that:

$$E_p = -\mu_p B$$

so using all we found we can show that:

$$-\mu_p B = \frac{4}{6}(\mu_d - 2\mu_u)B + \frac{1}{6}(-\mu_d B) + \frac{1}{6}(-\mu_d B)$$

divide with -B we get:

$$\mu_p = \frac{4}{6}(-\mu_d + 2\mu_u) + \frac{1}{6}(\mu_d) + \frac{1}{6}(\mu_d)$$

$$\mu_p = \frac{8}{6}\mu_u - \frac{2}{6}\mu_d = \frac{4}{3}\mu_u - \frac{1}{3}\mu_d$$