E732: Potential well with delta function

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The problem:

A particle is placed in a one dimensional potential well with boundary conditions equal to zero at $x = \pm\frac{L}{2}$.
The potential is $V(x,t) = \epsilon e^{-\frac{1}{2}(\frac{t}{\tau})^2}\delta(x)$
At a time $t \ll -\tau$ the particle was at the lowest energy state.

1. Using the perturbation theory (up to the first order) calculate the probability of finding the particle at energy state n at a time $\tau \ll t$
2. Define the condition where the calculation is valid in the last problem (careful).
3. Define the condition where the effect of the pulse will be adiabatic.

The solution:

1. First, we separate the potential to $W = \delta(x)$ and $f(t) = \epsilon e^{-\frac{1}{2}(\frac{t}{\tau})^2}$.
We will use the formula:

$$P_t(n|m) \approx |W_{nm}|^2 |FT(f(t))|^2$$

We are given that the particle starts at energy level $n = 1$ so we need to find $W_{n,1}$

$$W_{n,1} = \langle n|W|1\rangle = \int_{\frac{-L}{2}}^{\frac{L}{2}} \Psi^{n*}(x) W \Psi^1(x) dx$$

We remember the wave function of a particle inside a potential well

$$\Psi^n(x) = \sqrt{\frac{2}{L}}\cos(\frac{n\pi x}{L}) \qquad \text{for n=1,3,5,...}$$

$$\Psi^n(x) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L}) \qquad \text{for n=2,4,6,...}$$

Therefore the ground state is

$$\Psi^1(x) = \sqrt{\frac{2}{L}}\cos(\frac{\pi x}{L})$$

And for $W_{n,1}$

$$\langle \Psi^n|W|\Psi^1\rangle = \int_{\frac{-L}{2}}^{\frac{L}{2}} \Psi^{n*}(x) W \Psi^1(x) dx$$

Due to the delta function all the Sin states equal zero.
And all the Cos state will equal $\frac{2}{L}$.

$$W_{n,1} = \frac{2}{L} \qquad \text{for n=1,3,5,...}$$

$$W_{n,1} = 0 \qquad \text{for n=2,4,6,...}$$

And for $f(t)$

$$FT[\epsilon e^{-\frac{1}{2}(\frac{t}{\tau})^2}] = \sqrt{2\pi}\tau\epsilon e^{-2\pi^2\tau^2\omega^2}$$

We define that

$$\omega = E_n - E_1$$

$$E_n = \pi^2 n^2/2mL^2$$

$$E_1 = \pi^2 \ /2mL^2$$

$$P_t(n,1) = (\frac{2\epsilon}{L})^2 2\pi\tau^2 Exp\left(-\frac{4\pi^4\tau^2(n^2-1)}{2mL^2}\right) \qquad \text{for n=1,3,5,...}$$

$$P_t(n,1) = 0 \qquad \text{for n=2,4,6,...}$$

2. The condition is $|W| \ll \Delta$,and from that we derive $\epsilon \ll \frac{1}{ML}$ (it might be that the actual condition, which is $p \ll 1$ where p is the total transition probability, is weaker).

3. The condition is:

$$\left|\frac{df}{dt}\right| \ll \frac{\Delta^2}{|W|}$$

and we define that:

$$\left|\frac{df}{dt}\right| \approx \frac{\epsilon}{\tau}$$

Therefore:

$$\frac{1}{L}\frac{\epsilon}{\tau} \ll \left(\frac{1}{mL^2}\right)^2$$