E615: breaking symetry pertubation,degenerate removal

Submitted by: udi strobach

The problem:

A particle with spin 1 described by the hamiltonian

$$\mathcal{H} = 2\hat{S}_x^2 - 4\hat{S}_y^2 + 6\hat{S}_z^2 + \hat{1} + h\hat{S}_z$$

In the next three paragraph assume $h = 0$.
We define projectors operators as

$$P_i = \hat{1} - \hat{S}_i^2$$

(1) On which state the projection for exemple P_z is carried out ?
(2) What is the hamiltonian eigenstates (in the standard basis)?
(3) What is the eigenvalues ?

Magnetic field h was added on the z direction.

(4) Write the interaction part in the standart basis.
(5) Write the hamiltonian (including the interection part) in the eigenstates basis.
(6) What are the hamiltonian energy eigenvalues?
(7) Plot the energy as a function of the field h.

The particle was prepared in a linear polarization in z direction
The magnetic field turned on in adiabatic way so $h >> 1$.
Becouse of trouble in the system the magnetic field was diverted from the z direction.

(8)what is the spin state at the end of the process

The solution:

(1)

The projection carried out in the linear polarization basis

$$\hat{P}_z^2 |\bar{e}_z\rangle = |\bar{e}_z\rangle$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

(2)

The eigenstates are the linear polarization basis

$$|\bar{e}_x\rangle = \frac{1}{\sqrt{2}} \left(-|\Uparrow\rangle + |\Downarrow\rangle\right) \mapsto \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$|\bar{e}_y\rangle = \frac{1}{\sqrt{2}} \left(|\Uparrow\rangle + |\Downarrow\rangle\right) \mapsto \frac{1}{\sqrt{2}} \begin{pmatrix} i \\ 0 \\ i \end{pmatrix}$$

$$|\bar{e}_z\rangle = |\Updownarrow\rangle \mapsto \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

(3)

The energy eigenvalues

$$\mathcal{H}|\bar{e}_x\rangle = (2 \cdot 0 - 4 \cdot 1 + 6 \cdot 1 + 1)|\bar{e}_x\rangle = 3|\bar{e}_x\rangle$$

$$\mathcal{H}|\bar{e}_y\rangle = (2 \cdot 1 - 4 \cdot 0 + 6 \cdot 1 + 1)|\bar{e}_y\rangle = 9|\bar{e}_y\rangle$$

$$\mathcal{H}|\bar{e}_z\rangle = (2 \cdot 1 - 4 \cdot 1 + 6 \cdot 0 + 1)|\bar{e}_z\rangle = -1|\bar{e}_z\rangle$$

(4)

The interaction part in the standard basis

$$V^{'} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

(5)

by taking the eigenvalues from pharagraph 3 and adding the interaction we get

$$\mathcal{H} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & -1 \end{pmatrix} + h \cdot V$$

We need to transform the interaction matrix to the base of linear polarization
We do this with the transform matrix

$$T = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \end{pmatrix}$$

$$T^{\dagger} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & 0 & -\frac{i}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix}$$

$$V = T^{\dagger} V^{'} T = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathcal{H} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & -1 \end{pmatrix} + h \cdot \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

(6)

We now need to diagonalize the matrix in order to find the eigenvalues

$$\mathcal{H} = \begin{pmatrix} 3 & -ih & 0 \\ ih & 9 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$E_{1,2} = 6 \pm \sqrt{3^2 + h^2}$$

$$E_3 = -1$$

We can get to olmost the same solution by the perturbation theory

$$E_n = E_n^{(0)} + h \cdot E_n^{(1)} + h^2 \cdot E_n^{(2)} + ...$$

$$E^{(0)} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$E^{(1)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$E_1^{(2)} = \frac{|\langle \bar{e}_y | V | \bar{e}_x \rangle|^2}{E_1^{(0)} - E_2^{(0)}} = \frac{-1}{6}$$

$$E_1^{(2)} = \frac{|\langle \bar{e}_x | V | \bar{e}_y \rangle|^2}{E_2^{(0)} - E_1^{(0)}} = \frac{1}{6}$$

$$E_3^{(2)} = 0$$

$$E = \begin{pmatrix} 3 - \frac{h^2}{6} & 0 & 0 \\ 0 & 9 + \frac{h^2}{6} & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

After taking taylor expantion from the exact solution we can see that this solution is the same in the limit $h \rightarrow 0$

(7)

The plot at the left side is for $h << \Delta$ at the scound side the plot is for $h >> 1$ for the next pharagraph.

(8)

We can see from the plot of pharagraph 7 that there is some point where we will have degenerecy becouse for $h >> 1$ the energy will be linear in h so that we'll stay with 3 states $|\Uparrow\rangle$ and $|\Downarrow\rangle$ and $|\Updownarrow\rangle$ from the plot and from the adiabatic condition (the diversion of the magnetic field from the z axis) we can understand that due to degenerecy spliting our particle will be in the $|\Downarrow\rangle$ state at the end.