E604: First Order Perturbation Theory for an one dimension box with a displacing wall

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The problem:

A particle is placed in a one dimensional box of length L, such that $0 < x < L$. The purpose of this problem is to find the first order correction for the particle's energies, when we have a dL displacement of the wall, using the solution of the previous exercise and afterwards compare it to the exact solution.

The solution:

(1)From the previous exercise, the perturbed Hamiltonian can be writen as

$$\mathcal{H} = H_0 + V dL$$

where $V_{nm} = -\frac{\pi^2\hbar^2}{ML^3}nm$.

For the first order perturbation theory, we have

$$E_n = E^{(0)} + E^{(1)}dL + O(dL^2)$$

where $E^{(0)} = \frac{\pi^2\hbar^2}{2ML^2}n^2$ is the eigenvalue of the unperturbed Hamiltonian of a particle in a one dimensional box.

The first order correction of the energy is given by

$$E^{(1)} = -\frac{\pi^2\hbar^2}{ML^3}n^2$$

Therefore

$$E_n = \frac{\pi^2\hbar^2 n^2}{2ML^2}\left(1 - \frac{2dL}{L}\right) + O(dL^2)$$

(2)The exact solution for the energy levels a particle in an one dimensional box is

$$E_n = \frac{\pi^2\hbar^2 n^2}{2ML^2}$$

As we have $L \to L + dL$, we use Taylor expansion around $dL = 0$

$$(L + dL)^{-2} = \frac{1}{L^2} - \frac{2dL}{L^3} + O(dL^2)$$

We receive that

$$E_n = \frac{\pi^2\hbar^2 n^2}{2ML^2}\left(1 - \frac{2dL}{L}\right) + O(dL^2)$$

As one can see, we receive the same solution in both cases.