

E5668: Spin precession due to spin-orbit interaction

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The problem:

An electron, with mass M , charge e and gyromagnetic constant g , launched with energy E in a one dimensional conductor, in the direction of the X axis. The conductor passes through capacitor plates of length L . The capacitor creates an electric field \mathcal{E} in the Y direction. Likewise, there's a magnetic field B in that area in the Z direction. When the electron enters the region of interaction, its spin is being polarized in the direction of its motion.

Throughout the problem we may ignore the possibility of the particle returning from the interaction region. And also $\hbar = 1$.

In the first two paragraphs $v \approx (2E/M)^{1/2}$

- (1) What will be the direction ϕ of the spin, when the electron escapes the interaction region?
- (2) What does the magnetic field has to be, for the spin not to rotate?

In the next paragraphs $x = 0$ will be set as the entering point. Also giving an exact solution on the basis of the launching energy.

- (3) Write the particle's state in the region where the interaction takes place, in the standart basis $|x, m\rangle$, where $m = \uparrow, \downarrow$.
- (4) Give the exact answer to paragraph (1).

The solution:

- (1) The Hamiltonian of the motion is as follows:

$$\mathcal{H} = \frac{p^2}{2M} - g\frac{e}{2M}\mathbf{B} \cdot \mathbf{S} - \frac{e}{2M^2}(\mathcal{E} \times \mathbf{p}) \cdot \mathbf{S}$$
$$\mathcal{H} = \frac{p^2}{2M} - g\frac{eB_z}{2M}S_z + \frac{ev\mathcal{E}_y}{2M}S_z = \frac{p^2}{2M} - \frac{eS_z}{2M}(B_zg - v\mathcal{E}_y)$$

The time that it takes for the electron, to get out of the capacitor is:

$$t = \frac{L}{v} \approx L \left(\frac{M}{2E} \right)^{\frac{1}{2}}$$

Also we know that since S_z is the rotations generator around the z direction, it has to satisfy:

$$R(\bar{e}_z\phi) = e^{-i\phi S_z} = e^{-i\mathcal{H}t} = U(t)$$

The angle with which it escapes the interaction region is the angle at time t

$$\phi = -\frac{e}{2M}(B_zg - v\mathcal{E}_y) \cdot t = -\frac{eL}{2Mv}(B_zg - v\mathcal{E}_y)$$

- (2) If we don't want the spin to change, then we ask for $\phi = 0$ and get:

$$\phi = -\frac{eL}{2Mv}(B_zg - v\mathcal{E}_y) = 0$$

$$B_z = \frac{v\mathcal{E}_y}{g}$$

(3) With the addition of angular momentum we get:

$$|\psi\rangle = \sum \psi_{x,m} |x, m\rangle$$

Where $|x, m\rangle = |x\rangle \otimes |m\rangle$ and $m = \uparrow, \downarrow$

So we get:

$$|\psi\rangle = e^{ik_{\uparrow}x} \otimes |\uparrow\rangle + e^{ik_{\downarrow}x} \otimes |\downarrow\rangle$$

$$E = \frac{k^2}{2M} - \frac{eS_z}{2M}(B_z g - v\mathcal{E}_y); S_z = \pm \frac{1}{2}$$

$$\Rightarrow k_{\uparrow, \downarrow} = \sqrt{2ME \pm \frac{1}{2}e(B_z g - v\mathcal{E}_y)}$$

(4) By using the result of the last paragraph:

$$k_{\uparrow}^2 - k_{\downarrow}^2 = e(B_z g - v\mathcal{E}_y)$$

$$\Rightarrow \phi = -\frac{L}{2Mv}(k_{\uparrow}^2 - k_{\downarrow}^2)$$