**E601: Spin – Orbit Interaction**
**Submitted by: Chen Giladi, Yoav Pollack, Alon Oringn**

**The problem:**
Electron with mass m and a charge q is placed on one dimensional ring with radius R. There is a another point charge Q which is placed in the center of the ring, so there is a spin – orbit interaction
(1) Write the Hamiltonian $H(x, p, S)$ of the electron.
(2) What are the energies of the electron?
(3) What is the velocity of the electron in each energy level?
(4) Which magnetic Flux $\phi$ trough the ring will cancel out the effect of charge Q on the electron with spin up.
Use $\hbar = c = 1$and write the represent the final answers with the provided data.
**The solution:**
(1)We know that the spin – orbit term is $H_{spin-orbit} = -\frac{e}{2m^2}(\vec{\varepsilon} \times \vec{p}) \cdot \vec{S}$ where $\vec{\varepsilon} = -\frac{V'(r)}{r}\overrightarrow{r} \Rightarrow \vec{\varepsilon} = \frac{Q}{R^2} \cdot \widehat{r}$and using $\vec{p} = p_\varphi \widehat{\varphi}$, So our Hamiltonian is

$$H = \frac{\vec{p}^2}{2m} - \frac{e}{2m^2}(\vec{\varepsilon} \times \vec{p}) \cdot \vec{S} = \frac{1}{2m}(\frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2}) - \frac{e}{2m^2}(\frac{Q}{R^2} \cdot \widehat{r} \times \vec{p}) \cdot \vec{S}$$

Putting our ring on the XY plane and we know that $r = R$ we get the following Hamiltonian

$$H = \frac{p_\varphi^2}{2mR^2} - \frac{eQ}{2m^2R^2}S_z$$

(2) Using $[p_\varphi^2, S_z] = [H, S_z] = [p_\varphi^2, H] = 0$ we can write the energies of the electron as

$$E_{n,l} = \frac{1}{2mR^2}n^2 - \frac{eQ}{2m^2R^2}l$$

(3) The velocity can be found by using the Heisenberg equation of motion

$$\frac{d\varphi}{dt} = i[H, \varphi] = \frac{p_\varphi}{mR^2} + \frac{eQ}{2m^2R^2}S_z$$

So the velocity for the Eigen stats is $V_\varphi = \frac{n}{mR^2} + \frac{eQ}{2m^2R^2}l$
(4) By using the next Hamiltonian $H = \frac{1}{2m}(p_\varphi - \frac{e\phi}{2\pi R})^2 - \frac{eQ}{2m^2R^2}p_\varphi S_Z$
we get the condition on $\phi$ for which there is no effect of the interaction spin orbit

$$\phi = -\frac{Q}{2m}\frac{\pi}{R}$$