Quantum Mechanics II, Ex 4730

Submitted by: Roee Steiner 034744821

Given a spherical shell with radius R and a particle with mass M and charge e.
Notice that the standard variables which show the particle are (θ, φ, L_x, L_y, L_z)
In this question we have to assume that the particle can be excited from ground state to first energy level but not beyond so the state space is four dimensional |0>,|↑>,|↕>,|↓>.

1. What is the energy of every base.

We prepare the particle as ”concentrated” as possible in the ”north pole” of the shell.

2. Write the wave function $\psi(\theta,\varphi)$ of the particle

What you have written is not stationary. The particle oscillates between the North and the South pole.

3. What is the period of the oscillations?

We torn on a constant electric field **ε** in the Z direction

4. Write the electric perturbation in formal way with the standard variables.

5. What is the period time T of the fluctuation?

We torn on a constant magnetic field **B** in the X direction

6. Write the magnetic perturbation in formal way with the standard variables.

7. Find a value for B so in the period of T/2 where the particle will be in the first place.

$$Y_{00} = \frac{1}{\sqrt{4\pi}} \, , Y_{10} = \sqrt{\frac{3}{4\pi}} \cos\theta \, , Y_{1,\pm 1} \;\; \alpha \;\; \sin(\theta)$$

Paragraph 2 and 7 demands ”thinking”, the answer is simple and do not demand ”algebra”.

Solution:

1. The energy level for particle on sphere is $E_l = \frac{l(l+1)}{2MR^2}$.
So for the first state |0>, $l = 0$ so $E_0 = 0$.
For the three others state $l = 1$ so $E_1 = \frac{1}{MR^2}$.

2 We want to find a solution that gives us the best probability to find the particle in the North pole of the sphere. We need to use the spherical harmonics function.
Its means that the function will have a trigonometric variable, so the function will look like $\psi = \frac{1}{2}(1 + \cos\theta)$ And after we use the spherical harmonics and normalization we will get:

$$\psi = \frac{1}{2}\left(\sqrt{3}Y_{00} + Y_{10}\right)$$

3. We will write a time depended wave function:
$\psi(t) = \frac{1}{2}\left(\sqrt{3}Y_{00} + Y_{10}e^{-iE_1 t}\right)$
From here we can see easily that the frequency is E_1,so the period is

$T = \frac{2\pi}{E_1} = 2\pi MR^2$

4. $V = e\varepsilon\hat{z} = e\varepsilon R\cos\theta = \sqrt{\frac{4\pi}{3}}e\varepsilon RY_{10}$

5. We will find first the matrix elements of V that given by:
$\langle l', m'|V|l, m\rangle = \left\langle l', m' \middle| \sqrt{\frac{4\pi}{3}}e\varepsilon RY_{10} \middle| l, m\right\rangle = \frac{e\varepsilon R}{\sqrt{3}}\delta_{l1,m0}\delta_{l'0,m'0}$
So we can see that the system work like a two site system so the new Hamiltonian is:

$$H = \begin{pmatrix} 0 & \frac{e\varepsilon R}{\sqrt{3}} \\ \frac{e\varepsilon R}{\sqrt{3}} & \frac{1}{mR^2} \end{pmatrix} = \frac{1}{2mR^2}\hat{I} - \frac{1}{2mR^2}\sigma_z + \frac{e\varepsilon R}{\sqrt{3}}\sigma_x = \frac{1}{2mR^2}\hat{I} + \Omega S$$

Where $S = \frac{1}{2}\sigma_i$ and $\Omega = \left(\frac{2e\varepsilon R}{\sqrt{3}}, 0, \frac{-1}{mR^2}\right)$
According to "precession picture" (see lecture notes chapter 36), the frequency is:

$$\omega = |\Omega| = \sqrt{\frac{4}{3}(e\varepsilon R)^2 + \left(\frac{1}{mR^2}\right)^2}$$

So the period time is:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{4}{3}(e\varepsilon R)^2 + \left(\frac{1}{mR^2}\right)^2}}$$

6. From Zeeman we know that the impact of magnetic field is:

$$V_B = -\frac{eB}{2m}L_x$$

7. We can see that V_B react in the same way on the system as the electric perturbation
So in analogy we can see that the wave function will bee

$\Psi(t) = \frac{1}{2}\left(\sqrt{3}Y_{00} + e^{-i\Omega t}e^{iV_B t}Y_{10}\right)$
Because the time is two time lease its means that

$$|\Omega| + V_B = 2|\Omega|$$

So $$B = \frac{2m}{e}\sqrt{\frac{4}{3}(e\varepsilon R)^2 + \left(\frac{1}{4mR^2}\right)^2}$$