Quantum Mechanics II, Ex 471

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Given a spherical shell with radius R and a particle with mass M and charge q.in this question we have to assume that the particle can be excited from ground state to first energy level but not beyond so the state space is four dimensional.

we prepare the particle as "concentrated" as possible in the "north pole" of the shell.

1. write the wave function $\psi(\theta, \phi)$ of the particle

this question demands "thinking", the answer is simple and do not demand "algebra". the state you wrote is not stationary. the particle oscillate between the north and the south pole.

2. what is the period of the oscillations?

we define the of the system in the following:$P(t) =< \psi(t)|\hat{z}|\psi(t) >$

3. find the polarization P(t) as function of time

given

$$Y_{00} = \frac{1}{\sqrt{4\pi}}, \quad Y_{10} = \sqrt{\frac{3}{4\pi}} cos\theta, \quad Y_{1,\pm1} \propto sin(\theta)$$

answer:

1. We want to find a solution that gives us the best probability to find the particle in the north pole of the shell. we are only able to use the linear combination of $Y_{00}, Y_{1-1}, Y_{10}, Y_{11}$. we can see that the function $\psi = 1 + cos\theta$ gives us that probability. after normalization and using Y_{lm} we get:

$$\psi = \frac{\sqrt{3}}{2} Y_{00} + \frac{1}{2} Y_{10}$$

2. the energy level of each state will be $E_l = \frac{l(l+1)}{2mR^2}$ so the energy levels of the given problem are:

$$E_0 = 0$$

$$E_1 = \frac{1}{mR^2}$$

we will write the time depended wave function:

$$\psi = \frac{\sqrt{3}}{2} Y_{00} + \frac{1}{2} e^{-iE_1 t} Y_{10}$$

from here we can see easily that the frequency is $\omega = E_1 = \frac{1}{mR^2}$ and the period is

$$T = 2\pi m R^2$$

3. and now to make a sandwich...

$$P(t) =< \psi(t)|\hat{z}|\psi(t) >$$

$$z = R\sqrt{\frac{4\pi}{3}}Y_{10}$$

$$P(t) = R\sqrt{\frac{4\pi}{3}}\int_{-1}^{1} Y_{10}[\frac{\sqrt{3}}{2}Y_{00} + \frac{1}{2}e^{iE_1 t}Y_{10}][\frac{\sqrt{3}}{2}Y_{00} + \frac{1}{2}e^{-iE_1 t}Y_{10}]$$

after doing all the math the solution is:

$$P(t) = \frac{1}{2}R\cos(E_1 t)$$

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Remarks:


(3) The integration should be written in a more simple way,
    involving only two Y_{lm}'s.
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