E453: Operating Rotation Matrix on Spins

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The problem:

1) Write the standard representation of spin 1 with linear polarization in the $\hat{z}$ direction.
2) Calculate the standard representation of spin 1 with circular polarization in the $\hat{x}$ direction.
One measeures whether the spin is linear polarized in the $\hat{y}$ direction.
3) What is the probability of getting a positive result for the 1st preparation?
4) What is the probability of getting a positive result for the 2nd preparation?

The solution:

(1) The representation of spin one with linear polarization in the $\hat{z}$ direction is:

$$|\bar{e_z}\rangle = |\Updownarrow\rangle \rightarrow \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

(2) If we rotate the circular polarization state (of the standard basis) by 90 degrees round the y axis we get:

$$|\vec{e_x}\rangle = R(\frac{\pi}{2}\bar{e_y})\,|\Uparrow\rangle \rightarrow \begin{pmatrix} \frac{1}{2} & \frac{-1}{\sqrt{2}} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{2} \end{pmatrix}$$

(3) The probability of getting a positive result from the 1st preparation is:

$$|\langle\bar{e_y}|\bar{e_z}\rangle|^2 = 0$$

because of the 90 degree orthogonality of the linear basis.

(4) due to the freedem of choice for the axis system we can say that $|\langle\bar{e_y}|\vec{e_x}\rangle|^2 = |\langle\bar{e_z}|\vec{e_x}\rangle|^2$, and the probability of getting a positive result for the second preparation is:

$$|\langle\bar{e_z}|\vec{e_x}\rangle|^2 = \left|\begin{pmatrix}010\end{pmatrix} \cdot \begin{pmatrix} \frac{1}{2} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{2} \end{pmatrix}\right|^2 = \frac{1}{2}$$

We can confirm the results by direct calculations.