Ex3710: Electron in Hall Geometry - Double Well

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The problem:

A two dimensional rectangle is given. In the x direction ($0 < x < L$) there are periodic boundary conditions. In the y direction there is a slowly varying electric potential. $V(y) \approx 0$. This potential has local minimum along the lines $y = \pm W$. The rectangle is placed in a magnetic field, $\vec{B} = (0, 0, B(y))$. Two spinless electrons are placed in the system, with mass m and charge e. These electrons occupy the lowest landau states and their state is represented by the wave functions $\Psi^{\pm}(x, y)$.

In the following sections assume a homogenous magnetic field $B(y) = B_0$ in the z direction:

(1) Write the Hamiltonian of this model for an electron.
(2) Write down the effective potential $V^{\pm}(y)$ received after separation of variables.
(3) Write down the wave functions, $\Psi^{\pm}$(x,y), for the two electrons.

Now assume the magnetic field has been flipped at the lower part of the rectangle, $B(y < 0) = -B_0$.

(4) Write down and draw schematically the effective potential, $V^{eff}(y)$, for the new magnetic field.
(5) Write an approximate expression the new wave functions.
(6) Write an expression for the energy splitting of the new states, $\Delta = Ce^{-A}$.

Guidance:

- Use the notation: $\omega_B = \frac{eB_0}{mc}$.
- Use the notation: $\varphi = (\frac{m\omega_B}{\pi})^{\frac{1}{4}} e^{-\frac{1}{2}m\omega_B r^2}$
- In part 6, you can neglect the kinetic energy relatively to the barrier height.

The solution:

(1)
Since there are periodic boundary conditions in the x directions, we will choose Landau gauge which keeps the symmetry in the x direction.

$$\vec{A} = (-B_0 \cdot y, 0, 0)$$

Thus the Hamiltonian will be:

$$\mathcal{H} = \frac{1}{2m}(\hat{p_x} - \frac{e}{c} \cdot A_x(\hat{y}))^2 + \frac{1}{2m}\hat{p_y}^2 + V(y) = \frac{1}{2m}(\hat{p_x} + \frac{e}{c} \cdot B_0 \cdot y)^2 + \frac{1}{2m}\hat{p_y}^2 + V(y)$$

Since $[p_x, \mathcal{H}] = 0$ the Hamiltonian is block diagonal. We substitute: $\hat{p_x} = \frac{2\pi}{L}l$:

$$\mathcal{H}^l = \frac{1}{2m}(\frac{2\pi}{L}l + \frac{e}{c} \cdot B_0 \cdot y)^2 + \frac{1}{2m}\hat{p_y}^2 + V(y)$$

After rearranging:

$$\mathcal{H}^l = \frac{1}{2m}\hat{p_y}^2 + \frac{1}{2}m\omega_B^2(y - y_l)^2 + V(\hat{y})$$

Where: $y_l = \frac{2\pi}{B_0 L} l$ and $\omega_B = \frac{eB_0}{mc}$

(2)
It's given that $V(y)$ has local minimum values at $\pm W$, therefore the electrons will occupy the x momentum states so that $y_l = \pm W$:

$$l^{\pm} = \pm \frac{W B_0 L}{2\pi}$$

The effective potential is:

$$V^{\pm}(y) = \frac{1}{2} m\omega_B^2 (y \underbrace{-y_{l^{\pm}}}_{\mp W})^2 + V(\hat{y_{l^{\pm}}})$$

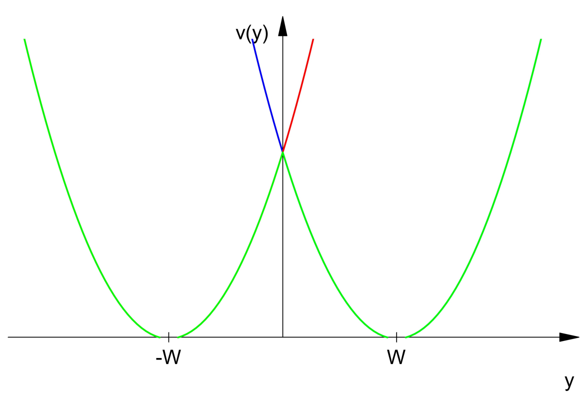
(3)
It's given that $V(y)$ is slowly varying so the Hamiltonian is one of an harmonic oscillator and a free particle:

$$\Psi^{\pm}(x, y) = \frac{1}{\sqrt{L}} e^{-i\frac{2\pi}{L} l^{\pm} x} \varphi_\nu (y - y_l^{\pm}) = \frac{1}{\sqrt{L}} e^{\mp i B_0 W x} \varphi_\nu (y - y_l^{\pm})$$

where φ_ν is an eigenstate of the harmonic oscillator. The two electrons will occupy the first Landau level, $\nu = 0$:

$$\Psi^{\pm}(x, y) = \frac{1}{\sqrt{L}} (\frac{m\omega_B}{\pi})^{\frac{1}{4}} e^{\mp i B_0 W x} e^{-\frac{1}{2} m\omega_B (y \mp W)^2}$$

(4)
We notice that the magnetic field changes it's direction but not it's magnitude, hence the wave function of the electrons will still be located at $\pm W$. However, since the effective potential had been changed, the eigenstates are different. When the electron is "going" from the positive side to negative side and vice versa it doesn't see an infinite well of the harmonic oscillator but another one.



Each one of these wells is one of an harmonic oscillator:

$$V^{eff}(y) = \frac{1}{2} m\omega_B^2 (y - W)^2$$

(5)

The problem is reduced to a two site problem where each site is an harmonic oscillator ($\nu = 0$) at $\pm W$. Also, the system is symmetric to reflections, thus the eigenstates are the symmetric and antisymmetric states:

$$\Psi^{\pm} \approx e^{-iB_0 W x} \frac{1}{\sqrt{2}} (\varphi^{+} \pm \varphi^{-})$$

where $\varphi^{\pm}$ are the states $\Psi^{\pm}$ mentioned in section 3.

(6)
In order for the problem to be reduced to a two sites problem, there must be a hoping amplitude between the two sites. The probability for tunneling to a near site is calculated by the WKB approximation:

$$g = e^{-2S}$$

Where:

$$S = 2 \int \sqrt{m^2 \omega_B^2 y^2} dy$$

The attempt rate is simply determined semi classically by the frequency of the harmonic oscillator. Hence according to Gammow formula one get:

$$\Delta = 2\omega_B \sqrt{g} = C e^{-A}$$

Where:

$$C = 2\omega_B$$

and:

$$A = m\omega_B W^2$$