Ex3640: Electrons in a 2 dimensional finite potential well with a magentic field

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The problem:

Given a 2 dimensional potential well of size $L \times L$ and of depth V_0 :

$$V(x,y) = \begin{cases} 0 & \text{if } x < -\frac{L}{2}, y < -\frac{L}{2}, \\ -V_0 & \text{if } -\frac{L}{2} < x < +\frac{L}{2}, -\frac{L}{2} < y < +\frac{L}{2}, \\ 0 & \text{if } x > +\frac{L}{2}, y > +\frac{L}{2}. \end{cases}$$

Hence,all of the electrons with energy $E > 0$ are escaping the well.Assume that the electrons which remain in the well have mass m ,charge e ,and have no spin.

(1) Write the condition $L_{min} < L$ of which at least one electron will could be in the well
For future calculations,assume $L_{min} << L$
(2) Calculate the number of electrons N_0 that can exist in the well
A homogeneous magnetic field B is created perpendicularly to the plane of the well.
In the next sections,known results regarding Landau levels can we used (without any proof)
(3) Find the maximal magnetic field $B = B_2$ so that two Landau levels will be filled in the well.
Denoting the number of particles from section (3) to be N_2:
(4) What is the ratio of $\frac{N_2}{N_0}$?.
(5) Generalize the result of section (4) for the case of n full Landau levels.

Guidance:
The boundary conditions at the edges of the well have negligible effect on the spectrum,
so that previous results that are derived from Landau‘s solution can be used.

The solution:

(1) The eigenenergies of a 2D finite well of depth V_0 are:

$$E_{n_x,n_y} = \frac{\pi^2}{2mL^2}(n_x^2 + n_y^2) - V_0$$

$$where : n_x, n_y > 0$$

In order to stay located in the well,the particle needs to have energy $E < 0$.The smallest energy level that can occupy one electron is $E_{1,1}$.substituting these conditions into the expression of the eigenenergies will give:

$$L > \frac{\pi}{\sqrt{mV_0}}$$

Therefore:

$$L_{min} = \frac{\pi}{\sqrt{mV_0}}$$

(2) If $L >> L_{min}$, then $E < 0$.Applying this condition on E_{n_x,n_y} will give:

$$n_x^2 + n_y^2 < \frac{2mL^2V_0}{\pi^2}$$

This equation describes a circle with center $(0, 0)$ and radius $R^2 = \frac{2mL^2V_0}{\pi^2}$.
Adding the constraint of $n_x, n_y > 0$,N_0 will be the quarter of this circle:

$$N_0 = \frac{mL^2V_0}{2\pi}$$

(3) Making use of known results regarding Landau levels,the energy of the n^{th} Landau level is:

$$E_\nu = (\frac{1}{2} + \nu)\omega_B - V_0$$

$$where : \omega_B = \frac{eB}{mc}, \nu = n - 1, \nu = 0, 1, 2, ...$$

For 2 Landau levels, $\nu = 1$ and then , plugging the constraint that $E < 0$ so that the electrons will stay located in the well,the maximal magnetic field B_2 is :

$$B_2 = \frac{2mcV_0}{3e}$$

(4) The number of electrons occupying each Landau level is defined as $g_{Landau} = \frac{L^2eB}{2\pi c}$.Due to the fact that the potential floor is "flat" the occupation in each Landau level is the same :
$N_n = n \cdot g_{Landau}$. Therefore:

$$N_2 = (2) \cdot g_{Landau} = \frac{4}{3}\frac{mL^2V_0}{2\pi} = \frac{4}{3}N_0$$

Now it is immediate that:

$$\frac{N_2}{N_0} = \frac{4}{3}$$

(5) By a short generalization of the previous sections, the maximal magnetic field B_n so that n Landau levels will be filled in the well:

$$B_n = \frac{mcV_0}{e(n - \frac{1}{2})}$$

The number of electrons $g_{B_nLandau}$ that can occupy each Landau level under this condition is :

$$g_{B_nLandau} = \frac{L^2eB_n}{2\pi c} = \frac{1}{n - \frac{1}{2}}N_0$$

Yielding the ratio:

$$\frac{N_n}{N_0} = \frac{n}{\frac{1}{2} + \nu} = \frac{n}{n - \frac{1}{2}}$$