E307: Magnetization of a ring with electrons

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The problem:

A ring with radius R is placed in a uniform magnetic field B.
There are 5 electrons with mass m, charge e and spin $\frac{1}{2}$.
Note that two electrons can't be at the same state.

Let E be the ground state energy
and $M(B) = -dE/dB$ be the magnetization
$B, R, m, e, g{\approx}2.0023$ are given.

If the electrons are spinless the magnetization is a periodic function with period B_0
Later we will assume that $B = lB_0 + \Delta B$
where l is an integer and $0<\Delta B \ll B_0$.

(1) Write an expression for B_0.
(2) Find $M(B)$ for $B = 0$.
(3) Write the addition to $M(B)$ for small magnetic field.
(4) Find $M(B)$ for large l when $\Delta B = 0$.
(5) Write the addition to $M(B)$ for large l when $\Delta B \neq 0$.
(6) Define large l.

The solution:

(1) The Hamiltonian can be written as:

$$H = \frac{1}{2m}(p - \frac{eRB}{2})^2$$

The eigenstates of the H are the momentum states $|k_n>$ where: $|k_n >= \frac{2\pi}{2\pi R}n$

The eigenvalues are: $En = \frac{1}{2MR^2}(n - \frac{eR^2B}{2})^2$

$$M(B) = -\frac{dE}{dB} = -\frac{e}{2M}(n - \frac{eR^2B}{2})$$

Now we can see that $M(B)$ is a periodic function and the period is:

$$B_0 = \frac{2}{eR^2} \ .$$

(2) Because of the spin of the electrons, the Hemiltonian include the zeeman term:

$$H = \frac{1}{2m}(p - \frac{eRB}{2})^2 - geB \cdot \frac{S}{2m}$$

The Zeeman term split the energy levels so that the energies which has the same n is not degenerated anymore.

Let us assume that B is parallel to z axis.

When B $\rightarrow$ 0 every two electrons can have the same n, therefore the total energy is:

$E_{total} = \frac{5e^2R^2\Delta B^2}{8m} - (\frac{e}{2m} + \frac{eg}{4m})\Delta B + \frac{3}{2mR^2}$

and $M(B) = \frac{e}{2m}(1 - \frac{5\Delta B}{B_0}) + \frac{eg}{4m}$

(3) the addition to $M(B)$ for small magnetic field is the suseptability:

$\chi = \frac{dM(B)}{dB} = -\frac{5e}{2mB_0}$

(4) When l is large enough, all the electrons would have spin $|\uparrow>$ because of the Zeeman splitting,

Therefore:

$E_{total} = \frac{1}{2mR^2}(\frac{5e^2R^4\Delta B^2}{4} + 10) - \frac{5eg\Delta B}{4m}$

and $M(B) = \frac{e}{2m}(0 - \frac{5\Delta B}{B_0}) + \frac{5eg}{4m}$

(5) the addition to $M(B)$ for small magnetic field is the suseptability:

$\chi = \frac{dM(B)}{dB} = -\frac{5e}{2mB_0}$

(6) l is large when the diffrence between the fifth energy level with spin $|\uparrow>$ to the lowest energy level with spin $|\downarrow>$ is smaller than $\frac{egB}{2m}$:

$\frac{2}{mR^2} < \frac{egB}{2m}$, where: $B = lB_0 = \frac{2l}{eR^2}$

$\frac{2}{g} < l$