E306: Magnetic susceptibility in an Aharonov-Bohm system

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The problem:

A particle with mass m and charge e is placed in a one dimensional ring of length L, the flux through the ring is Φ. The partical is in the ground state.

(1) Find the magnetic susceptibility of the system, $\chi_n = -\frac{d^2 E_n}{d\Phi^2}$.
(2) Explain in what sense the effect is diamagnetic.
(3) How the result is effected if we add a scattering device to the system?

Fermions are defined as particles obeying Pauli principal, electrons are fermions with spin hulf.

(4) Do question (1) for 3 spinless fermions.
(5) for 6 fermions which are electrons?

The solution:

(1) We know that for Aharonov-Bohm system the energy is:

$$E_n = \frac{\left(\frac{\pi n}{L} - \frac{e\Phi}{L}\right)^2}{2m}$$

thus using n=0 and substituting the above equation into defenition for susceptibility we get:

$$\chi_0 = -\frac{d^2 E_0}{d\Phi^2} = -\frac{d^2}{d\Phi^2}\frac{1}{2m}(\frac{e\phi}{L})^2 = -\frac{e^2}{mL^2}$$

(2) From the above calculations we get $\chi_n = const < 0$ it indicates that the system is diamagnetic - negative responce to magnetic field. In our system the meaning is raise in energy (while at the same energy level) due to a slight raise in flux.

(3) The dependence of $I(\Phi)$ on the flux becomes weaker and weaker as the transmission of the ring becomes smaller. Consequently the diamagnetic effect becomes smaller. For very large scatterer (ring with zero transmission) the energy levels become flat, and $I(\Phi)$ and χ become both zero.

(4) The particles obey paulis principal, meaning there will be only one particle per energy level (we can regard then as spin 1 particles), the total energy will be:

$$E = E_1 + E_2 + E_3$$

and the susceptibility:

$$\chi = \chi_1 + \chi_2 + \chi_3 = 3\chi_0 = -\frac{3e^2}{mL^2}$$

(5) for electrons we'll get 2 particales per energy level:

$$E = 2E_1 + 2E_2 + 2E_3$$

and the susceptibility:

$$\chi = 6\chi_0 = -\frac{6e^2}{mL^2}.$$