E266: Scattering matrix - The Mach-Zhender Interferometer

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The problem:

Describe a 4×4 matrix of a semi-reflecting mirror while t is the transmit The Mach-Zehnder Interferometer is built from two semi-reflecting mirrors and two regular mirrors.
A regular mirror has a return amplitude of 1 so that the only effect to a beam that hits it, is a phase change of π.
A beam of particles is split at the first semi-reflecting mirror to two beams that go through two different optical courses so that one grows a difference of ϕ in its phase from the other.
They reunite in the second semi-reflecting mirror and the particles can exit in one of two channels.

- Derive an expression to the transmit
- Make sure that the sum of the probability s is one.

The solution:

There are a few passable ways to write the matrix of the semi-reflecting mirror here are two of them each describes a different mirror , the first symmetric and the second is not symmetric. (each matrix will gives a different result but we can build an interferometer from both)

$$\mathcal{S} = \begin{pmatrix} 0 & r & it & 0 \\ r & 0 & 0 & it \\ it & 0 & 0 & r \\ 0 & it & r & 0 \end{pmatrix} \quad or \quad \begin{pmatrix} 0 & r & t & 0 \\ r & 0 & 0 & t \\ t & 0 & 0 & -r \\ 0 & t & -r & 0 \end{pmatrix}$$

From $S^\dagger S = 1$ we get:

(1) $\quad t^2 + r^2 = 1$

Were r,t are real and we will work with the first one.
The amplitude for each channel equals the sum over paths that went in it.
For channel one there are only two paths and the amplitude is:

$$A_1 = [itre^{i\phi_1} + irte^{i\phi_2}]A_0$$

Where $\phi_1 = k * L_1$ and $\phi_2 = k * L_2$ and $A_0 = 1$ is the amplitude of the beam before it is split.
And similarly for channel two the amplitude is:

$$A_2 = [r^2e^{i\phi_2} - t^2e^{i\phi_1}]A_0$$

From here we get that the probabilitys are:

$$|A_1|^2 = r^2t^2(2 + e^{i\phi} + e^{-i\phi}) = 2r^2t^2(1 + cos(\phi))$$

$$|A_2|^2 = r^4 + t^4 - r^2t^2e^{i\phi} - r^2t^2e^{-i\phi} = r^4 + t^4 - 2r^2t^2cos(\phi)$$

Were $\phi = \phi_2 - \phi_1$.
And with the use of (1) it is clear that $|A_1|^2 + |A_2|^2 = 1$.