E166: Josephson junction and current

Submitted by: Yair Vered , Arnon Mizrahy

The problem:

A capacitor assembled from two superconducting plates. In a superconductor the charge carriers are cooper pairs, which can be refered as bosons with the charge e*=2e.
At a neutral state $|n=0\rangle$ some of the pairs are at the left plate and some at the right one.
The charged state $|n\rangle$ obtained by moving n pairs from the left plate to the right plate.
In order to describe the system we use the basis$|n\rangle$, $n=$ integer
WE define the translation operator D as: $D|n\rangle = |n+1\rangle$
Assume that the shift Amplitude(per time unit)between the plates(by tunnling) is Ω, and the capacitance of the capacitor is C.

1. Define the eigenvalues $exp(i\varphi)$ and the eigenstates $|\varphi\rangle$ of D.

2. Write the Hamiltonian of the system using the c.c operatores φ,N.

3. Define the current operator J as $\frac{d\langle n\rangle}{dt}$.

4.How does the capacitance effect the current as function of time?

The solution:

(1) The problem is similar to a sites system, where D is the translation operator

$$D|\varphi\rangle = e^{(i\varphi)}|\varphi\rangle$$

(2)The operator N can be identified as the position operator, and operator φ as the momentum operator.

$$H = E_0 + \Omega D + \Omega^* D^\dagger$$

assuming Ω is real,and the average energy of the capacitor is $\frac{Q^2}{2C}$ we'll get:

$$H = \frac{(e^*N)^2}{2C} + \Omega(e^{i\varphi} + e^{-i\varphi}) = \frac{(e^*N)^2}{2C} + 2\Omega\cos\varphi$$

(3)

$$J = \frac{\partial\langle N\rangle}{\partial t} = i\left[2\Omega\cos\varphi, N\right] = i(-i2\Omega\sin\varphi) = 2\Omega\sin\varphi$$

(4)

$$\frac{\partial J}{\partial t} = i\left[H, J\right] = \frac{(e^*)^2}{2C}\Omega\left[N^2, D + D^\dagger\right] =$$

$$-\frac{(e^*)^2}{2C}\Omega\left[(DN - ND)N + N(DN - ND) - \left(D^\dagger N - ND^\dagger\right)N - N\left(D^\dagger N - ND^\dagger\right)\right] =$$

$$\frac{(e^*)^2\Omega}{C}\left(N cos\varphi + cos\varphi N\right)$$

The current decreased with the increase of the capacitance.