E140: Two particles in two sites, oscillations

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The problem:

Two different particles a and b are given in two different sites. The system is symetric to transparent and the jump amplitude per time unit is Δ. (assume Δ is real without the generality limitation). The standard basis is: $|x_a, x_b\rangle = |1,1\rangle, |1,2\rangle, |2,1\rangle, |2,2\rangle$.

(1) Write the Hamiltonian H in the standard basis.
Note that in an infinite time step only one particle can move between sites. The Probability for a simultanian movemant is zero.
(2) Write the matrix presentation of the operator T which switches the particles.
(3) Define a set of states $|i = L, R, S, A\rangle$ in which T_{ij} is diagnol.
From now on assume the particles are identicle with spin zero (bosons!).
(4) Write the Hamiltonian H_{ij} in the basis $|i = L, R, S\rangle$.
(5) Write in the same basis the transparent operator R_ij.
(6) Define a set of states in which R is diagnol.
(7) Find the eigensates of the hamiltonian. write them in the standard basis.
(8) What are the eigenenergies ?
The system is prepared at $t = 0$ to the symetric state S.
(9) The number of particles in the left site is measured. what are the possible results ?
(10) What is the period of the system oscillations ?
(11) What is the answer to (9) after half a period ?

The solution:

(1)
The Hamiltonian in the standard basis is

$$\mathcal{H} = \begin{pmatrix} 0 & \Delta & \Delta & 0 \\ \Delta & 0 & 0 & \Delta \\ \Delta & 0 & 0 & \Delta \\ 0 & \Delta & \Delta & 0 \end{pmatrix}$$

(2)
The operator T which switches the particles is

$$\mathcal{T} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(3)
A set of states in which T_{ij} is diagnol:
$|L\rangle = |1,1\rangle$
$|R\rangle = |2,2\rangle$
$|S\rangle = \frac{1}{\sqrt{2}}(|1,2\rangle + |2,1\rangle)$

$|A\rangle = \frac{1}{\sqrt{2}}\left(|1,2\rangle - |2,1\rangle\right)$

(4)
the Hamiltonian in the basis $|i = L, R, S\rangle$ is

$$\mathcal{H} = \begin{pmatrix} 0 & 0 & \Delta \\ 0 & 0 & \Delta \\ \Delta & \Delta & 0 \end{pmatrix}$$

(5)
the transparent operator R is

$$\mathcal{R} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(6)
A set of states in which R is diagnol:
$|S\rangle = \frac{1}{\sqrt{2}}\left(|1,2\rangle + |2,1\rangle\right)$
$|+\rangle = \frac{1}{\sqrt{2}}\left(|1,1\rangle + |2,2\rangle\right)$
$|-\rangle = \frac{1}{\sqrt{2}}\left(|1,1\rangle - |2,2\rangle\right)$

(7)
The eigensates of the hamiltonian are
$|E_1\rangle = |-\rangle = \frac{1}{\sqrt{2}}\left(|1,1\rangle - |2,2\rangle\right)$
$|E_2\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle + |S\rangle\right) = \frac{1}{\sqrt{2}}\left(|1,1\rangle + |2,2\rangle + |1,2\rangle + |2,2\rangle\right)$
$|E_3\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle - |S\rangle\right) = \frac{1}{\sqrt{2}}\left(|1,1\rangle + |2,2\rangle - |1,2\rangle - |2,2\rangle\right)$

(8)
The eigenenergies are
$E_1 = 0$
$E_2 = 2\Delta$
$E_3 = -2\Delta$

(9)
If the system is in the symetric state and we measure how many particles are in the left site the result will be 1.

(10)
The period of the system oscillations is
$T = \frac{\pi}{2\Delta}$

(11)
If the system is prepared in the state $|S\rangle$ after half a period $T = \frac{\pi}{4\Delta}$ it will be in the state $|+\rangle$, and then if we measure the number of particles in the left site the possible results are: $0, 2$.

`Remarks:`

`(4) Missing numerical factor in the Hamiltonian.`