Thermalization of mesoscopic subsystems

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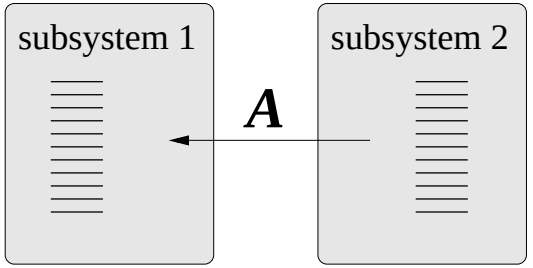
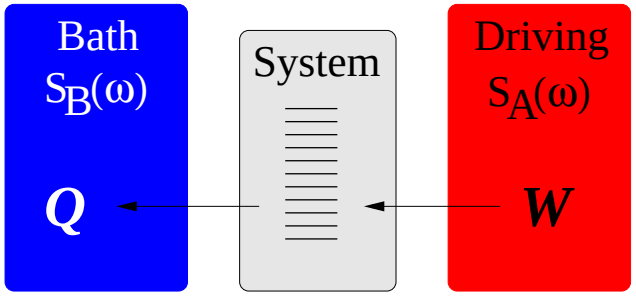


[Ref: PRL 2013]



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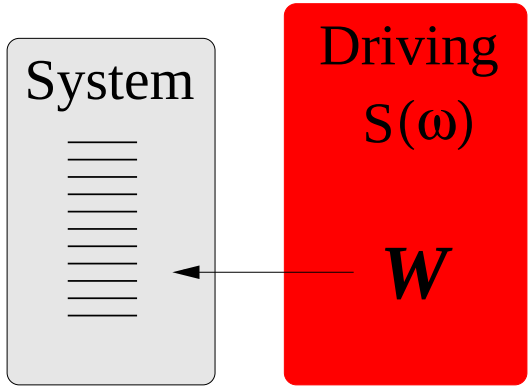
The fluctuation-diffusion-dissipation relation

Rate of energy absorption (work):

$$A(\varepsilon) \;=\; \partial_\varepsilon D_\varepsilon + \beta(\varepsilon) D_\varepsilon, \qquad \dot{W} = \langle A \rangle$$

$$D_\varepsilon \;=\; \int_0^\infty \frac{d\omega}{2\pi}\, \omega^2\, \tilde{C}_\varepsilon(\omega)\, \tilde{S}(\omega)$$



Derivation:

$$\frac{\partial \rho}{\partial t} \;=\; \frac{\partial}{\partial \varepsilon}\left(g(\varepsilon) D(\varepsilon) \frac{\partial}{\partial \varepsilon}\left(\frac{1}{g(\varepsilon)} \rho \right) \right) \;=\; -\frac{\partial}{\partial \varepsilon}\left(A(\varepsilon)\rho - \frac{\partial}{\partial \varepsilon}\left[D(\varepsilon)\rho \right] \right)$$

M. Wilkinson (1988), based on the diffusion picture of **Ott (1979)**

C. Jarzynski (1995) - adding FPE perspective.

D. Cohen (1999) - adding FDT perspective + addressing the quantum case.

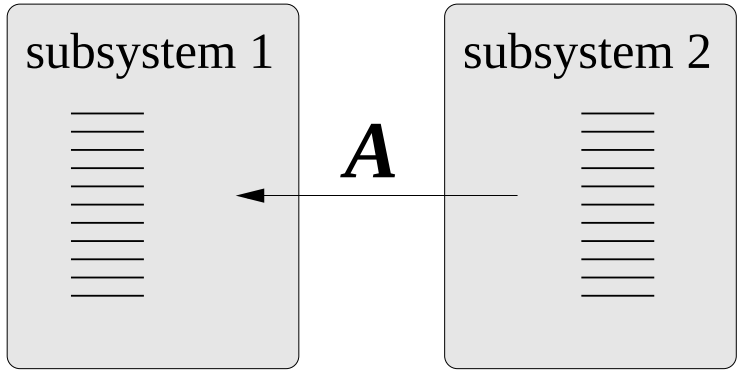
G. Bunin, L. D'Alessio, Y. Kafri, A. Polkovnikov (2011) - adding NFT based derivation.

Thermalization of two subsystems

Rate of energy transfer [FPE version]:

$$A(\varepsilon) \;=\; \partial_\varepsilon D_\varepsilon + (\beta_1 - \beta_2) D_\varepsilon$$

$$D_\varepsilon \;=\; \int_0^\infty \frac{d\omega}{2\pi}\, \omega^2\, \tilde{S}^{(1)}(\omega)\, \tilde{S}^{(2)}(\omega)$$



Derivation:

The diffusion is along constant energy lines: $\varepsilon_1 + \varepsilon_2 = \mathcal{E}$

The proper Liouville measure is: $g(\varepsilon) = g_1(\varepsilon) g_2(\mathcal{E} - \varepsilon)$

Note: After canonincal preparation of the two subsystems:

$$\langle A(\varepsilon) \rangle \;=\; \left(\frac{1}{T_1} - \frac{1}{T_2} \right) \langle D_\varepsilon \rangle$$

MEQ version: **Hurowitz, Cohen (EPL 2011)**

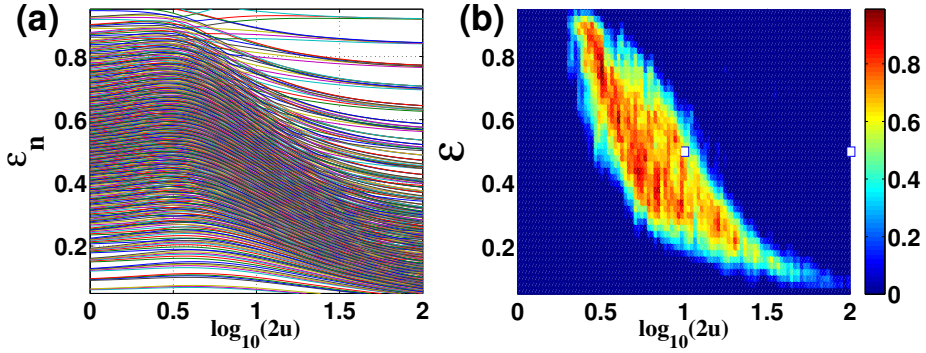
NFT version: **Bunin, Kafri (arXiv 2012)**

The minimal model for a subsystem

The FPE description makes sense if each subsystem is chaotic and hence ergodic.

Minimal models: Billiard; 2deg oscillator; 3site Bose-Hubbard model.

$$\mathcal{H} \;=\; \frac{U}{2}\sum_{i=0,1,2} a_i^\dagger a_i^\dagger a_i a_i + \frac{K}{2}\sum_{i=1,2}\left(a_i^\dagger a_0 + a_0^\dagger a_i\right)$$

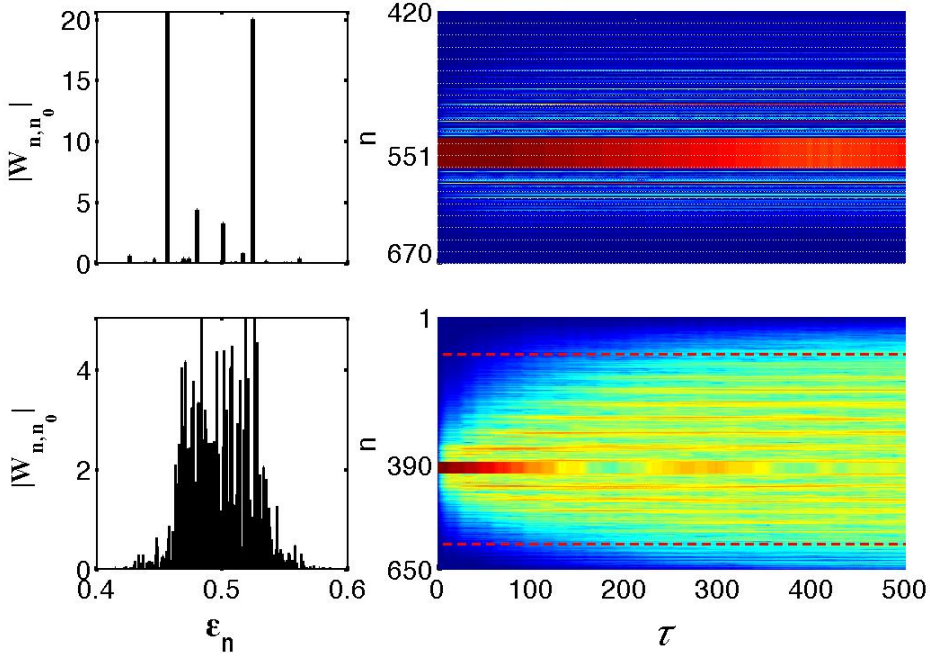


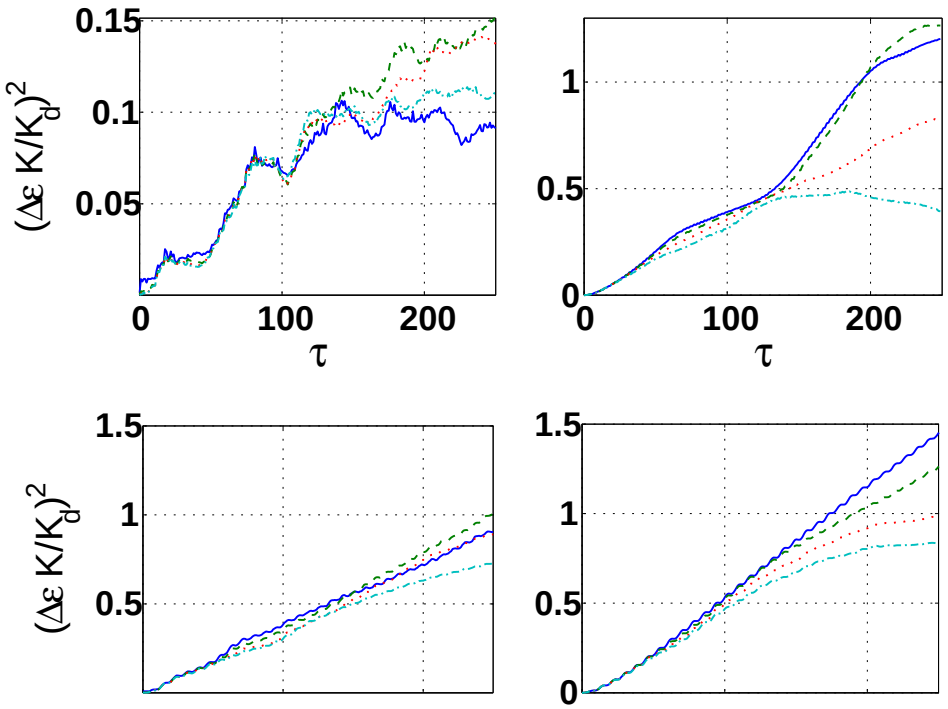
$$K = K_0 + \varepsilon f(t)$$

$$\mathcal{H} \;=\; \mathcal{H}_0 + f(t)W$$

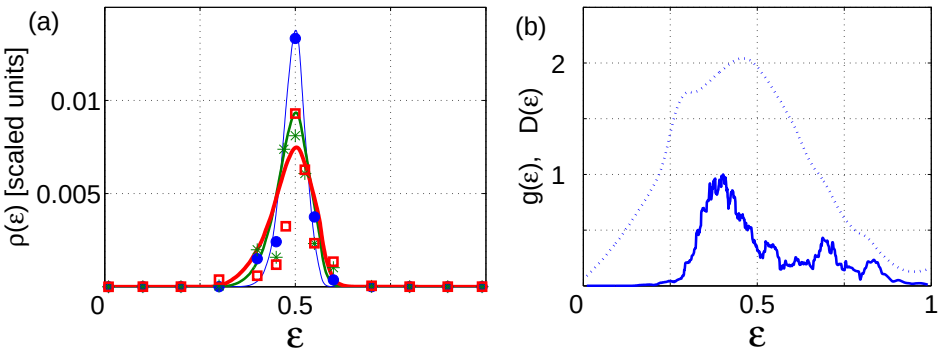
Note on linear response: Driven integrable system (e.g. "kicked rotor") - quasi-linear behavior shows up only for large driving amplitude $\varepsilon > \varepsilon_c$.

Demonstration of diffusion: driven Bose-Hubbard trimer





Quantum vs Classical



Quantum chaos $\rightsquigarrow$ QCC

[Cohen (PRL 1999)]

Originally demonstrated for RMT model

[Cohen, Kottos (PRL 2000)]

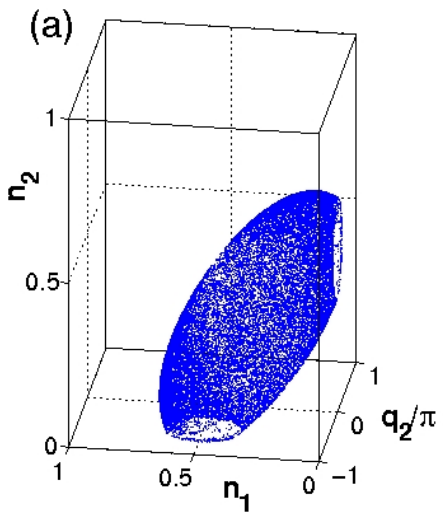
Complexity of phase space - stickiness - beyond FPE

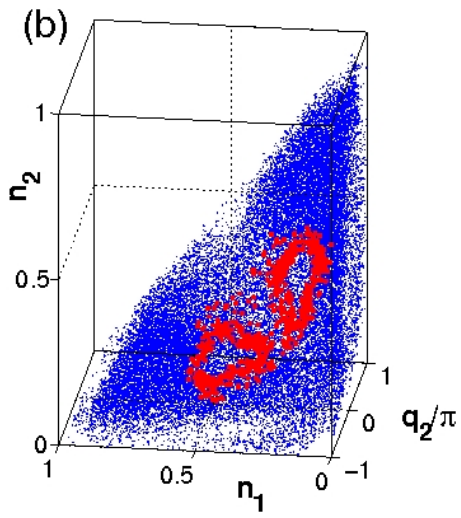
The minimal Fokker-Planck description of thermalization:

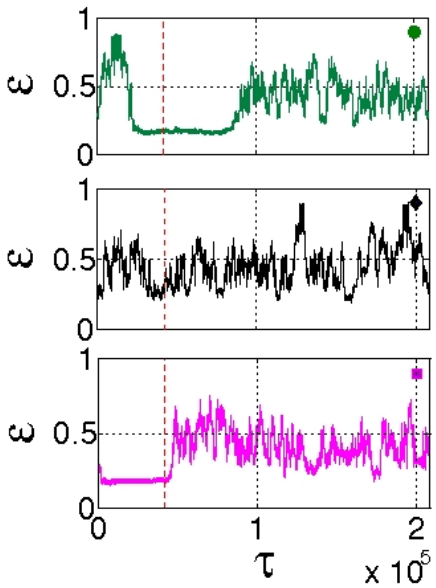
$$\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial \varepsilon}\left(g(\varepsilon)D(\varepsilon)\frac{\partial}{\partial \varepsilon}\left(\frac{1}{g(\varepsilon)}\rho\right)\right) \qquad g(\varepsilon) = g_1(\varepsilon)\ g_2(\mathcal{E}-\varepsilon)$$

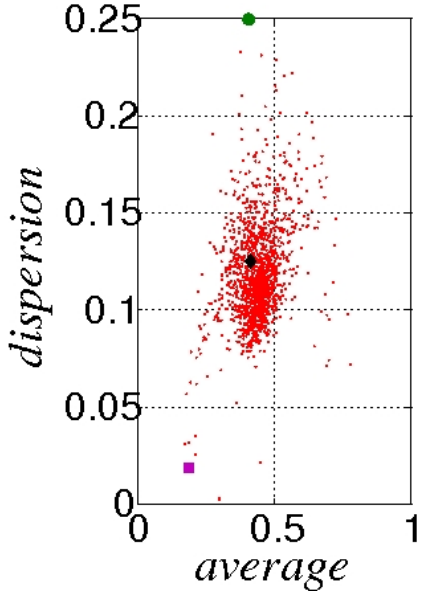
Complexity of phase space might affect the thermalization.

BEC trimer: long dwell times in sticky regions are reflected in $\varepsilon(t)$



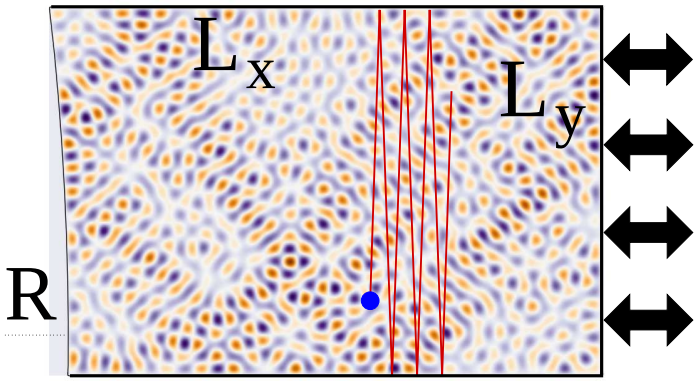


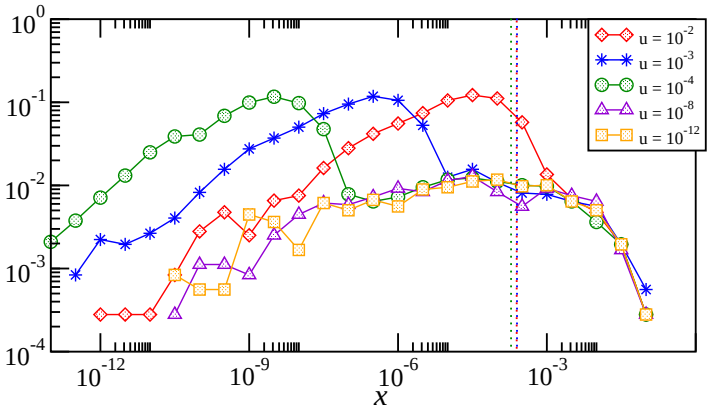




Complexity of phase space - sparsity - beyond LRT

$$\mathcal{H}_{\text{total}} = \text{diag}\{E_n\} - f(t)V_{nm}$$





[log-wide distribution]

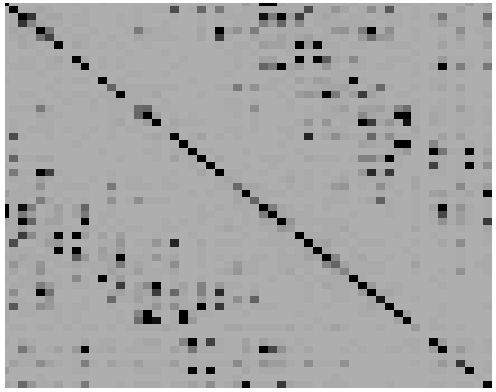
Stotland, Budoyo, Peer, Kottos, Cohen (2008),

Stotland, Cohen, Davidson (2009),

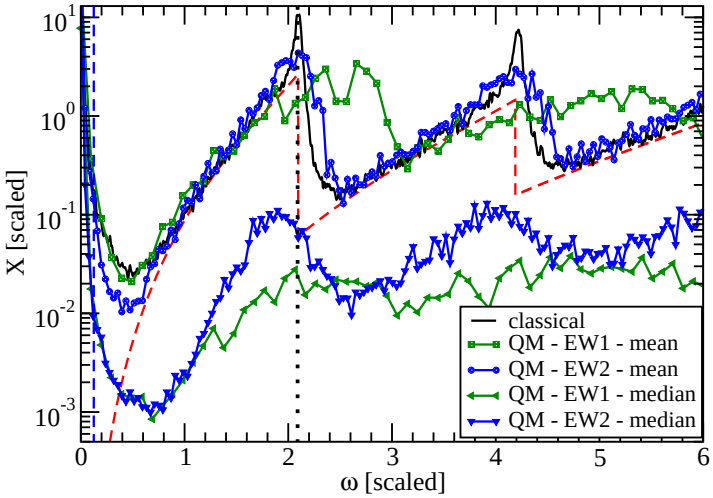
Stotland, Kottos Cohen (2010),

Stotland, Pecora, Cohen (2010,2011)

$$w^{\nu}_{n,m} = \nu\,|V_{nm}|^2$$



[not a Gaussian matrix...]



[median $\ll$ mean]

LRT, SLR, NLR

LRT = linear response theory

$$D[\tilde{S}(\omega)] = \int_0^\infty \frac{d\omega}{2\pi}\, \omega^2\, \tilde{C}(\omega)\, \tilde{S}(\omega)$$

$$D\Big[\lambda \tilde{S}(\omega)\Big] = \lambda\, D\Big[\tilde{S}(\omega)\Big]$$

$$D\Big[\tilde{S}^{(1)} + \tilde{S}^{(2)}\Big] = D\Big[\tilde{S}^{(1)}\Big] + D\Big[\tilde{S}^{(2)}\Big]$$

SLR = semilinear response

e.g. due to "series addition" [Cohen, Kottos, Schanz, JPA 2006; Wilkinson, Mehlig, Cohen, EPL 2006]

$$D[\tilde{S}(\omega)] = \left[\int_0^\infty d\omega\, \frac{\tilde{P}(\omega)}{\tilde{S}(\omega)}\right]^{-1}$$

e.g. due to "variable range hopping" [de Leeuw, Cohen, PRE 2012]

$$D[\tilde{S}(\omega)] \approx \mathrm{EXP}_d\left(\frac{1}{s}\right)\, \mathrm{e}^{-\frac{1}{s}}\, D_{\mathrm{LRT}}$$

[VRH for $s \ll 1$, LRT for large s]

NLR = nonlinear response

e.g. due to non-Ohmic power spectrum [Sela, Aisenberg, Kottos, Cohen, IPA-FTC 2010]

$$D_\varepsilon[\tilde{S}(\omega)] \sim \left\|\tilde{S}(\omega)\right\|^{\frac{3}{4-\sigma}}$$

[NLR for $\sigma \neq 1$, LRT for $\sigma = 1$]

Conclusions

1. BEC trimers are the minimal building blocks for thermalization
2. The generic package deal: diffusion, LRT and QCC.
3. FPE based FD phenomenology for mesoscopic thermalization
4. Beyond FPE - statistics of dwell times due to sticky dynamics
5. Beyond LRT - non Ohmic fluctuations - nonlinear response
6. Beyond LRT - sparsity - resistor network picture - semilinear response
7. FD phenomenology for sparse (glassy) systems **[Hurowitz, Cohen, EPL 2011]**