Fluctuation dissipation phenomenology away from equilibrium

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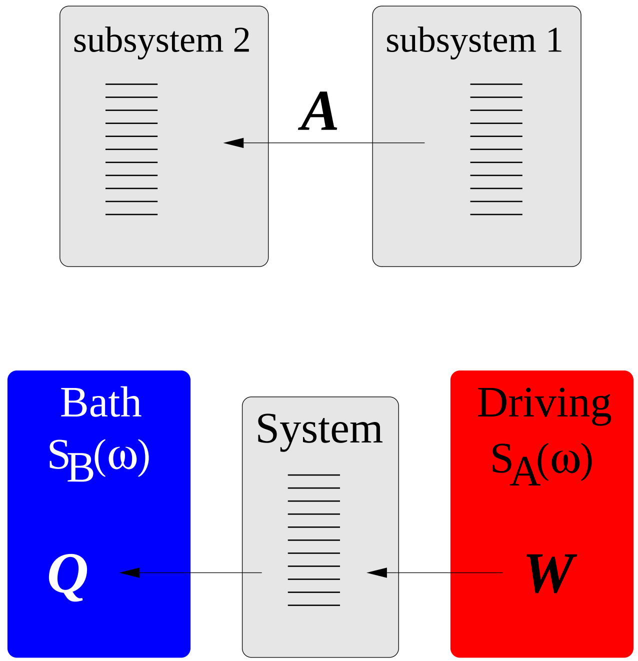

Daniel Hurowitz (BGU) [1]

Igor Tikhonenkov (BGU) [2]

Amichay Vardi (BGU) [2]

James R. Anglin (Kaiserslautern) [2]

[1] Hurowitz, Cohen (EPL 2011).

[2] Tikhonenkov, Vardi, Anglin, Cohen (arXiv 2012).

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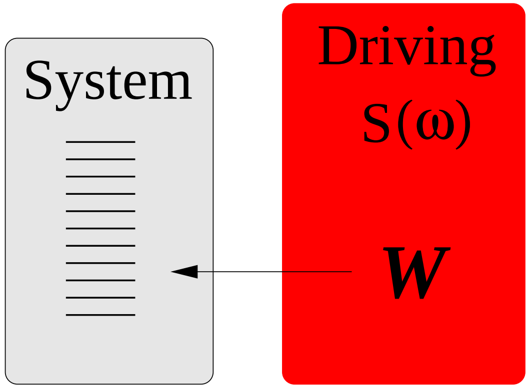
http://www.bgu.ac.il/∼dcohen

The fluctuation-diffusion-dissipation relation

Rate of energy absorption (work):

$$A(\varepsilon) \;=\; \partial_\varepsilon D_\varepsilon + \beta(\varepsilon) D_\varepsilon, \qquad \dot{W} = \langle A \rangle$$

$$D_\varepsilon \;=\; \int_0^\infty \frac{d\omega}{2\pi}\,\omega^2\,\tilde{C}_\varepsilon(\omega)\,\tilde{S}(\omega)$$



Derivation:

$$\frac{\partial \rho}{\partial t} \;=\; \frac{\partial}{\partial \varepsilon}\left(g(\varepsilon)D(\varepsilon)\frac{\partial}{\partial \varepsilon}\left(\frac{1}{g(\varepsilon)}\rho\right)\right) \;=\; -\frac{\partial}{\partial \varepsilon}\left(A(\varepsilon)\rho - \frac{\partial}{\partial \varepsilon}\left[D(\varepsilon)\rho\right]\right)$$

M. Wilkinson (1988), based on the diffusion picture of **Ott (1979)**

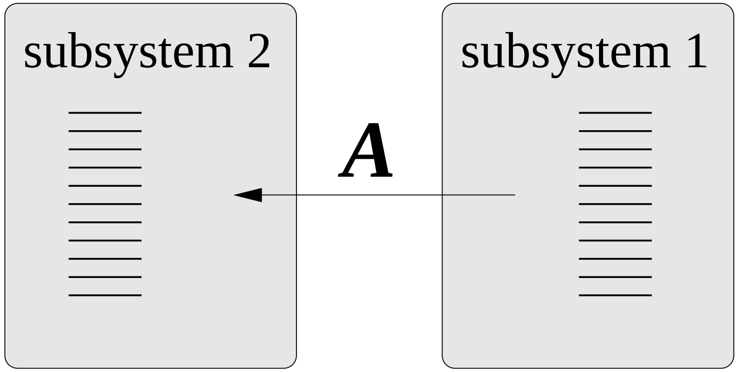
C. Jarzynski (1995) - adding FPE perspective.

D. Cohen (1999) - adding FDT perspective + addressing the quantum case.

G. Bunin, L. D'Alessio, Y. Kafri, A. Polkovnikov (2011) - adding NFT based derivation.

Thermalization of two subsystems

Rate of energy transfer:



$$A(\varepsilon) \;=\; \partial_\varepsilon D_\varepsilon + (\beta_1 - \beta_2) D_\varepsilon,$$

$$D_\varepsilon \;=\; \int_0^\infty \frac{d\omega}{2\pi}\, \omega^2\, \tilde{S}^{(1)}(\omega)\, \tilde{S}^{(2)}(\omega)$$

After canonincal preparation of the two subsystems:

$$\langle A(\varepsilon)\rangle \;=\; \left(\frac{1}{T_1} - \frac{1}{T_2}\right)\langle D_\varepsilon\rangle$$

MEQ version: **Hurowitz, Cohen (EPL 2011)**

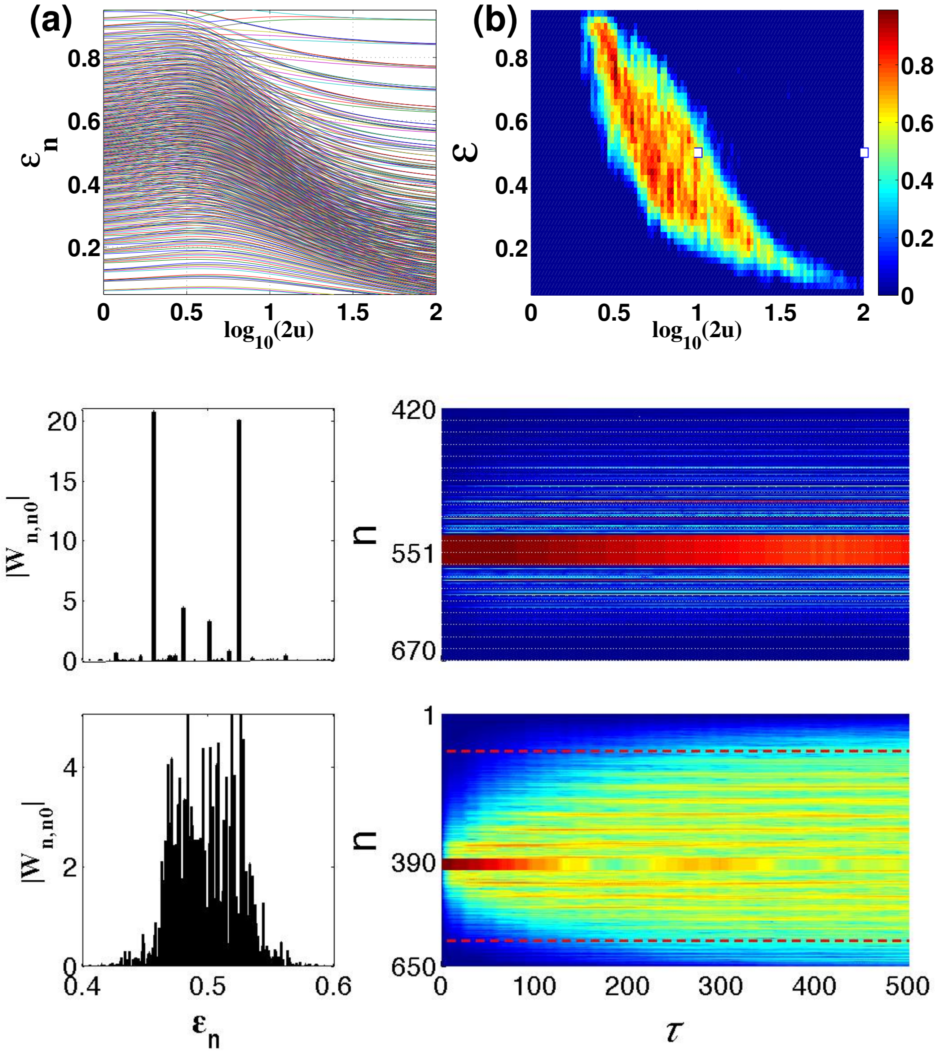
NFT version: **Bunin, Kafri (arXiv 2012)**

Derivation:

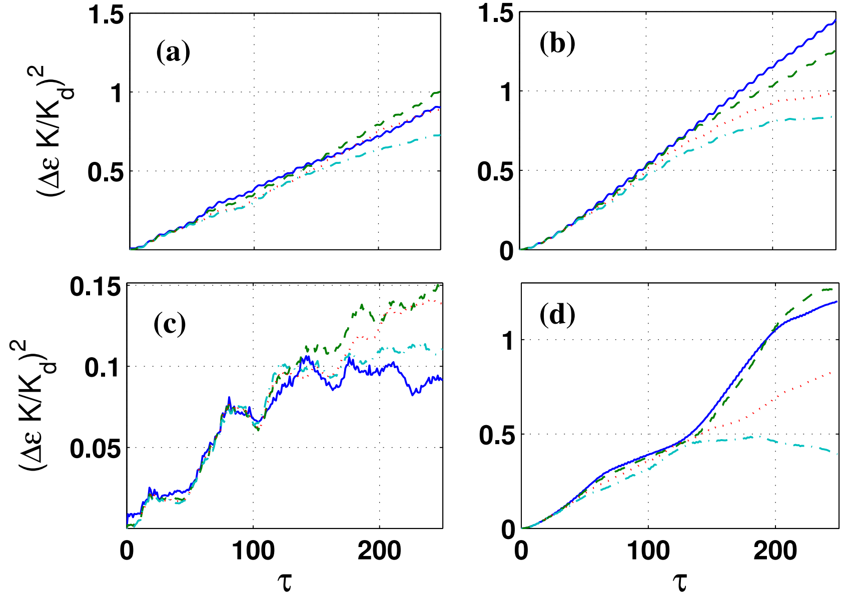
The diffusion is along constant energy lines: $\varepsilon_1 + \varepsilon_2 = \mathcal{E}$

The proper Liouville measure is: $g(\varepsilon) = g_1(\varepsilon) g_2(\mathcal{E} - \varepsilon)$

Demonstration of diffusion: driven Bose-Hubbard trimer



[Tikhonenkov, Vardi, Anglin, Cohen (arXiv 2012)]



Quantum vs Classical

Quantum chaos ⇝ QCC

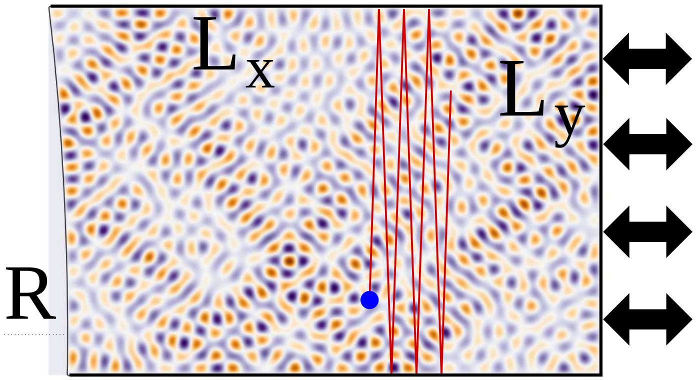
[Cohen (PRL 1999)]

Originally demonstrated for RMT model

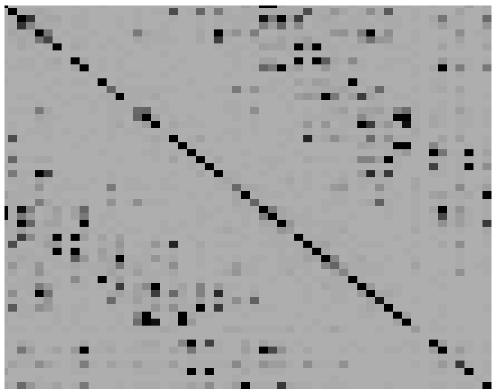
[Cohen, Kottos (PRL 2000)]

The "sparsity" of weakly chaotic driven systems

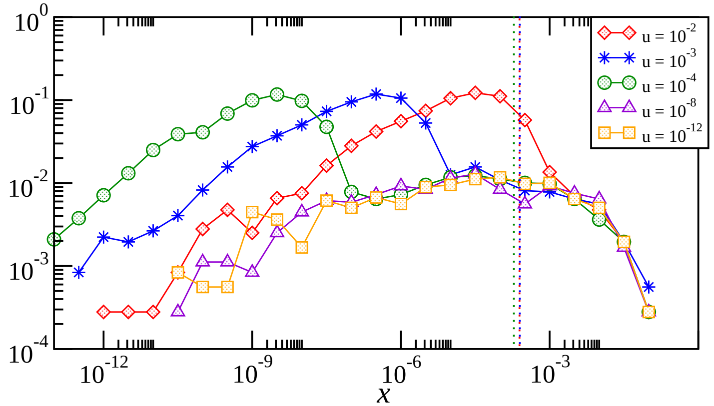
$$\mathcal{H}_{\text{total}} = \text{diag}\{E_n\} - f(t) V_{nm}$$



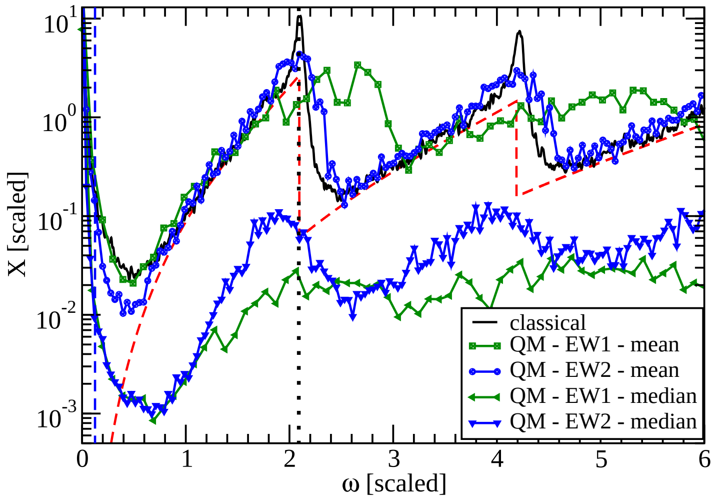
$$w^{\nu}_{n,m} \;=\; \nu \, |V_{nm}|^2$$



[not a Gaussian matrix...]



[log-wide distribution]



[median ≪ mean]

Stotland, Budoyo, Peer, Kottos, Cohen (2008),

Stotland, Cohen, Davidson (2009),

Stotland, Kottos Cohen (2010),

Stotland, Pecora, Cohen (2010,2011)

The NESS of a "sparse" system

$$w_{nm} \quad = \quad w^{\beta}_{nm} + w^{\nu}_{nm} \quad = \quad w^{\beta}_{nm} + \nu g_{nm}$$

$$\frac{w^{\beta}_{nm}}{w^{\beta}_{mn}} \;=\; \exp\left[-\frac{E_n - E_m}{T_B}\right], \qquad\qquad g_{nm} \;=\; g_{mn}$$

w^{ν} by themselves - induces diffusion / ergodization

w^{β} by themselves - leads to equilibrium

Combined - leads to **NESS**

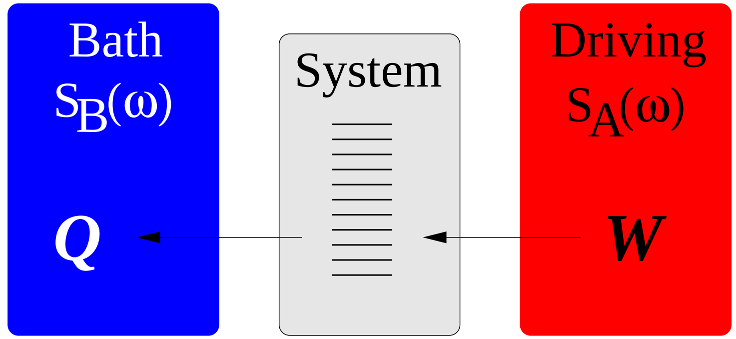
Linear response and traditional FD: $\nu \times \{g\} \ll \{w^{\beta}\}$

Glassy response and Sinai physics: [within a wide crossover regime]

Semi-linear response and Saturation: $\nu \times \{g\} \gg \{w^{\beta}\}$

FD phenomenology for a "sparse" system

$$w_{nm} = w^{\beta}_{nm} + w^{\nu}_{nm} = w^{\beta}_{nm} + \nu g_{nm}$$



$$\dot{W} = \text{rate of heating} = \frac{D(\nu)}{T_{\text{system}}}$$

$$\dot{Q} = \text{rate of cooling} = \frac{D_B}{T_B} - \frac{D_B}{T_{\text{system}}}$$

Hence at the NESS:

$$T_{\text{system}} = \left(1 + \frac{D(\nu)}{D_B}\right) T_B$$

$$\dot{Q} = \dot{W} = \frac{1/T_B}{D_B^{-1} + D(\nu)^{-1}}$$

Experimental way to extract response:

$$D(\nu) = \frac{\dot{Q}(\nu)}{\dot{Q}(\infty) - \dot{Q}(\nu)} D_B$$

$D(\nu)$ exhibits LRT to SLRT crossover

$$D(\nu) = \left[\overline{\left(\frac{w_n}{w_\beta + w_n}\right)}\right] \left[\overline{\left(\frac{1}{w_\beta + w_n}\right)}\right]^{-1}$$

$$D_{[\text{LRT}]} = \overline{g_n}\ \nu \qquad \text{[weak driving]}$$

$$D_{[\text{SLRT}]} = [\overline{1/g_n}]^{-1} \nu \qquad \text{[strong driving]}$$

Expressions above assume n.n. transitions only.

Conclusions

(*) Wigner ($\sim$ 1955): "The perturbation is represented by a random matrix whose elements are taken from a Gaussian distribution." Not always...

1. "weak quantum chaos" $\Longrightarrow$ log-wide distribution, "sparsity" and "texture"
2. The heating $\sim$ a percolation process.
3. Resistors network calculation to get the response coefficient.
4. RMT modeling $\rightsquigarrow$ generalization of the VRH estimate.
5. Experimental fingerprint: semi-linear response characteristics.

6. SLRT applies if the driving is stronger then the background relaxation.
7. The stochastic NESS has glassy characteristics (wide distribution of microscopic temperatures).
8. Definition of effective NESS temperature, and extension of the F-D phenomenology.
9. For very strong driving - quantum saturation of the NESS temperature ($T \to T_{\infty}$).

10. Topological aspects: The emergence of the Sinai regime.
11. Topological term in the formula for the heating rate.

12. Applications: beyond the "Drude formula" and beyond the "Wall formula".

Perspective and references

The classical LRT approach: Ott, Brown, Grebogi, Wilkinson, Jarzynski, Robbins, Berry, Cohen

The Wall formula (I): Blocki, Boneh, Nix, Randrup, Robel, Sierk, Swiatecki, Koonin

The Wall formula (II): Barnett, Cohen, Heller [1] - regarding g_c

Semi Linear response theory: Cohen, Kottos, Schanz... [2-6]

Billiards with vibrating walls: Stotland, Cohen, Davidson, Pecora [7,8] - regarding g_s

Sparsity: Austin, Wilkinson, Prosen, Robnik, Alhassid, Levine, Fyodorov, Chubykalo, Izrailev, Casati

Random networks: Mott; Miller, Abrahams; Ambegaokar, Halperin, Langer; Pollak [...]

Random site model: Alexander, Bernasconi, Schneider, Orbach; Amir, Oreg, Imry [...]

Extensions related to: Sinai; Derrida,Pomeau; Burlatsky, Oshanin, Mogutov, Moreau, [...]

[1] A. Barnett, D. Cohen, E.J. Heller (PRL 2000, JPA 2000)

[2] D. Cohen, T. Kottos, H. Schanz (JPA 2006)

[3] S. Bandopadhyay, Y. Etzioni, D. Cohen (EPL 2006)

[4] M. Wilkinson, B. Mehlig, D. Cohen (EPL 2006)

[5] A. Stotland, R. Budoyo, T. Peer, T. Kottos, D. Cohen (JPA/FTC 2008)

[6] A. Stotland, T. Kottos, D. Cohen (PRB 2010)

[7] A. Stotland, D. Cohen, N. Davidson (EPL 2009)

[8] A. Stotland, L.M. Pecora, D. Cohen (EPL 2010, PRE 2011)

[9] D. Hurowitz, D. Cohen (EPL 2011)

[10] Yaron de Leeuw, D. Cohen (PRE 2012)