Fluctuation dissipation phenomenology away from equilibrium

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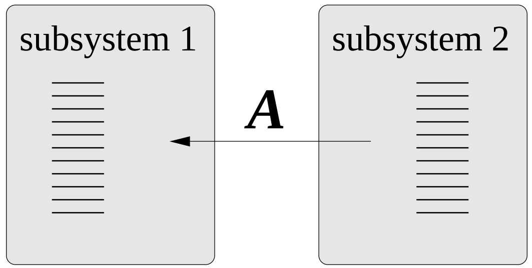

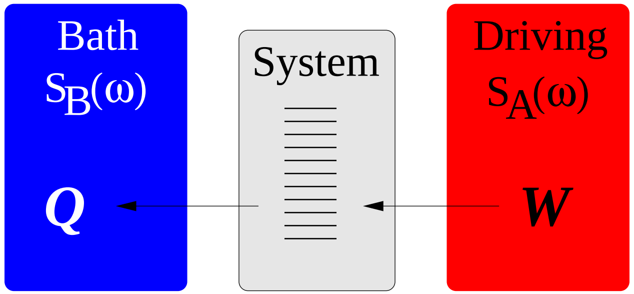

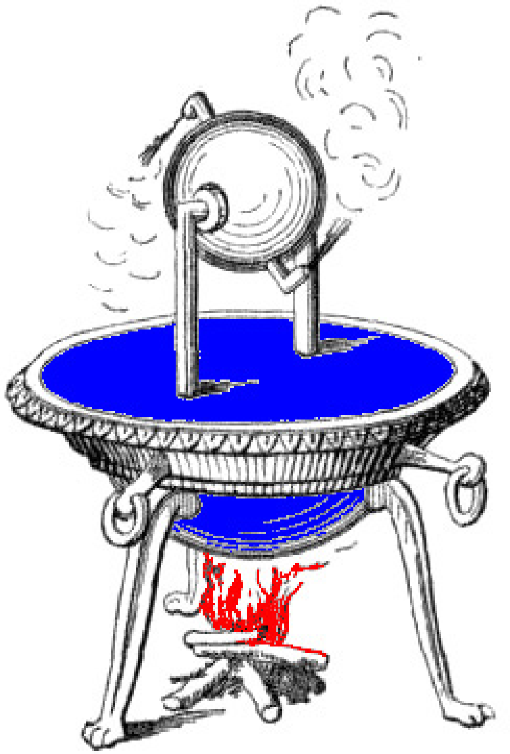

[1] Hurowitz, Cohen (EPL 2011).

[2] Tikhonenkov, Vardi, Anglin, Cohen (PRL 2013).

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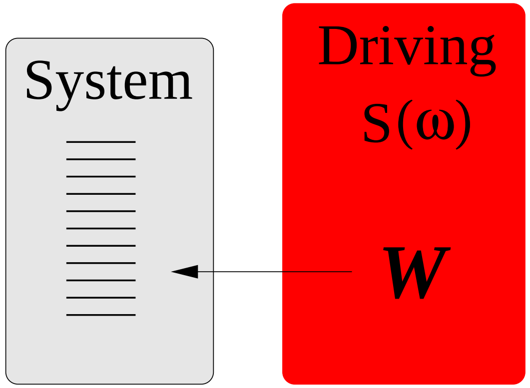
The fluctuation-diffusion-dissipation relation

Rate of energy absorption (work):

$$A(\varepsilon) \;=\; \partial_\varepsilon D_\varepsilon + \beta(\varepsilon) D_\varepsilon, \qquad \dot{W} = \langle A \rangle$$

$$D_\varepsilon \;=\; \int_0^\infty \frac{d\omega}{2\pi}\, \omega^2\, \tilde{C}_\varepsilon(\omega)\, \tilde{S}(\omega)$$



Derivation:

$$\frac{\partial \rho}{\partial t} \;=\; \frac{\partial}{\partial \varepsilon}\left(g(\varepsilon) D(\varepsilon) \frac{\partial}{\partial \varepsilon}\left(\frac{1}{g(\varepsilon)}\rho\right)\right) \;=\; -\frac{\partial}{\partial \varepsilon}\left(A(\varepsilon)\rho - \frac{\partial}{\partial \varepsilon}\left[D(\varepsilon)\rho\right]\right)$$

M. Wilkinson (1988), based on the diffusion picture of **Ott (1979)**

C. Jarzynski (1995) - adding FPE perspective.

D. Cohen (1999) - adding FDT perspective + addressing the quantum case.

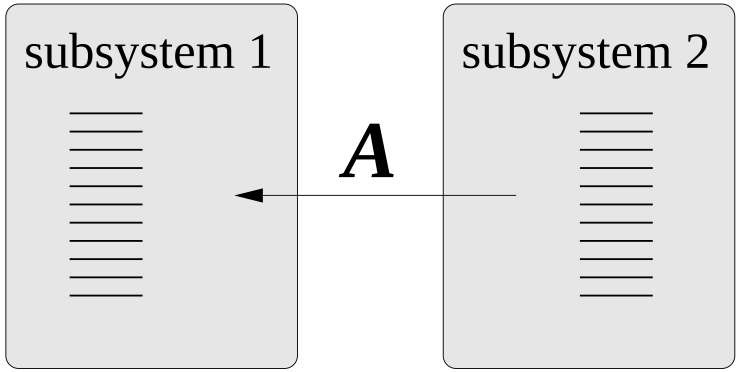
G. Bunin, L. D'Alessio, Y. Kafri, A. Polkovnikov (2011) - adding NFT based derivation.

Thermalization of two subsystems

Rate of energy transfer [FPE version]:

$$A(\varepsilon) \;=\; \partial_\varepsilon D_\varepsilon + (\beta_1 - \beta_2) D_\varepsilon$$

$$D_\varepsilon \;=\; \int_0^\infty \frac{d\omega}{2\pi}\, \omega^2\, \tilde{S}^{(1)}(\omega)\, \tilde{S}^{(2)}(\omega)$$



Derivation:

The diffusion is along constant energy lines: $\varepsilon_1 + \varepsilon_2 = \mathcal{E}$

The proper Liouville measure is: $g(\varepsilon) = g_1(\varepsilon) g_2(\mathcal{E} - \varepsilon)$

Note: After canonincal preparation of the two subsystems:

$$\langle A(\varepsilon) \rangle \;=\; \left(\frac{1}{T_1} - \frac{1}{T_2}\right) \langle D_\varepsilon \rangle$$

MEQ version: **Hurowitz, Cohen (EPL 2011)**

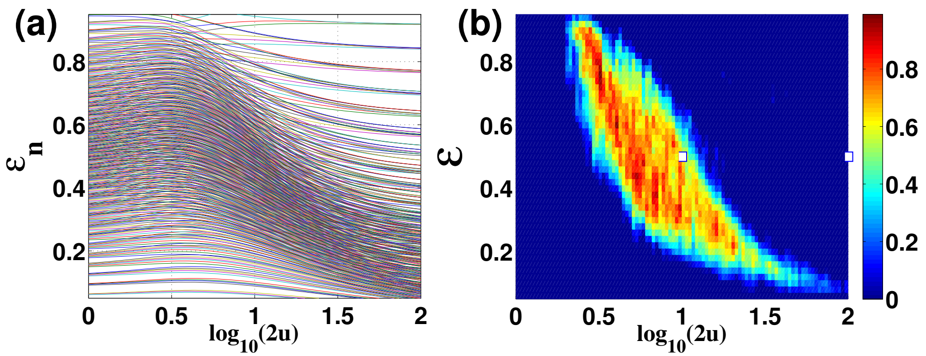
NFT version: **Bunin, Kafri (arXiv 2012)**

The minimal model for a subsystem

The FPE description makes sense if each subsystem is chaotic and hence ergodic.

Minimal models: Billiard; 2deg oscillator; 3site Bose-Hubbard model.

$$\mathcal{H} \;=\; \frac{U}{2}\sum_{i=0,1,2} a_i^\dagger a_i^\dagger a_i a_i + \frac{K}{2}\sum_{i=1,2}\left(a_i^\dagger a_0 + a_0^\dagger a_i\right)$$

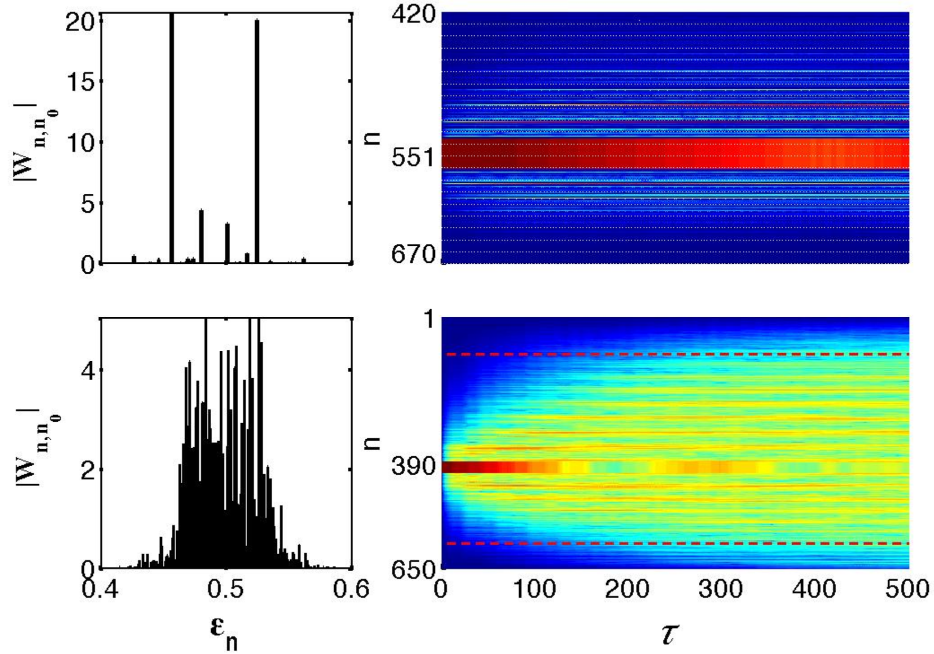


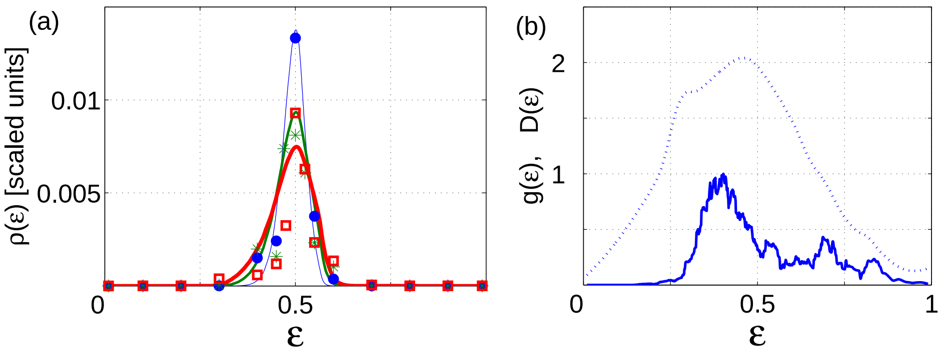
$K = K_0 + \varepsilon f(t)$

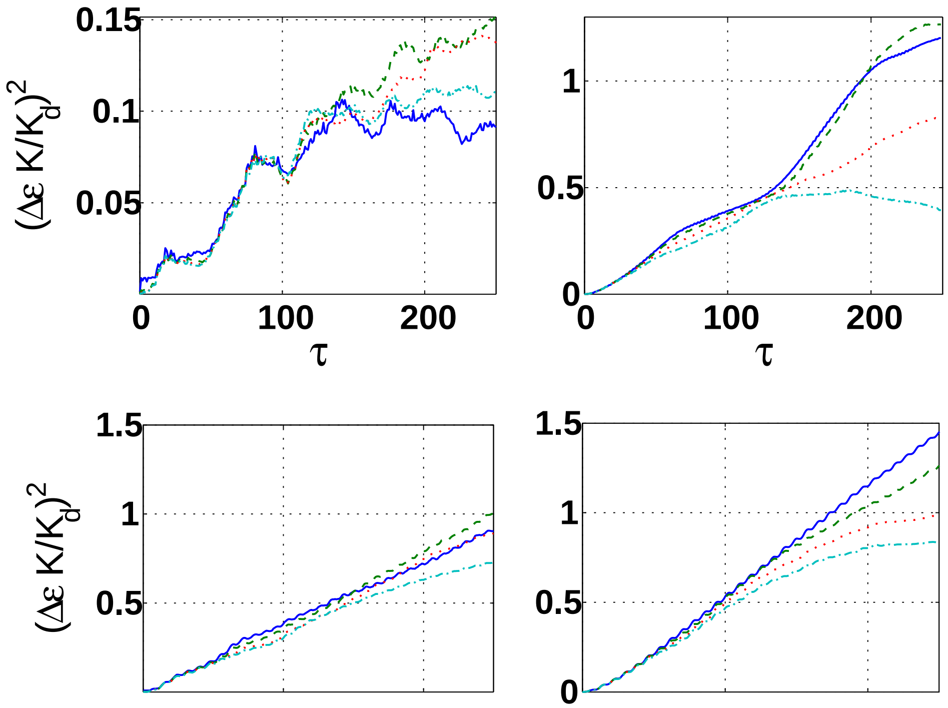
$$\mathcal{H} \;=\; \mathcal{H}_0 + f(t)W$$

Note on linear response: Driven integrable system (e.g. "kicked rotor") -
quasi-linear behavior shows up only for large driving amplitude $\varepsilon > \varepsilon_c$.

Demonstration of diffusion: driven Bose-Hubbard trimer







Quantum vs Classical

Quantum chaos $\rightsquigarrow$ QCC

[Cohen (PRL 1999)]

Originally demonstrated for RMT model

[Cohen, Kottos (PRL 2000)]

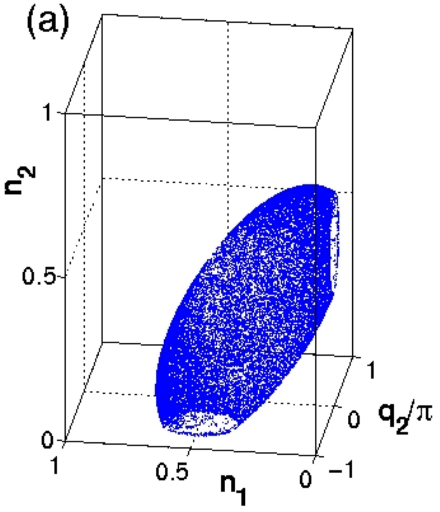
Complexity of phase space - stickiness - beyond FPE

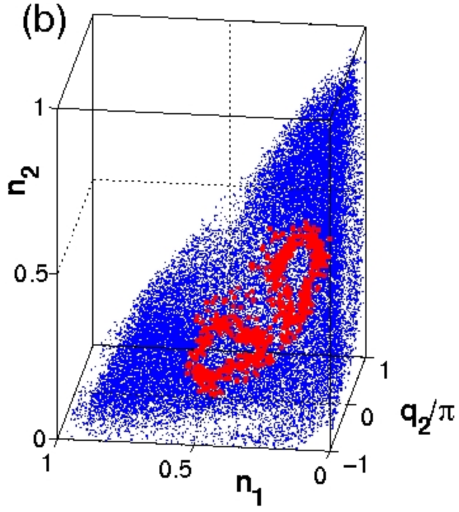
The minimal Fokker-Planck description of thermalization:

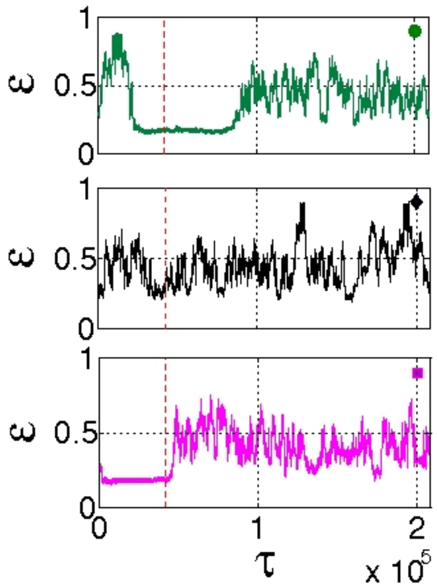
$$\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial \varepsilon}\left(g(\varepsilon) D(\varepsilon) \frac{\partial}{\partial \varepsilon}\left(\frac{1}{g(\varepsilon)}\rho\right)\right) \qquad g(\varepsilon) = g_1(\varepsilon)\ g_2(\mathcal{E}-\varepsilon)$$

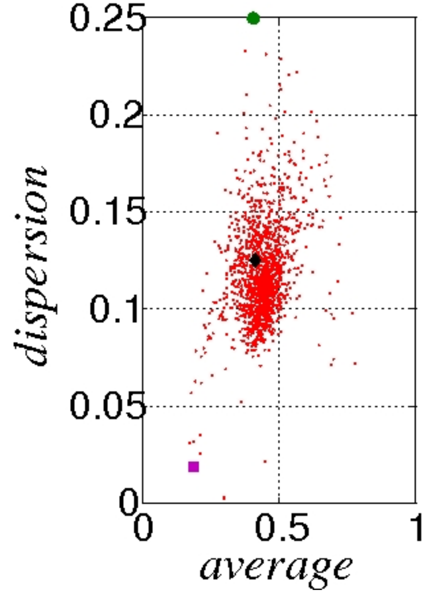
Complexity of phase space might affect the thermalization.

BEC trimer: long dwell times in sticky regions are reflected in $\varepsilon(t)$



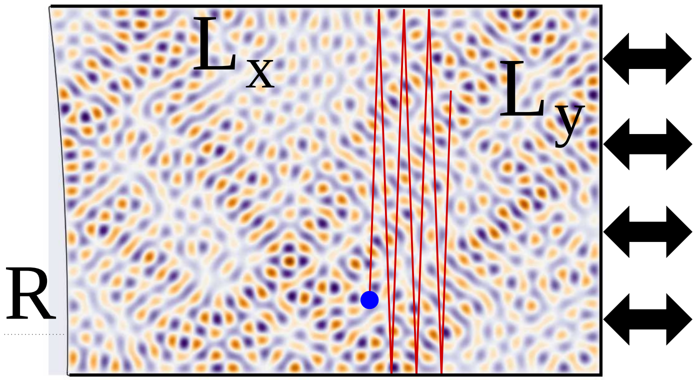


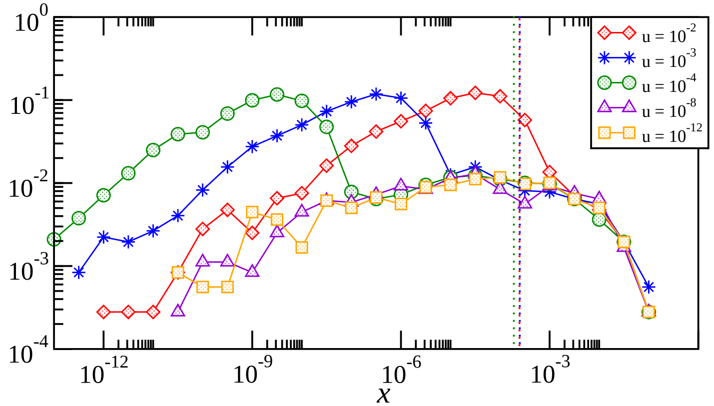




Complexity of phase space - sparsity - beyond LRT

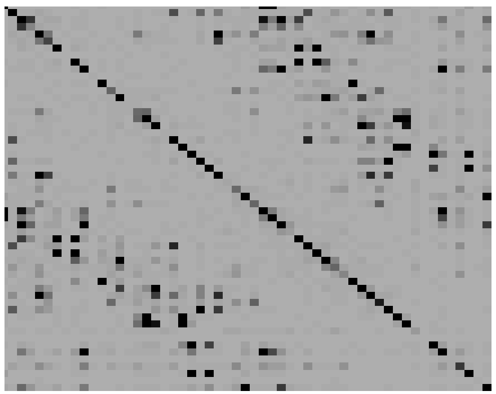
$$\mathcal{H}_{\text{total}} = \text{diag}\{E_n\} - f(t)V_{nm}$$



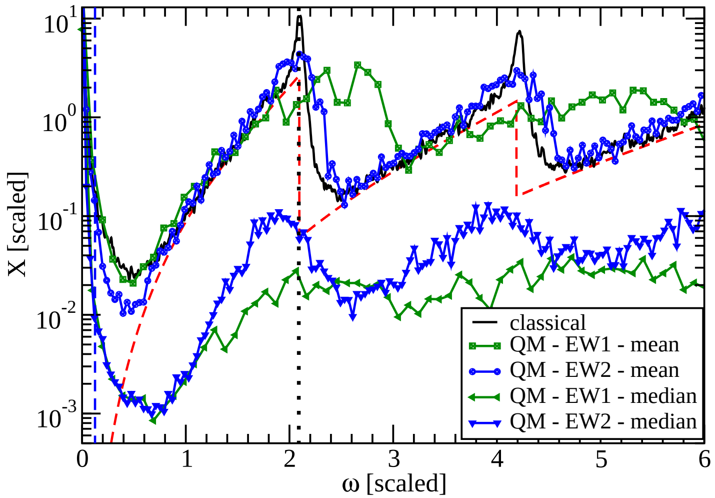


[log-wide distribution]

$$w^{\nu}_{n,m} \;=\; \nu\, |V_{nm}|^2$$



[not a Gaussian matrix...]



[median $\ll$ mean]

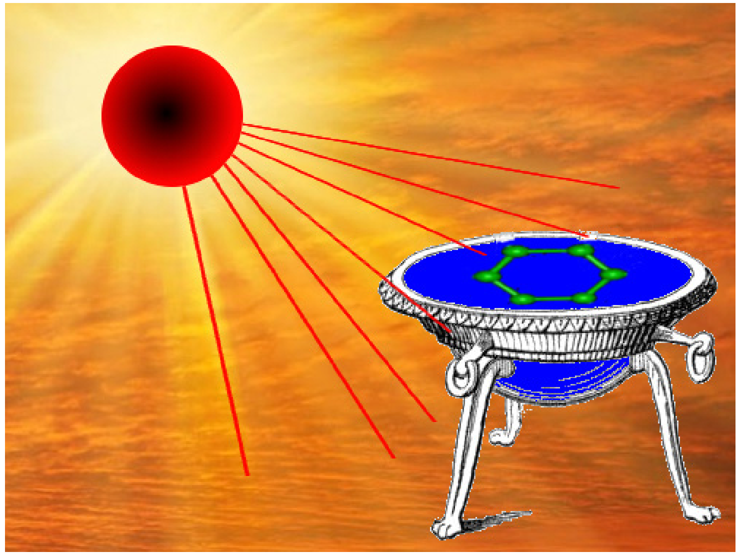
Stotland, Budoyo, Peer, Kottos, Cohen (2008),

Stotland, Cohen, Davidson (2009),

Stotland, Kottos Cohen (2010),

Stotland, Pecora, Cohen (2010,2011)

The NESS of a "sparse" system



$$w_{nm} \;=\; w^{\beta}_{nm} + w^{\nu}_{nm} \;=\; w^{\beta}_{nm} + \nu g_{nm}$$

Cold bath: $$\frac{w^{\beta}_{nm}}{w^{\beta}_{mn}} \;=\; \exp\left[-\frac{E_n - E_m}{T_B}\right]$$

Hot source: $$g_{nm} \;=\; g_{mn}$$

w^{ν} by themselves - induces diffusion / ergodization

w^{β} by themselves - leads to equilibrium

Combined - leads to **NESS**

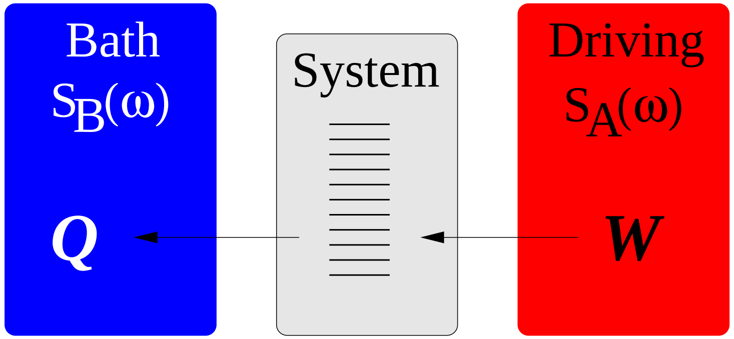
Linear response and traditional FD: $\nu \times \{g\} \;\ll\; \{w^{\beta}\}$

Glassy response and Sinai physics: [within a wide crossover regime]

Semi-linear response and Saturation: $\nu \times \{g\} \;\gg\; \{w^{\beta}\}$

FD phenomenology for a "sparse" system

$$w_{nm} \quad = \quad w^{\beta}_{nm} + w^{\nu}_{nm} \quad = \quad w^{\beta}_{nm} + \nu g_{nm}$$



$$\dot{W} \quad = \quad \text{rate of heating} \;=\; \frac{D(\nu)}{T_{\text{system}}}$$

$$\dot{Q} \quad = \quad \text{rate of cooling} \;=\; \frac{D_B}{T_B} - \frac{D_B}{T_{\text{system}}}$$

Hence at the NESS:

$$T_{\text{system}} \quad = \quad \left(1 + \frac{D(\nu)}{D_B}\right) T_B$$

$$\dot{Q} \;=\; \dot{W} \quad = \quad \frac{1/T_B}{D_B^{-1} + D(\nu)^{-1}}$$

Experimental way to extract response:

$$D(\nu) \;=\; \frac{\dot{Q}(\nu)}{\dot{Q}(\infty) - \dot{Q}(\nu)}\, D_B$$

$D(\nu)$ exhibits LRT to SLRT crossover

$$D(\nu) \quad = \quad \left[\overline{\left(\frac{w_n}{w_\beta + w_n}\right)}\right] \left[\overline{\left(\frac{1}{w_\beta + w_n}\right)}\right]^{-1}$$

$$D_{[\text{LRT}]} \quad = \quad \overline{g_n}\; \nu \qquad \text{[weak driving]}$$

$$D_{[\text{SLRT}]} \quad = \quad [\overline{1/g_n}]^{-1} \nu \qquad \text{[strong driving]}$$

Expressions above assume n.n. transitions only.

Conclusions

1. BEC trimers are the minimal building blocks for thermalization
2. The generic package deal: diffusion, LRT and QCC.
3. FPE based FD phenomenology for mesoscopic thermalization
4. Beyond FPE - statistics of dwell times due to sticky dynamics
5. Beyond LRT - sparsity - resistor network picture - semilinear response
6. FD phenomenology for sparse (glassy) systems