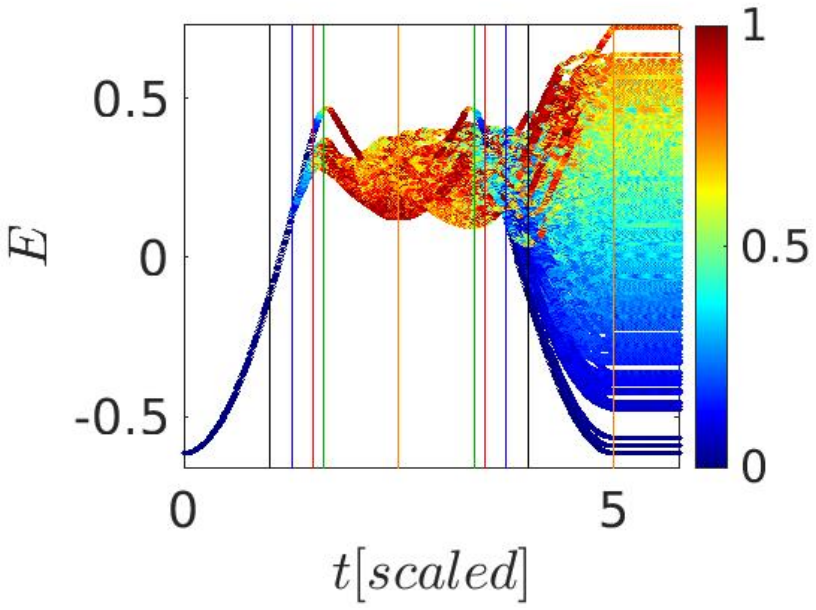
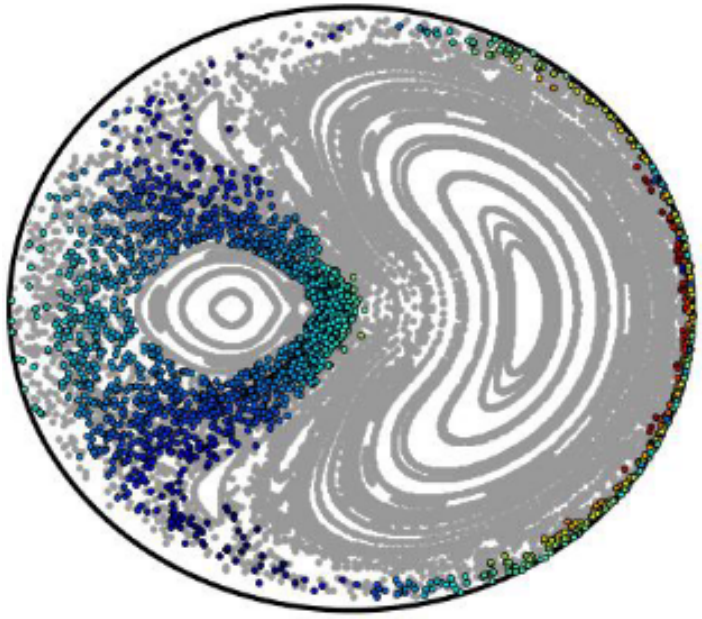
Quantum irreversibility of quasistatic protocols

Doron Cohen, Ben-Gurion University





Yehoshua Winsten, DC,
Chaos-assisted depletion and quantum irreversibility of quasistatic protocols, arXiv (2022).
Quasistatic transfer protocols for atomtronic superfluid circuits, Scientific Reports 11, 3136 (2021).

Related publications:

R. Burkle, A. Vardi, DC, J.R. Anglin, Hamiltonian Hysteresis, Phys. Rev. Lett. 123, 114101 (2019).

A. Dey, DC, A. Vardi, Sweep through chaos, Phys. Rev. Lett. 121, 250405 (2018).

Other related publications, notably by Geva Arwas, are listed in the last slide.

The piston paradigm for irreversibility

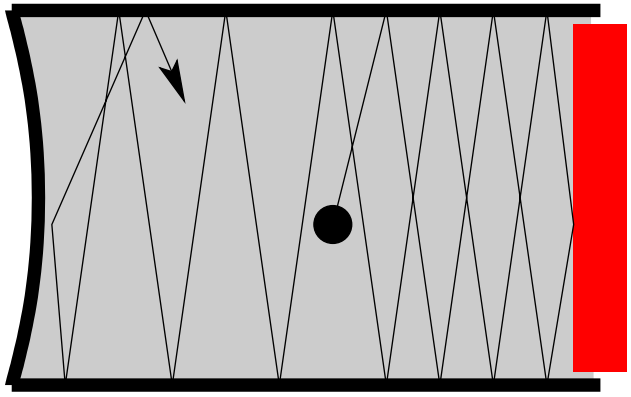
$H[\boldsymbol{r}, \boldsymbol{p}; X(t)]$

The Ott-Wilkinson-Kubo picture:

$\mathrm{Var}(E) \approx 2D_E\, t$ (diffusion)

$D_E \propto \dot{X}^2$

$\dot{E} \propto \dot{X}^2$ (dissipation)



$X(t)$ = piston position

- Chaos - motion of particles $(\boldsymbol{r}, \boldsymbol{p})$ inside the box is chaotic
- Quasi static limit - the piston position (X) is varied very slowly

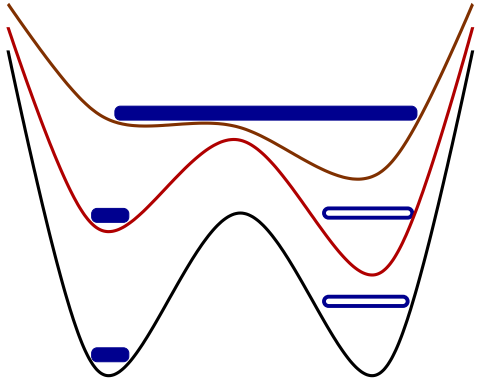
Common wisdom: The quasistatic limit is adiabatic, hence reversible.

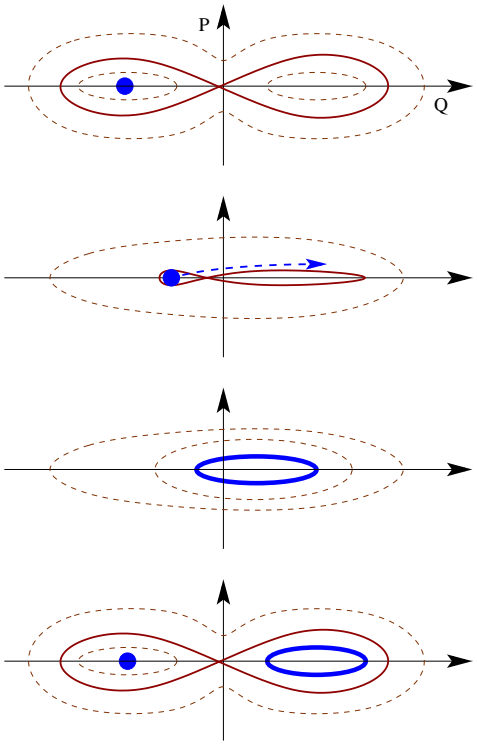
Fine print: Kubo picture requires the assumption of global chaos at any moment.

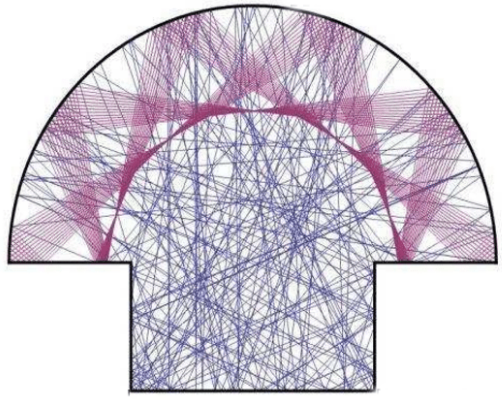
Implications: Kubo picture fails in general due to structural changes in phase-space

Irreversibility due to structural changes in phase-space

Separatrix crossing:
Adiabaticity is broken
in the quasi static limit.







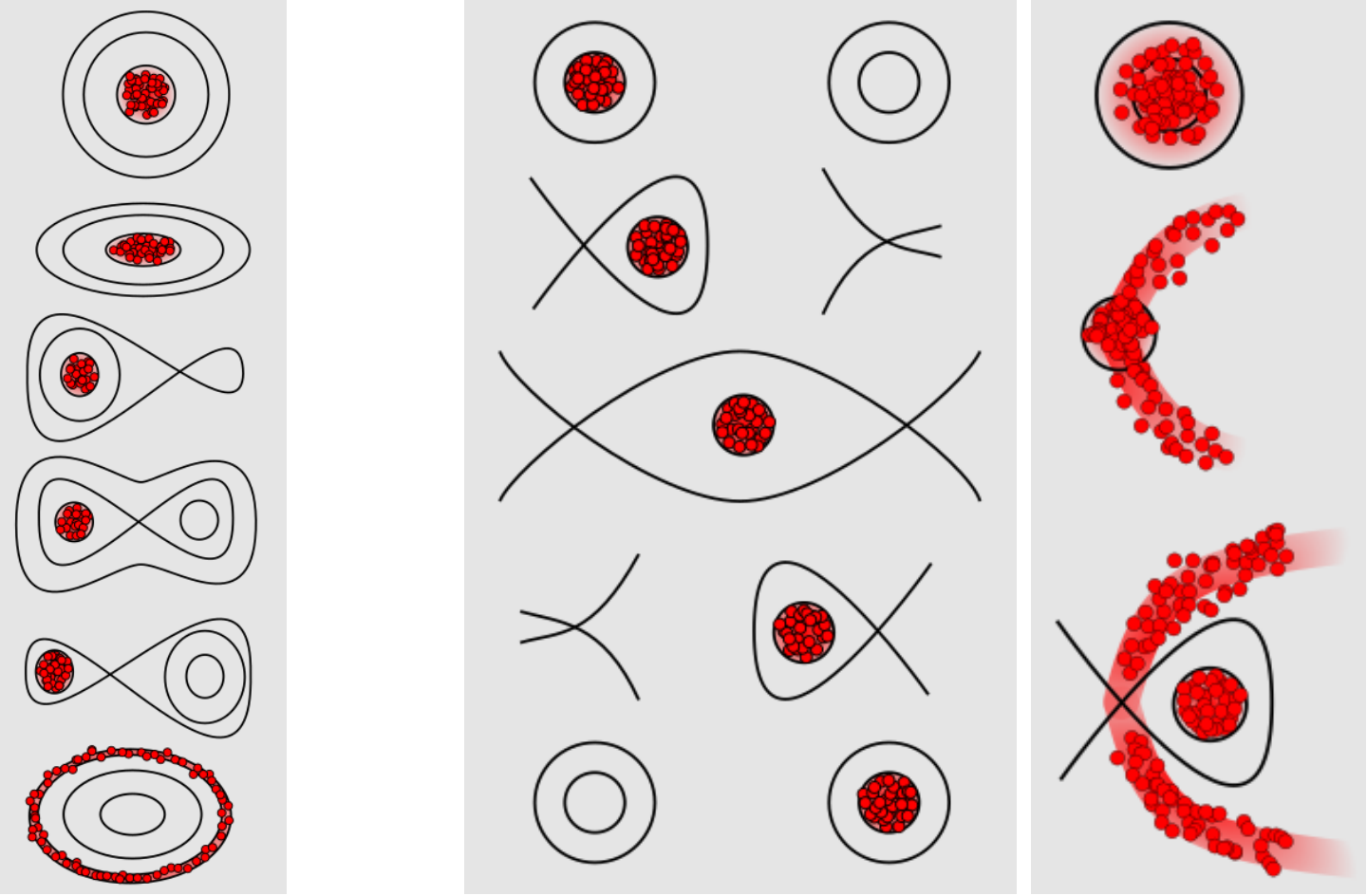
Oscillating mushrooms,
Gelfreich, Rom-Kedar, Turaev
(JPA 2014).
Figure by Paul Tupper.

Irreversibility in the quasistatic limit is expected generically for systems with mixed chaotic phase-space.

The chaotic sea can either expand or get fragmented. Islands can be swallowed by the chaotic sea; or split; or merge.

Quantum signature?

Dynamics in Phase space



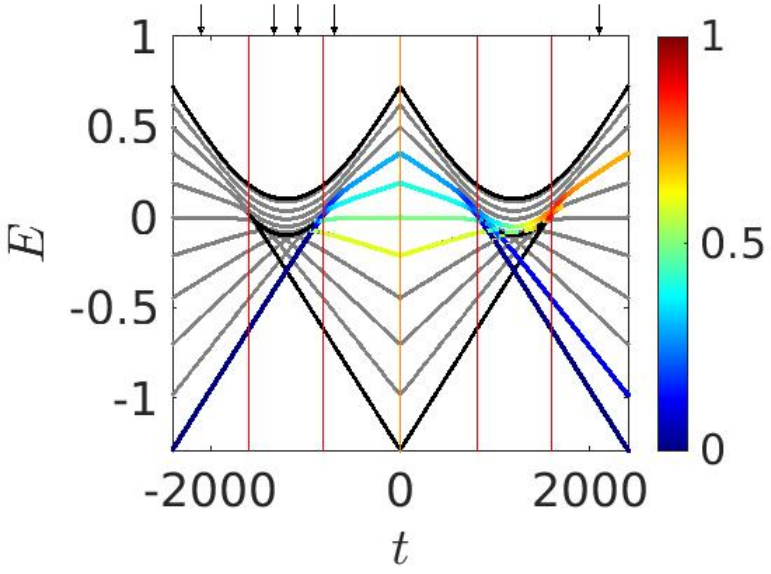
Problem of interest:
Depletion of an orbital.

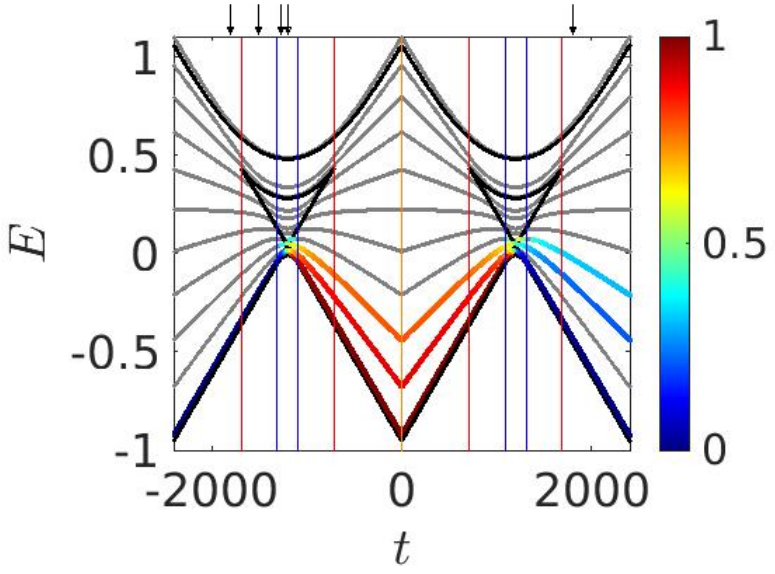
Competing mechanisms:
Relay shuttling;
Chaos-assisted depletion.

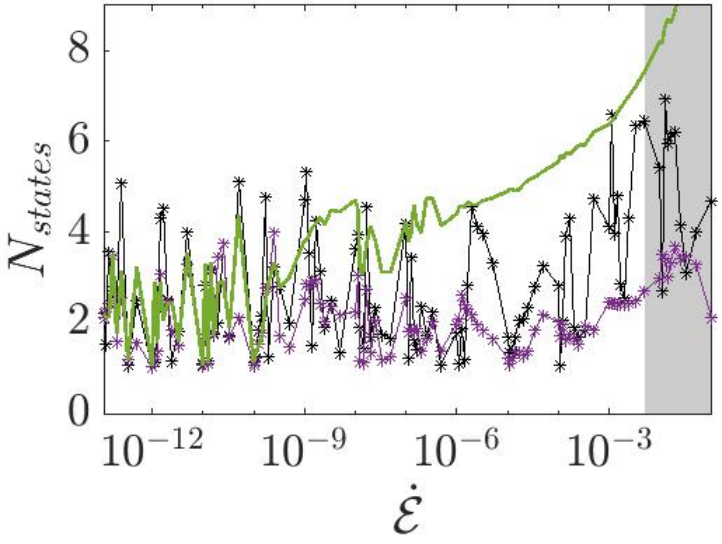
Observations:
Irreversibility;
Breakdown of QCC.

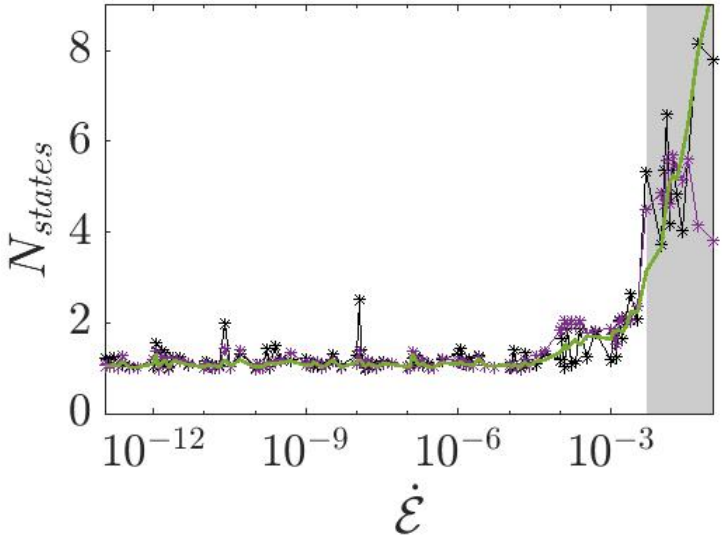
- Adiabatic shuttling (if no bifurcations are involved)
- Diabatic ejection (saddle-node bifurcation) - The nonlinear Landau-Zener transition
- Relay shuttling (pitchfork bifurcation)
- Chaos-assisted depletion (if we have a surrounding chaotic region)

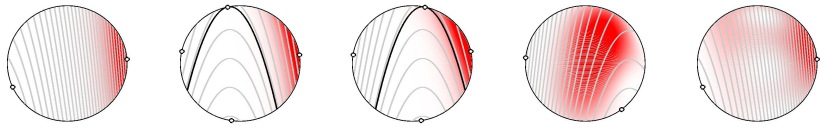
Simulations for Adiabatic Ejection and Relay Shuttling

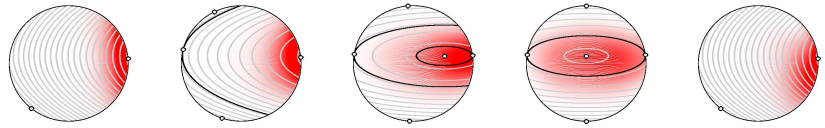








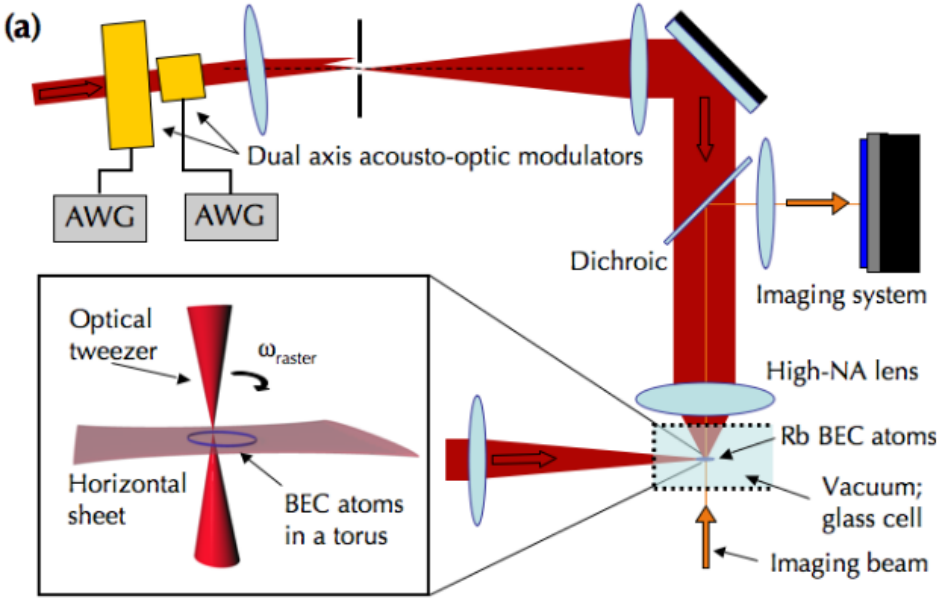




Atomtronics

Roadmap on Atomtronics: State of the art and perspective

L. Amico,[1,2,3,4,a] M. Boshier,[5] G. Birkl,[6] A. Minguzzi,[7] C. Miniatura,[4,2,8,9,10] L.-C. Kwek,[2,4,11] D. Aghamalyan,[12,2] V. Ahufinger,[13] D. Anderson,[14,15] N. Andrei,[16] A. S. Arnold,[17] M. Baker,[18] T.A. Bell,[18] T. Bland,[19,20] J.P. Brantut,[21] D. Cassettari,[22] W.J. Chetcuti,[23,1] F. Chevy,[24] R. Citro,[25] S. De Palo,[26] R. Dumke,[2,9,4] M. Edwards,[27] R. Folman,[28] J. Fortagh,[29] S. A. Gardiner,[30] B.M. Garraway,[31] G. Gauthier,[18] A. Günther,[29] T. Haug,[2] C. Hufnagel,[2] M. Keil,[28] P. Ireland,[22] M. Lebrat,[32] W. Li,[2,8] L. Longchambon,[33] J. Mompart,[13] O. Morsch,[34] P. Naldesi,[35,3] T.W. Neely,[18] M. Olshanii,[36] E. Orignac,[37] S. Pandey,[38] A. Pérez-Obiol,[39] H. Perrin,[33] L. Piroli,[40] J. Polo,[41] A.L. Pritchard,[18] N. P. Proukakis,[19] C. Rylands,[42] H. Rubinsztein-Dunlop,[18] F. Scazza,[43] S. Stringari,[44] F. Tosto,[2] A. Trombettoni,[45,46] N. Victorin,[35] W. von Klitzing,[38] D. Wilkowski,[4,2,47,9] K. Xhani,[19,48] and A. Yakimenko[49]



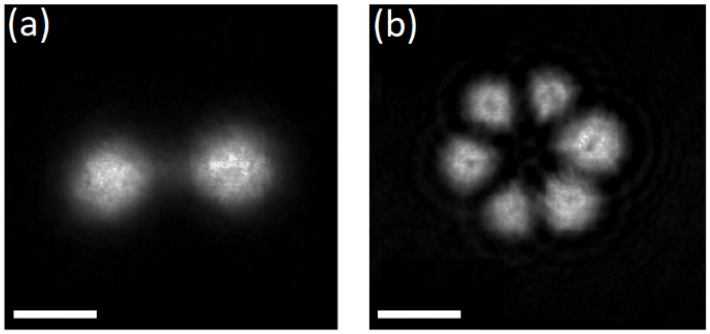


FIG. 1. Example geometries of time averaged optical dipole potentials - (a) BEC trapped in dumbbell potential, with two reservoirs connected through a channel of tunable length and width (b) Ring lattice of BECs. The scale bar on each image indicates 50 μm.

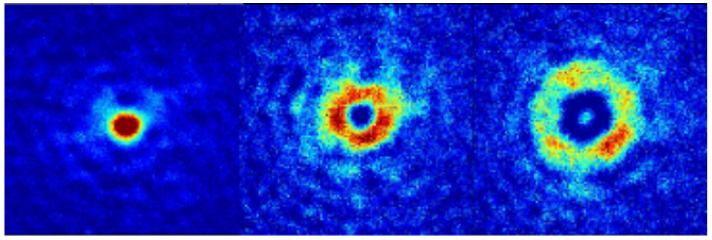


FIG. 15. Time-of-flight images of (a): a non-rotating ring, (b) and (c): rotating rings with different circulations. The rotation is imparted by a rotating 7 µm-waist blue-detuned vertical Gaussian beam.

Toroidal or Lattice rings can be "painted". Currents can be measured.

Stability condition for superflow?

Geve Arwas, DC (SREP 2015, NJP 2016, PRB 2017, PRA 2019).

The Bose Hubbard Hamiltonian

The system consists of N bosons in L sites.
Optionally we can add a gauge-field Φ.

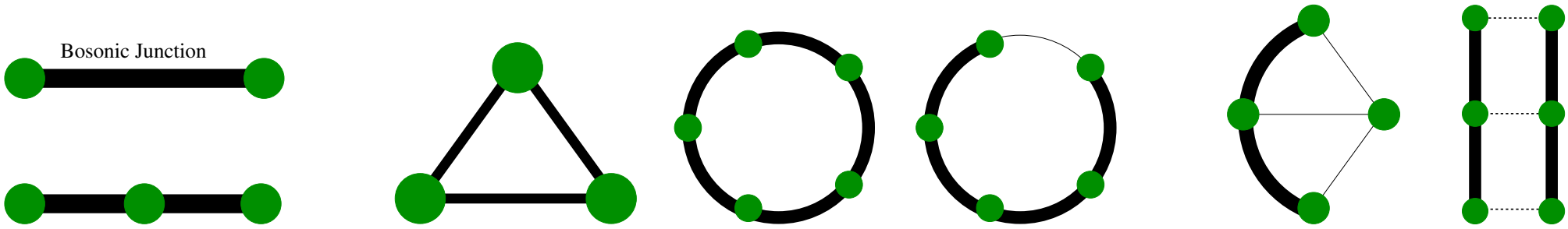
$$\mathcal{H}_{\rm BHH} = \frac{U}{2}\sum_{j=1}^{L} a_j^\dagger a_j^\dagger a_j a_j - \sum_{\langle i,j\rangle}\frac{K_{ij}}{2} a_i^\dagger a_j$$

$$u \equiv L\frac{NU}{K} \qquad \text{[classical, stability, supefluidity, self-trapping]}$$

$$\gamma \equiv \frac{LU}{NK} = \frac{u}{N^2} \qquad \text{[quantum, Mott-regime]}$$

The two dimensionless parameters have a well defined value also in the GP/continuum limit.

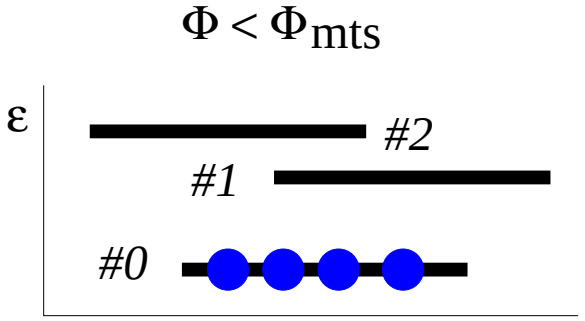
$L = 2, 3, 4, 5, 6$

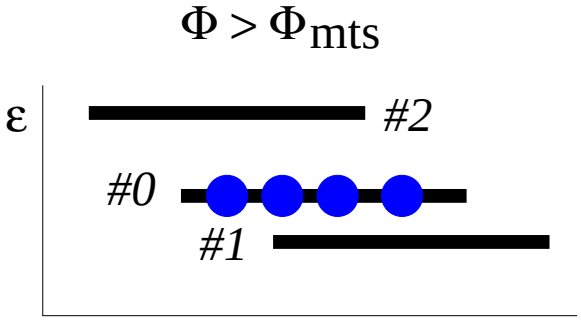


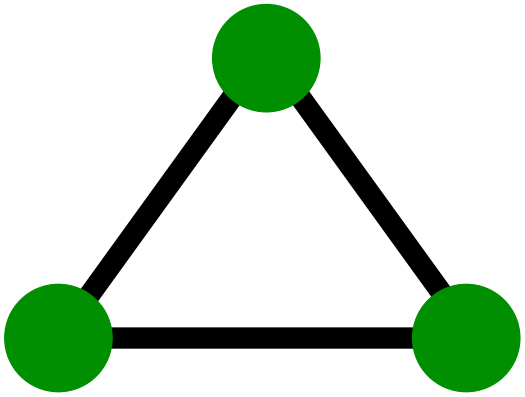
The Bose Hubbard Ring Circuit

In the rotating reference frame we have a Coriolis force,
which is like magnetic field $\mathcal{B} = 2\mathsf{m}\Omega$.
which implies an effective flux $\Phi = \text{area} \times \mathcal{B}$

$$\mathcal{H} = \sum_{j=1}^{L} \left[\frac{U}{2} a_j^\dagger a_j^\dagger a_j a_j - \frac{K}{2} \left(\mathrm{e}^{i(\Phi/L)} a_{j+1}^\dagger a_j + \mathrm{e}^{-i(\Phi/L)} a_j^\dagger a_{j+1} \right) \right] \qquad L = 3$$







$$\mathcal{H} = \sum_k \epsilon_k(\Phi)\, b_k^\dagger b_k + \frac{U}{2L} \sum_{k_4 k_3 k_2 k_1}{}' b_{k_4}^\dagger b_{k_3}^\dagger b_{k_2} b_{k_1}$$

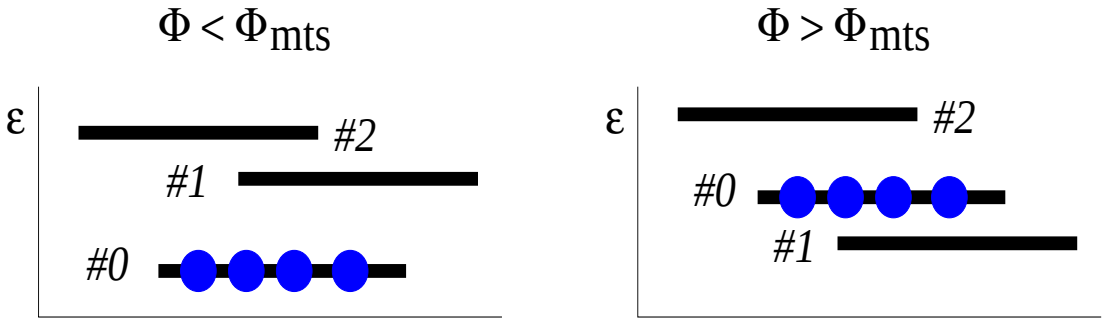
Quasistatic protocol for an atomtronic superfluid circuit

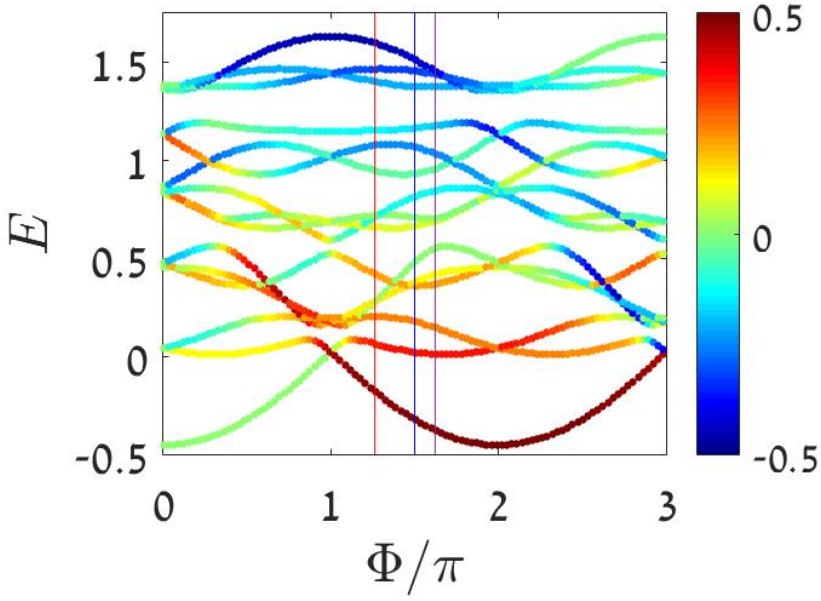
(1) All the particles are condensed into the lowest momentum orbital that has a zero winding number.
(2) The rotation frequency Φ is gradually changed from 0 to 2.5π, aka sweep process.
(3) Optional: reversed sweep back to $\Phi = 0$.
(4) The final state of the system is probed; the momentum distribution is measured.

What is the fate of the evolving many-body state?

$n = (n_1 + n_2) =$ depletion coordinate

$M = (n_1 - n_2) =$ occupation imbalance





(u=2.3, N=4, dim=15 levels).

What is the fate of the evolving many-body state?

Possible answers:

(1) Without interaction: No depletion.

(2) Quantum adiabaticity: Depletion at $\Phi_{\rm mts}$.

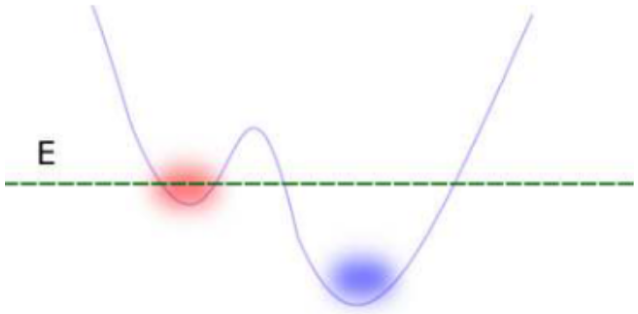
(3) Landau Criterion: Depletion at $\Phi_{\rm stb}$.

(4) Bogolyubov stability analysis: Depletion starts at $\Phi_{\rm dyn}$.

(5) Bogolyubov (integrable) approximation: Depletion at $\Phi_{\rm swp}$.

(6) Beyond Bogolyubov (mixed chaotic phase-space)... ???

(7) Quantum chaos... ???



The naive two orbital approximation implies no interaction.
Therefore the manybody Landau-Zener paradigm does not apply.

Landau criterion asks whether the the condensate is energetically metastable (local minimum of the energy landscape).

Bogolyubov approximation keeps only pair-creation events

and therefore $M = (n_1 - n_2)$ is constant of motion.

Semiclassical Hamiltonian

We set $b_k = \sqrt{n_k} e^{i\varphi_k}$, where $n_0 + n_1 + n_2 = N$.

$n = \frac{n_1+n_2}{2}$ = the depletion coordinate [Initially $n = 0$]

$M = \frac{n_1-n_2}{2}$ = the population imbalance

$$\mathcal{H}(\varphi, n; \phi, M) = \mathcal{H}^{(0)}(\varphi, n; M) + \left[\mathcal{H}^{(+)} + \mathcal{H}^{(-)}\right]$$

$$\mathcal{H}^{(0)}(\varphi, n; M) = \mathcal{E}n + \mathcal{E}_\perp M - \frac{U}{3}M^2 + \frac{2U}{3}(N-2n)\left[\frac{3}{4}n + \sqrt{n^2 - M^2}\cos(\varphi)\right]$$

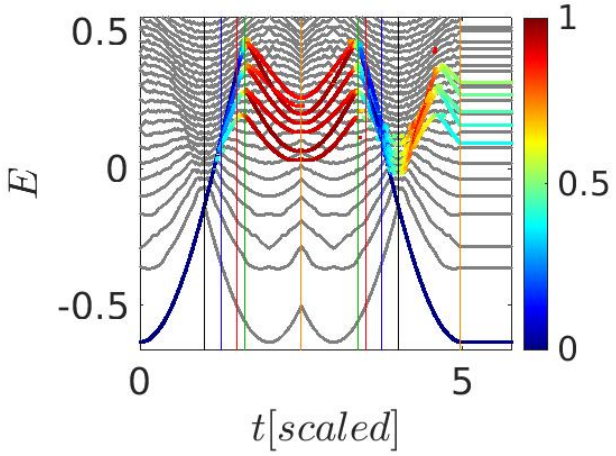
$$\mathcal{H}^{(\pm)} = \frac{2U}{3}\sqrt{(N-2n)(n\pm M)}(n\mp M)\cos\left(\frac{3\phi\mp\varphi}{2}\right)$$

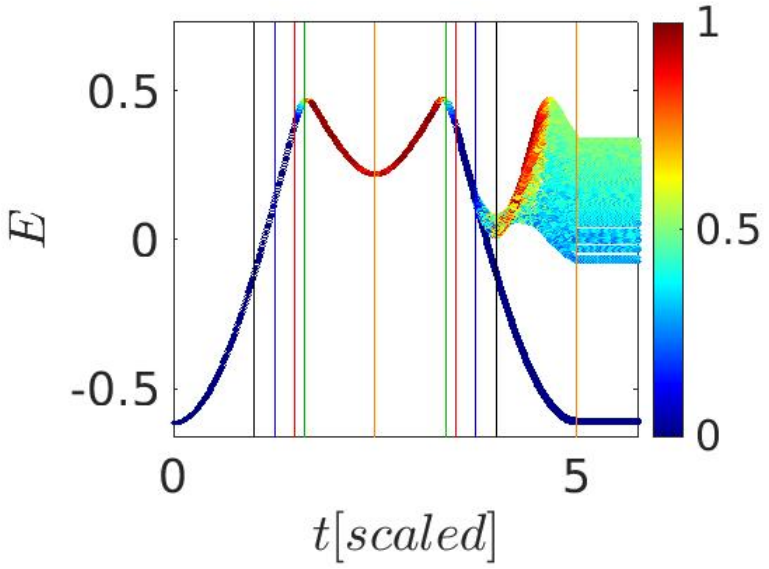
The current has the following expression in terms of (n, M):

$$I = -\frac{\partial\mathcal{H}}{\partial\Phi} = \left(n - \frac{N}{3}\right)K\sin\frac{\Phi}{3} + \frac{M}{\sqrt{3}}K\cos\frac{\Phi}{3}$$

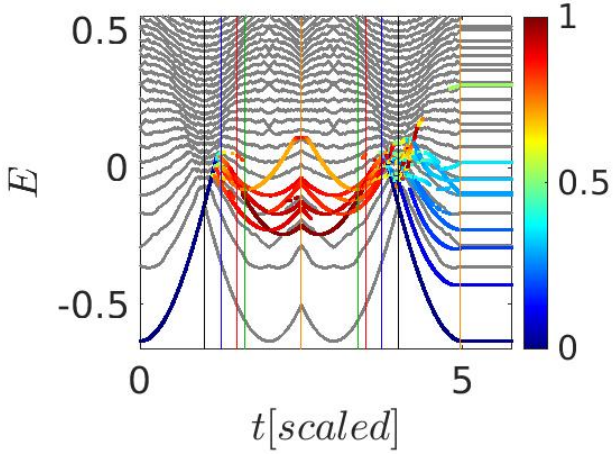
Simulations

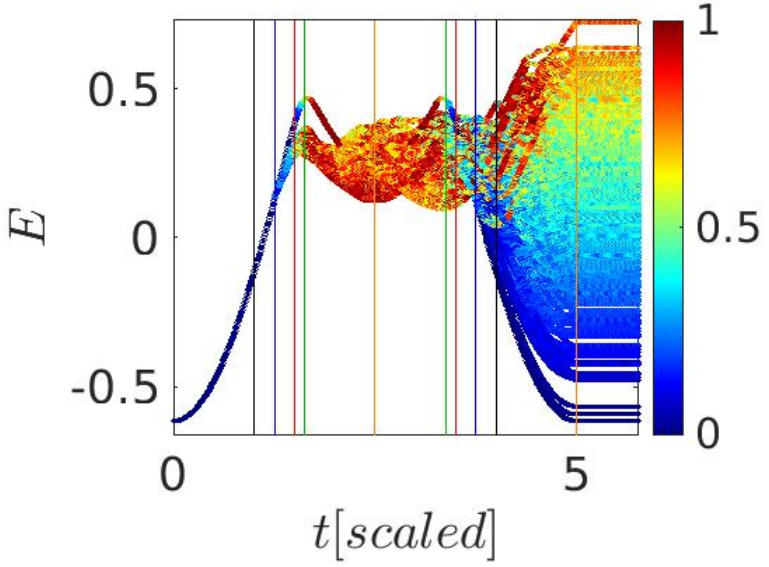
Slow:





Very Slow:





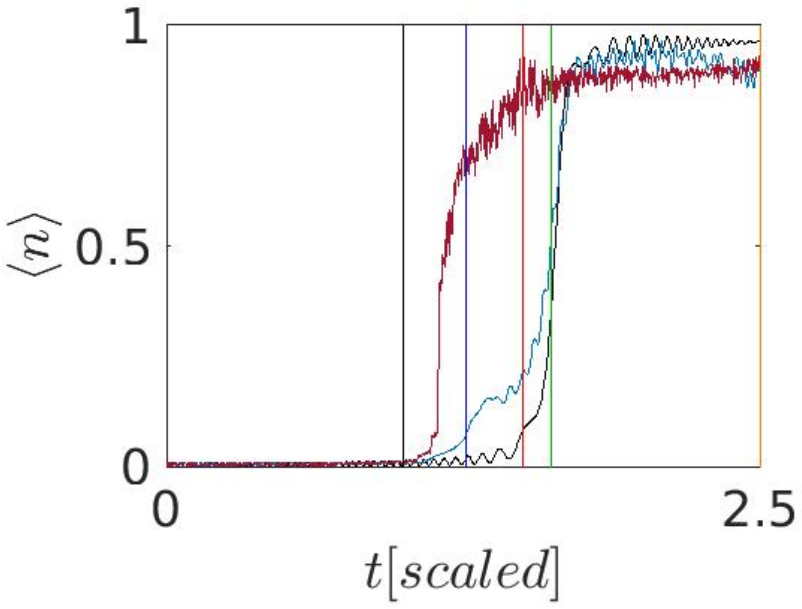
- Thresholds:

$(\Phi_{\mathrm{mts}}, \Phi_{\mathrm{stb}}, \Phi_{\mathrm{dyn}}, \Phi_{\mathrm{swp}})$

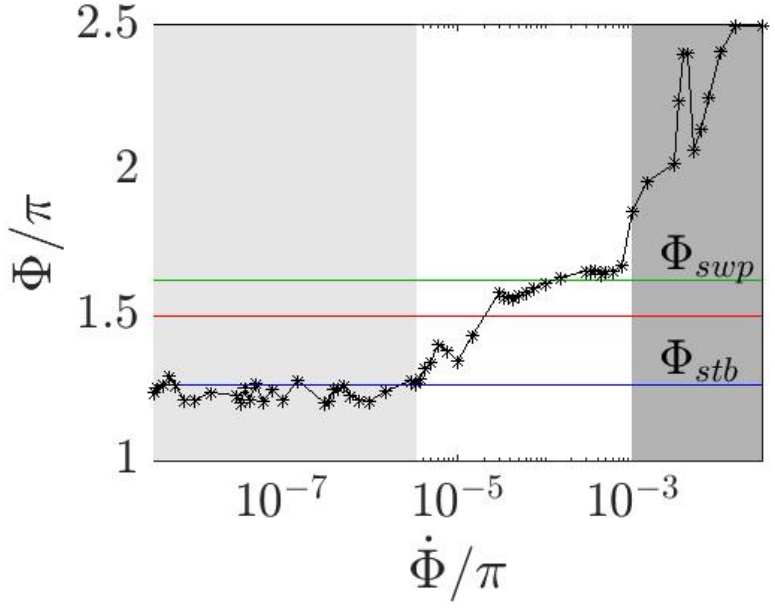
- Branching
- Depletion time
- UQF
- Irreversibility?
- QCC?

Time of Depletion vs Sweep Rate

Depletion versus time:



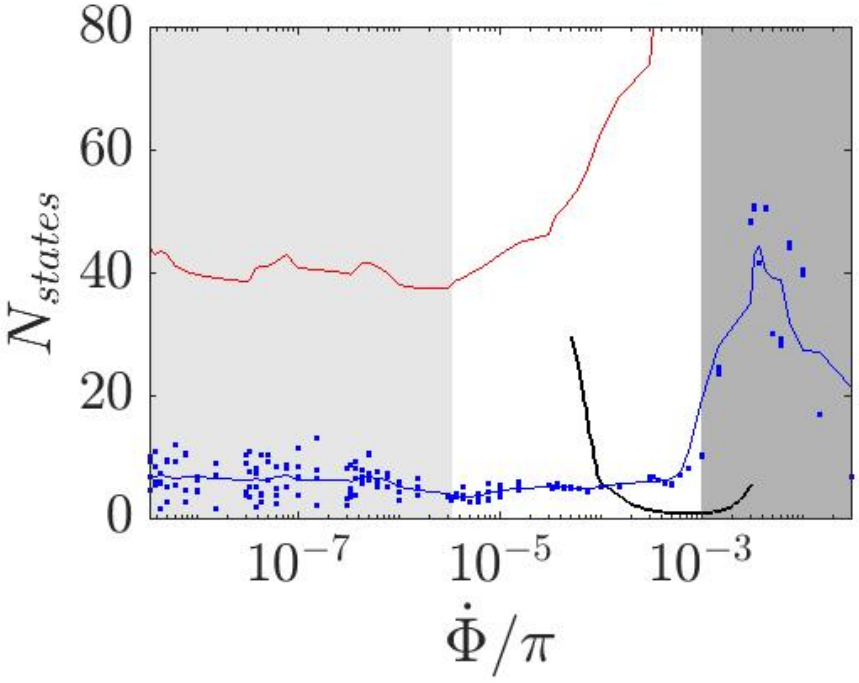
Φ at the time of depletion:



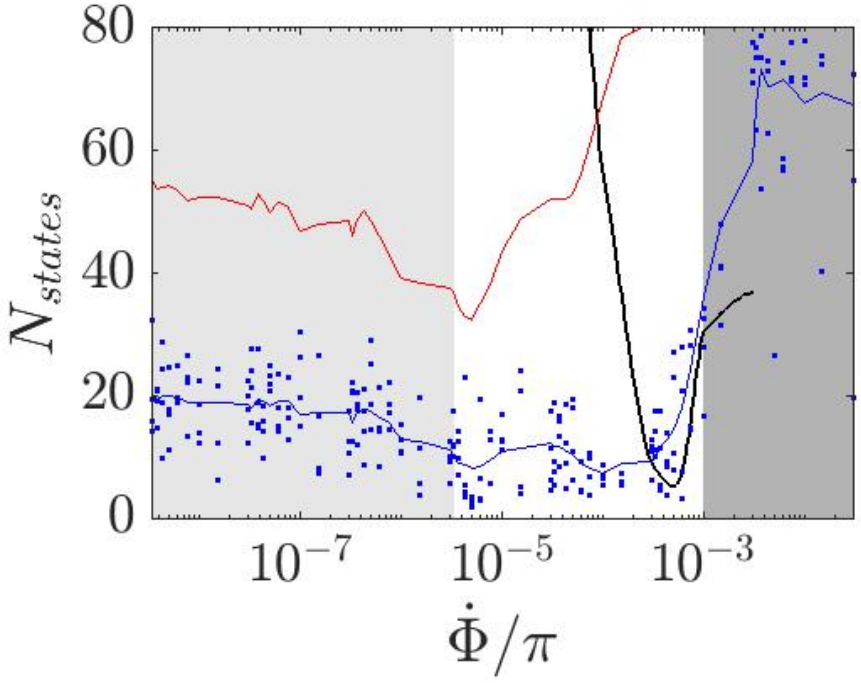
- Slow sweep: depletion at Φ_{dyn}, indicating Relay-Shuttling.
- Very Slow sweep: depletion at Φ_{stb}, indicating Chaos-Assisted mechanism.
- Note agreement of Relay-Shuttling with the Bogolyubov approximation.

Irreversibility vs Sweep Rate

Spreading after the forward sweep:

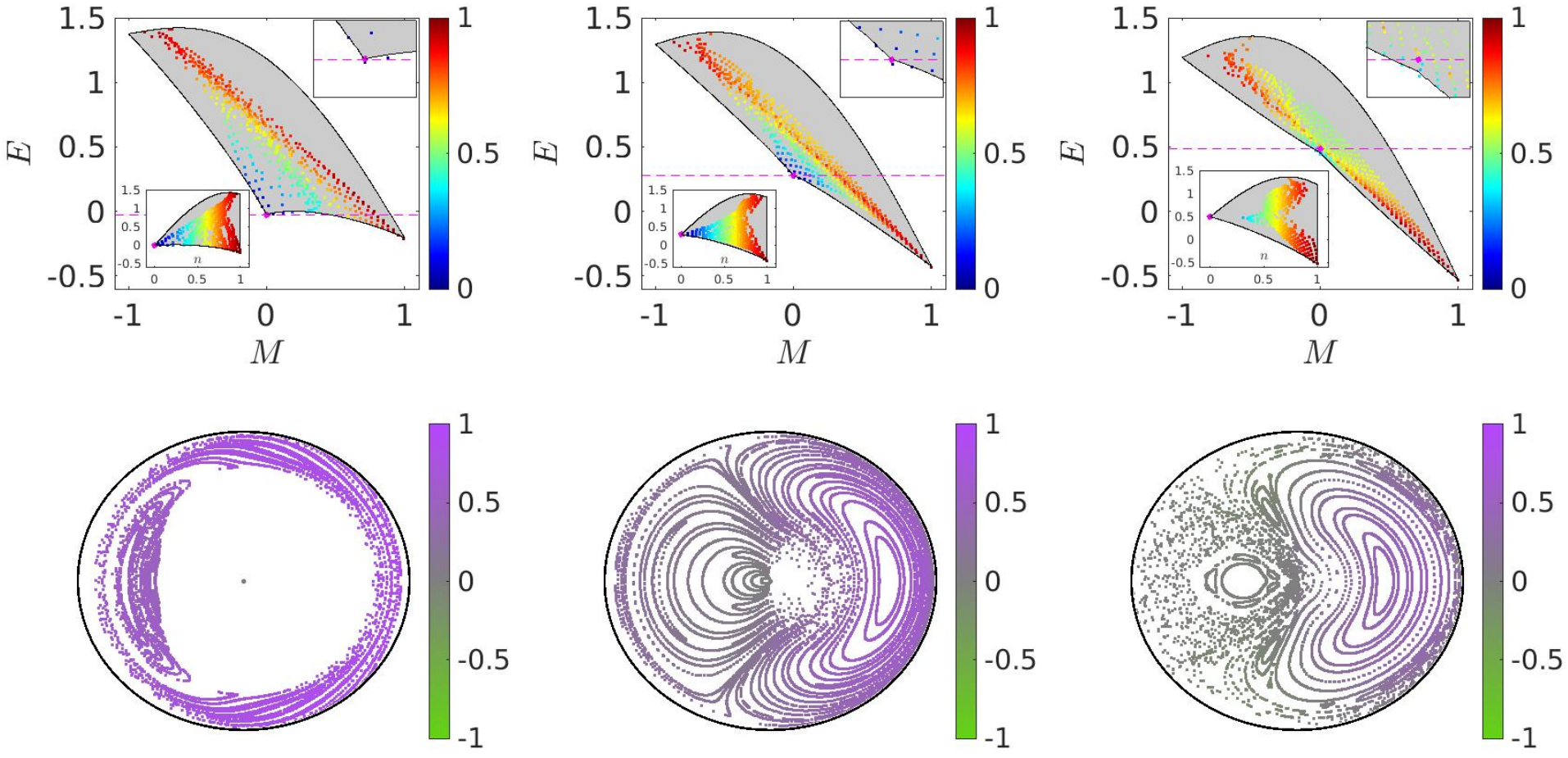


Spreading after the reversed sweep:



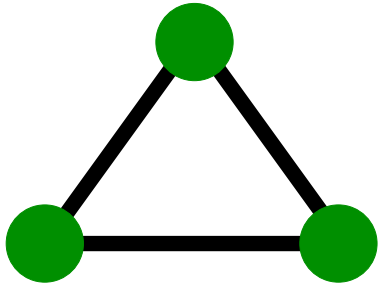
- Optimal sweep rate in the semiclassical simulations.
- Universal Quantum Fluctuations (UQF) in the chaos-assisted-depletion regime.
- Breakdown of Quantum to Classical Correspondence (QCC).

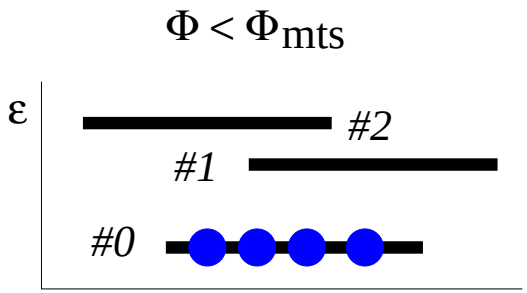
Phase space structure

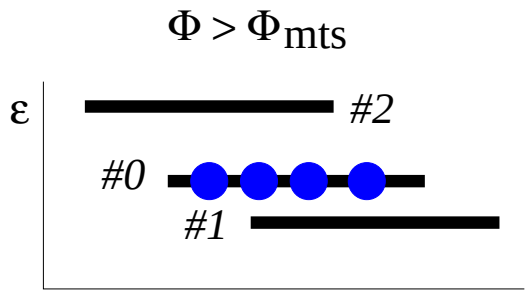


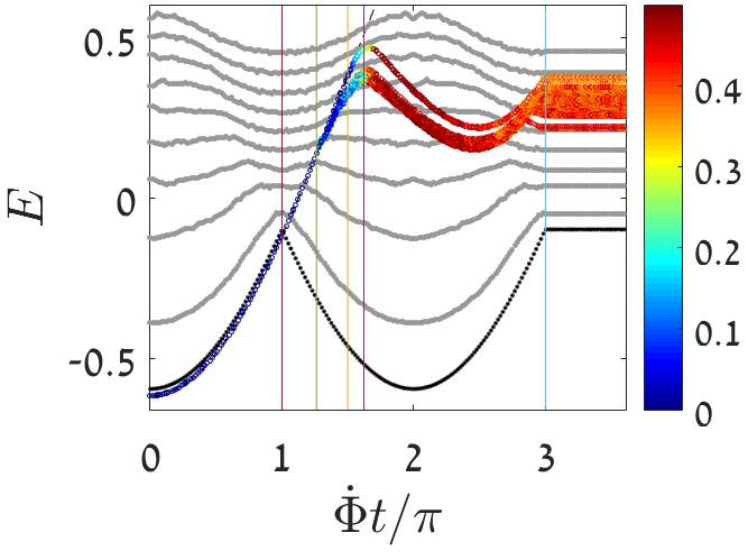
Thresholds: $(\Phi_{\text{mts}}, \Phi_{\text{stb}}, \Phi_{\text{dyn}}, \Phi_{\text{swp}})$

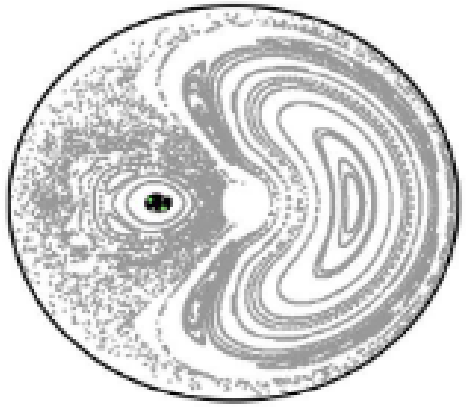
Chaos-assisted depletion

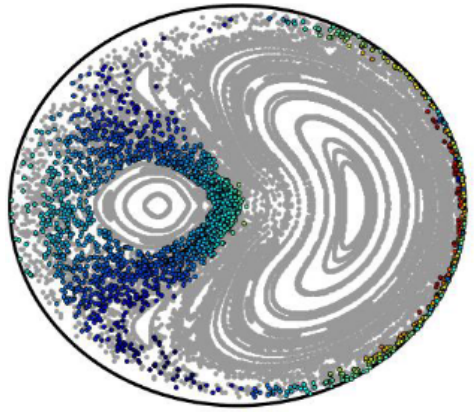












Optimal sweep: The cloud is shuttled by a fixed-point (PO) that has bifurcated from the center.

Slow sweep: The cloud spreads through the corridor, and after that shuttled by an outer torus.

Thresholds

The central SP is the global minimum of the energy landscape up to

$\Phi_{\mathrm{mts}} = \pi$

The SP is still a local minimum up to

$\Phi_{\mathrm{stb}} = 3\arccos\left(\frac{1}{6}\left(\sqrt{u^2+9}-u\right)\right)$

The SP becomes an unstable saddle at

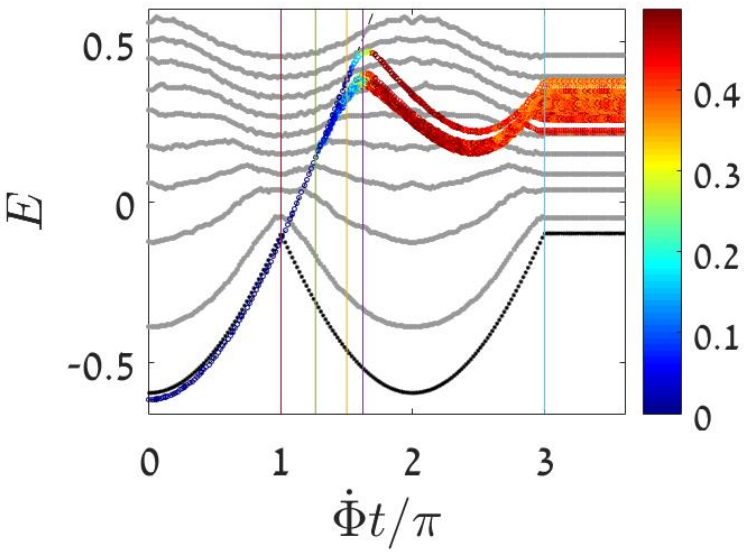
$\Phi_{\mathrm{dyn}} = \frac{3}{2}\pi$

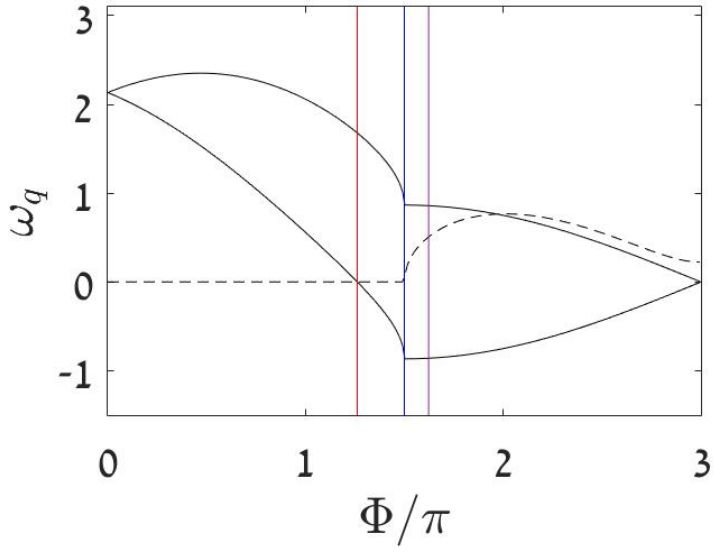
The SP becomes connected to the periphery at the swap transition

$\Phi_{\mathrm{swp}} = 3\arccos\left(-\frac{1}{18}u\right)$

The Bogolyubov frequencies:

$\omega_{\pm} = \pm\frac{\sqrt{3}}{2}\sin\frac{\Phi}{3} + \sqrt{\left(\frac{3}{2}\cos\frac{\Phi}{3}\right)^2 + u\cos\frac{\Phi}{3}}$

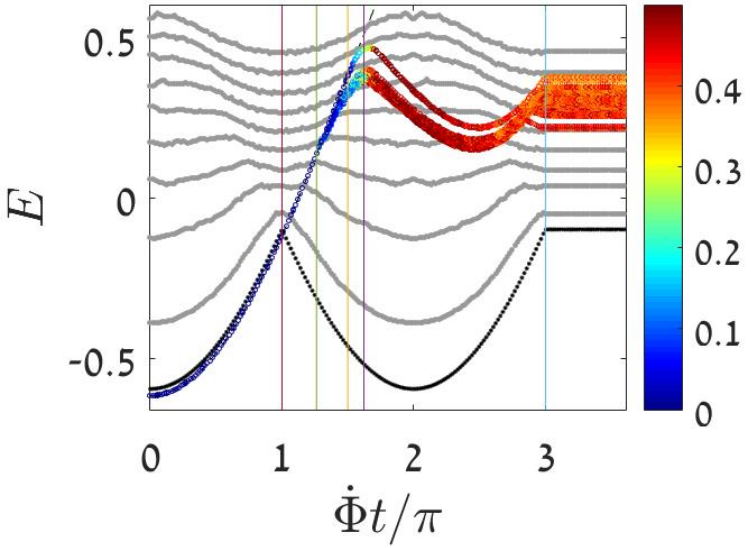


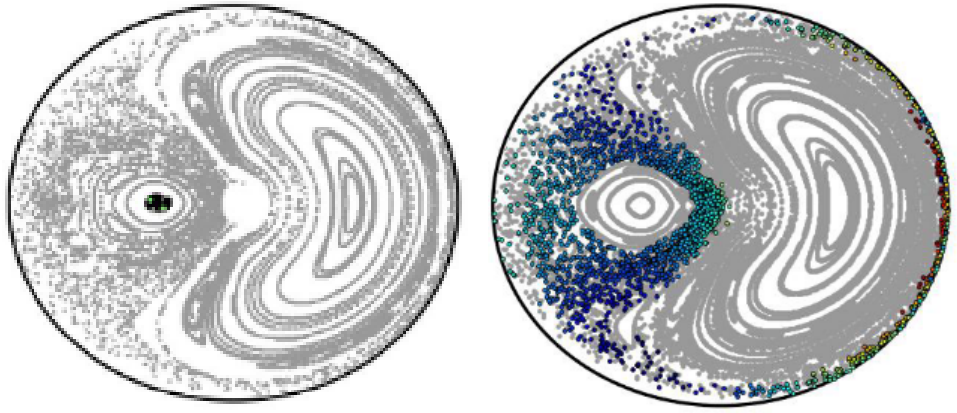


$\mathcal{H} \approx E_0 + \sum_q \omega_q c_q^\dagger c_q$

Main messages I

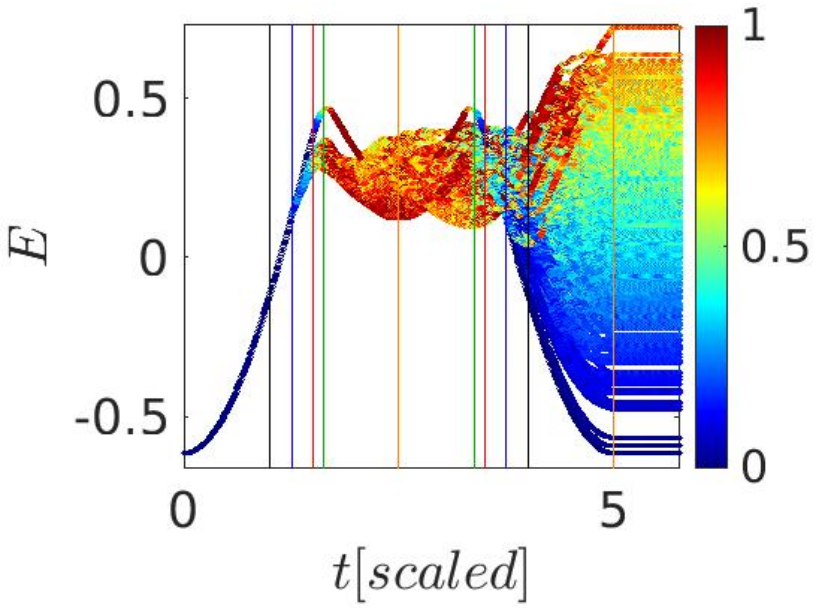
- A quasi-static protocol is in general not adiabatic,
  and hence not reversible, due to mixed-chaotic dynamics.
- It is implied that slowness is bad for adiabaticity.
- We have considered a protocol whose aim is to transfer condensed particles
  from a source orbital to a target orbital.
- Two competing mechanisms: adiabatic shuttling versus chaos-assisted depletion.
- The irreversible chaos-assisted depletion mechanism dominates in the quasi-static limit.
- An implied optimal sweep rate for the performance of the transfer protocol.

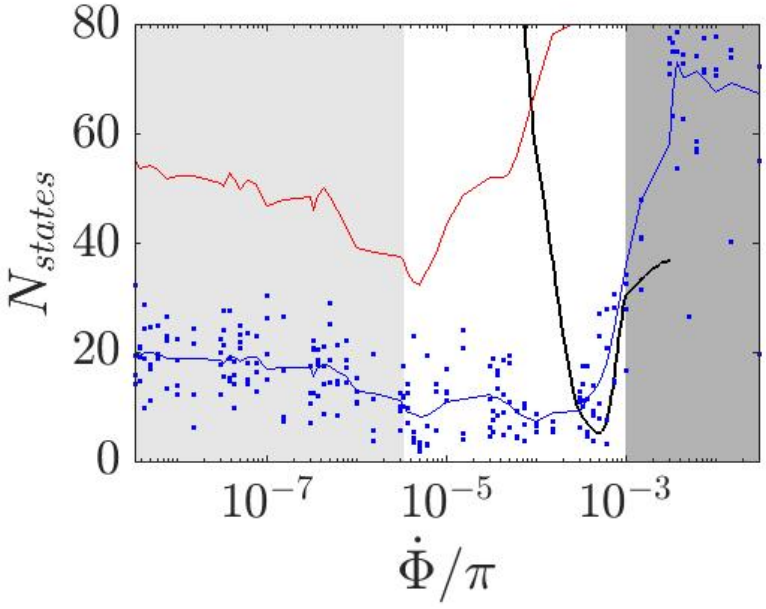




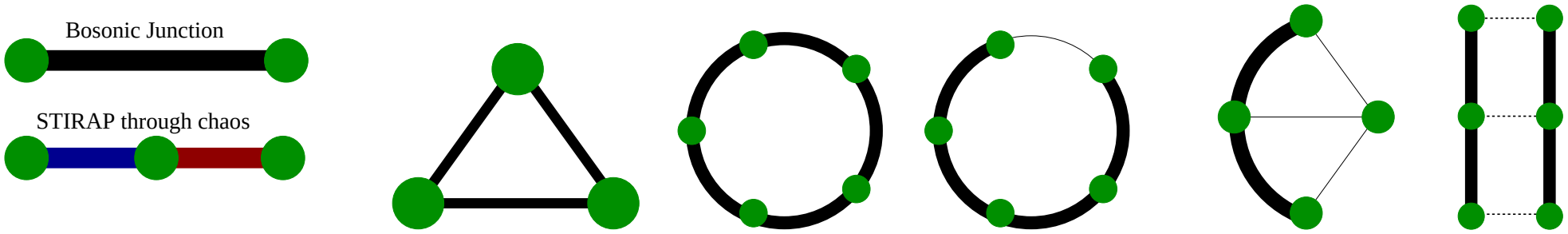
Main messages II

- Semiclassical fingerprints in the Quantum dynamics.
- Universal Quantum Fluctuations (UQF) in chaos-assisted-depletion regime.
- Breakdown of Quantum to Classical Correspondence (QCC).
- Limitations of the two-orbital approximation.
- Limitations of the Bogolyubov approximation.
- Limitations of the Semiclassical approximation.





Credits / related studies



Dimer (L=2): Bosonic Josephson junction; Pendulum physics [1a].
Driven dimer: Landau-Zener dynamics [1b]; Kapitza effect [1c]; Zeno effect [1d]; Scars [1e].
Rings (L>2): Superfluidity [2a]; SF-Mott transition [2b].
Driven trimer: Many body STIRAP [3a]; Hamiltonian Hysteresis [3b]; Quasistatic protocols [3c].
Coupled subsystems (L>3): Minimal model for Thermalization [4a,4b].

[1a] Chuchem, Smith-Mannschott, Hiller, Kottos, Vardi, DC (PRA 2010).
[1b] Smith-Mannschott, Chuchem, Hiller, Kottos, DC (PRL 2009).
[1c] Boukobza, Moore, DC, Vardi (PRL 2010).
[1d] Khripkov, Vardi, DC (PRA 2012)
[1e] Khripkov, DC, Vardi (JPA 2013, PRE 2013).
[2a] Arwas, DC (SREP 2015, NJP 2016, PRB 2017, PRA 2019).
[2b] Arwas, DC, Hekking, Minguzzi (PRA 2017).
[3a] Dey, DC, Vardi (PRL 2018, PRA 2019).
[3b] Burkle, Vardi, DC, Anglin (PRL 2019).
[3c] Winsten, DC (SREP 2021, arXiv 2022).
[4a] Tikhonenkov, Vardi, Anglin, DC, (PRL 2013).
[4b] Khripkov, Vardi, DC (NJP 2015, PRE 2018, PRA 2020).