Some open problems in Quantum Stirring [1,2]

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$GIF, $ISF

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[1] Gilad Rosenberg, DC [JPA 2006]

[2] DC, Tsampikos Kottos, Holger Schanz [PRE(R) 2005]

Adiabatic transport and beyond:
Thouless (1983); Berry, Robbins (JPA 1993) - Geometric magnetism;
Avron, Sadun, Raveh, Zur (1988) - networks driven by fluxes;
Wilkinson (1988-1995) - Kubo for quantized chaotic systems?;
DC (1999-2003) - Generalized Kubo approach to analyze closed driven systems.

Open systems, S matrix formalism:
The Buttiker Pretre Thomas [BPT] formula (1994); Brouwer (1998);
Avron, Elgart, Graf, Sadun - ... The snow plow pump;
Shutenko, Aleiner, Altshuler (PRB 2000) - quantization?;
Levinson, Entin-Wohlman, Wolfle (2000) - The double barrier pump.

Pumping / Stirring in closed systems:
DC (2002-2003) - The Kubo approach + the double barrier pump;
DC (PRB-Rapid, 2003) - from closed to open systems;
Moskalets, Buttiker (PRB-Rapid, 2003); Sela, DC (JPA 2006) - the double barrier pump;
DC, Kottos, Schanz (PRE-Rapid, 2005); Rosenberg, DC (JPA 2006) - the snow plow pump, stirring;
Aunola, Toppari (2003); Mottonen, Pekola, Vartiainen, Brosco, Hekking (2006) - Cooper pair pumping.

Driven Systems

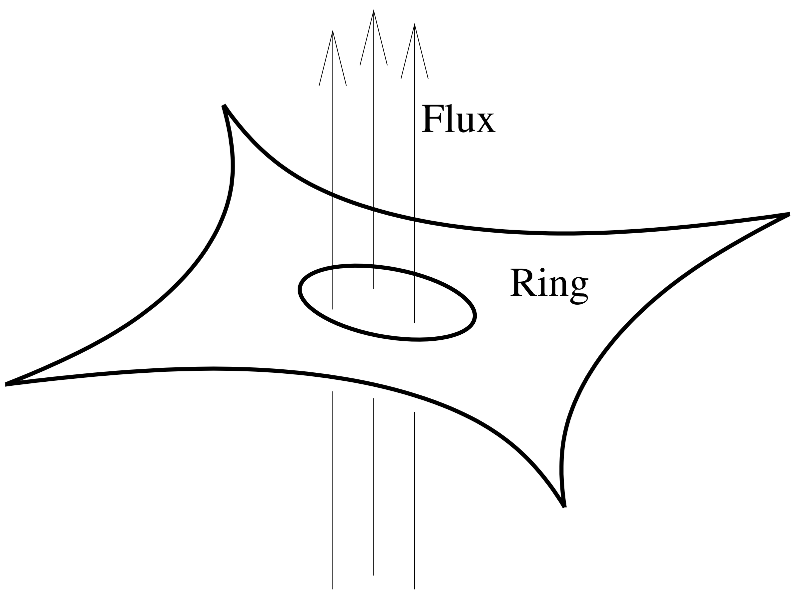
Non interacting “spinless” electrons.

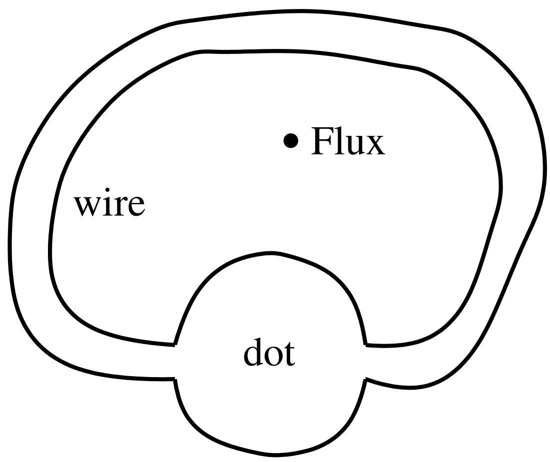
Held by a potential (e.g. AB ring geometry).

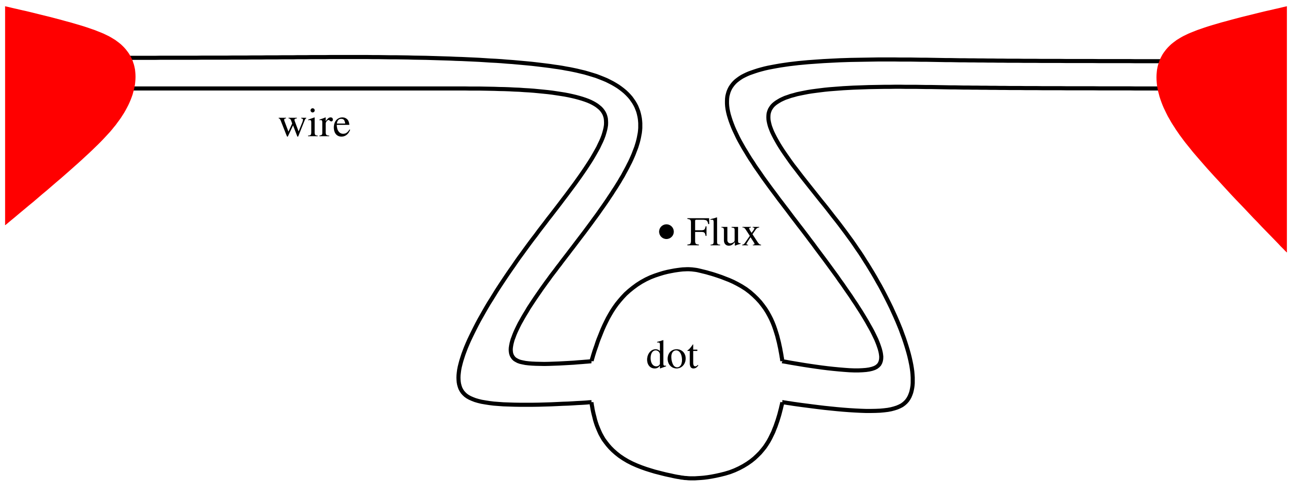
$$\mathcal{H} = \mathcal{H}(\boldsymbol{r}, \boldsymbol{p};\ X_1(t), X_2(t), X_3(t))$$

$X_1,\ X_2$ = shape parameters

$X_3 = \Phi = (\hbar/e)\phi$ = magnetic flux







"Ohm law"

For one parameter driving by EMF

$$\begin{aligned} I &= \mathcal{G}^{33} \times (-\dot{X}_3) \\ dQ &= -\mathcal{G}^{33}\, dX_3 \end{aligned}$$

For driving by changing another parameter

$$\begin{aligned} I &= -\mathcal{G}^{31}\, \dot{X}_1 \\ dQ &= -\mathcal{G}^{31}\, dX_1 \end{aligned}$$

For two parameter driving

$$\begin{aligned} I &= -\mathcal{G}^{31}\dot{X}_1 - \mathcal{G}^{32}\dot{X}_2 \\ dQ &= -\mathcal{G}^{31}\, dX_1 - \mathcal{G}^{32}\, dX_2 \\ Q &= -\oint \boldsymbol{G} \cdot dX \end{aligned}$$

and in general

$$\langle F^k \rangle = -\sum_j \mathcal{G}^{kj}\, \dot{X}_j$$

The Kubo formula approach [DC, PRB 2003]

$$\langle \mathcal{I} \rangle = -\boldsymbol{G}\dot{X}$$

$$\boldsymbol{G} = \int_0^\infty K(t)\,t\,dt$$

$$\boldsymbol{G} = \varrho(E_F) \int_0^\infty C(t)\,dt$$

$$\boldsymbol{G} = \sum_n f(E_n) \sum_{m(\neq n)} \frac{2\hbar \mathrm{Im}\,[\mathcal{I}_{nm}\mathcal{F}_{mn}]}{(E_m - E_n)^2 + (\Gamma/2)^2}$$

$$\boldsymbol{G} = \frac{e}{2\pi i} \mathrm{trace}\left(P_{\mathrm{A}} \frac{\partial S}{\partial x_j} S^{\dagger} \right)$$

$$K(t) = \frac{i}{\hbar} \langle [\mathcal{I}(t), \mathcal{F}(0)] \rangle_{\mathrm{F}}$$

$$C(\tau) = \frac{1}{2} (\langle \mathcal{I}(t)\mathcal{F}(0) \rangle_{\mathrm{E}} + cc)$$

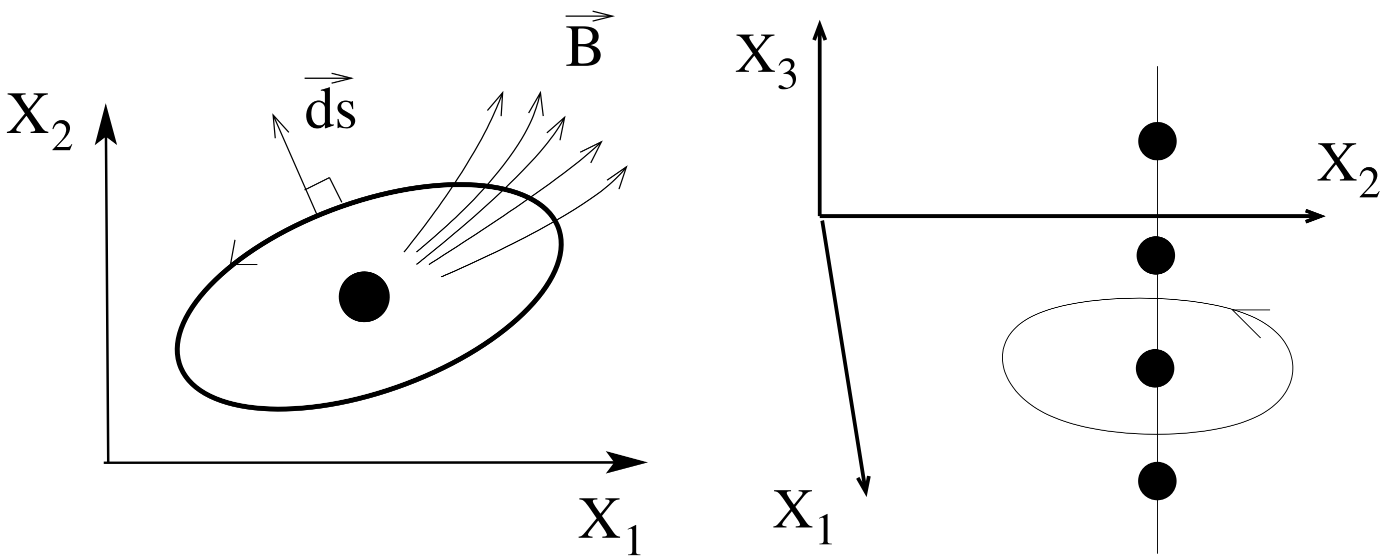
$$\mathcal{F} = -\frac{\partial \mathcal{H}}{\partial X}$$

How do we calculate Q

$$Q = \oint_{\text{cycle}} I dt = -\oint (G^{31} dX_1 + G^{32} dX_2)$$

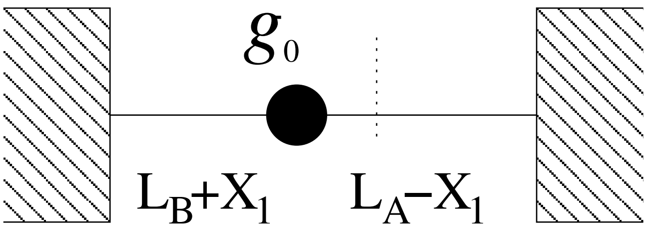
$$\begin{aligned} \vec{ds} &= (dX_2, -dX_1, 0) \\ \vec{\boldsymbol{B}} &= (-G^{32}, G^{31}, 0) \end{aligned}$$

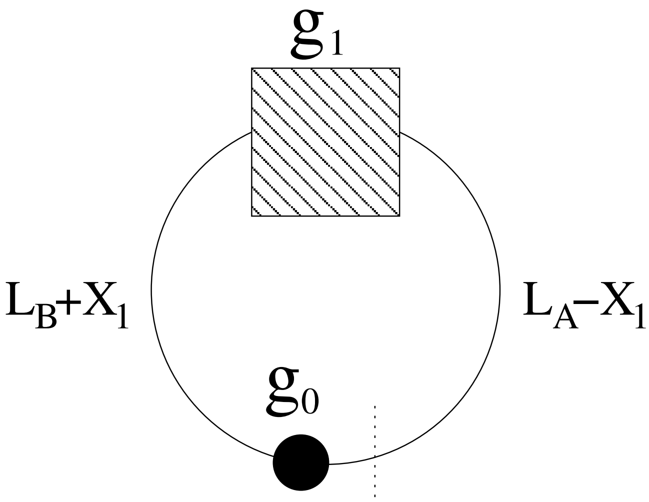
$$Q = \oint \boldsymbol{B} \cdot \vec{ds} = \iint \sigma(X_1, X_2) dX_1 dX_2$$

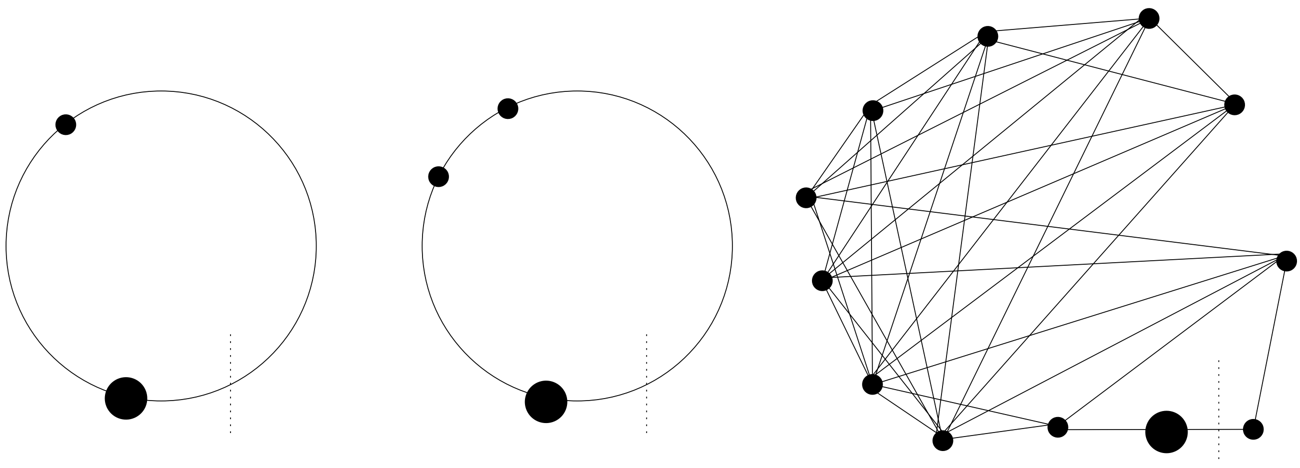


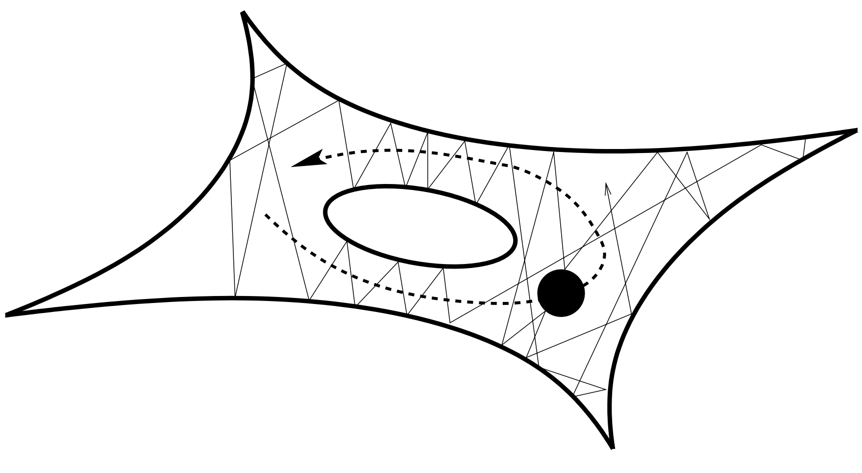
Note: $\text{BerryPhase} = \oint \boldsymbol{A} \cdot d\vec{X}$

Quantum stirring of particles in closed devices

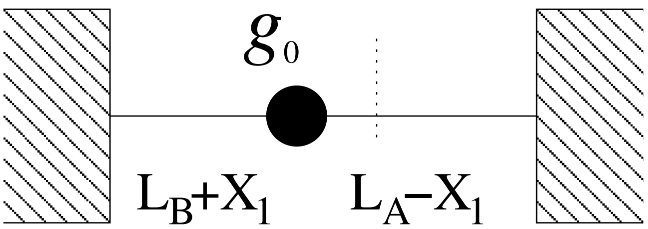




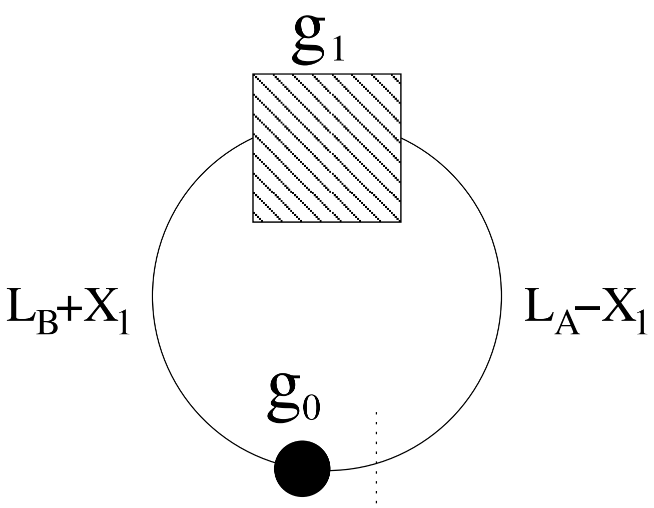




Stirring - "classical" reasoning

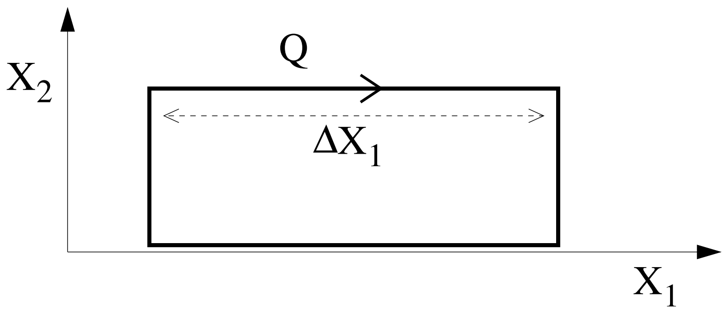


$$dQ = (1 - g_0)\,\frac{e}{\pi} k_F\, dX$$

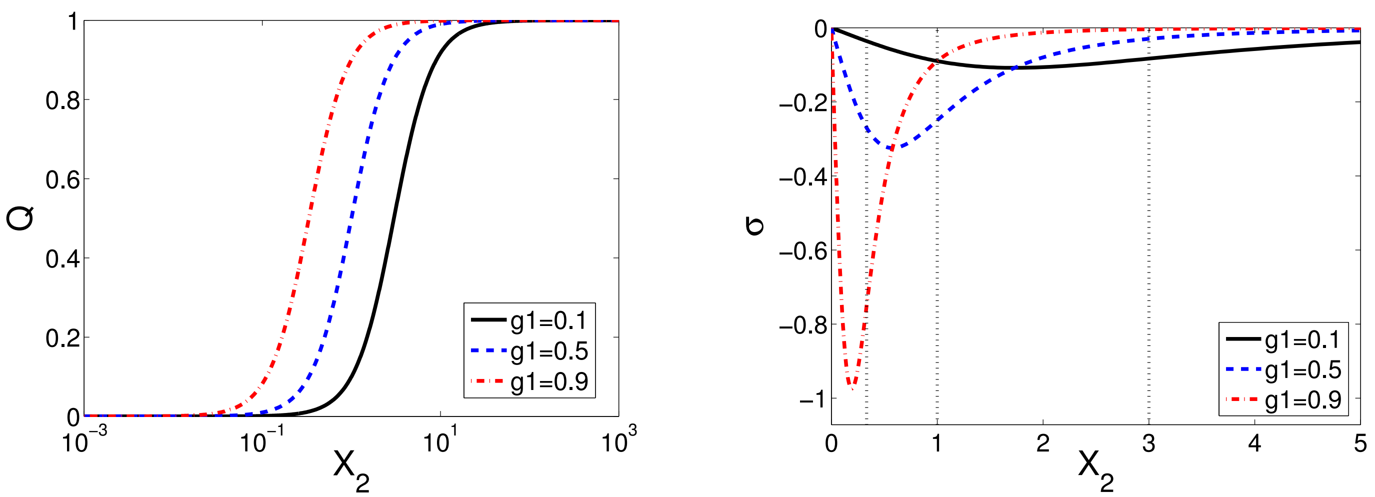


$$dQ = \left[\frac{(1-g_0)g_1^{cl}}{g_0 + g_1^{cl} - 2g_0 g_1^{cl}}\right] \frac{e}{\pi} k_F\, dX$$

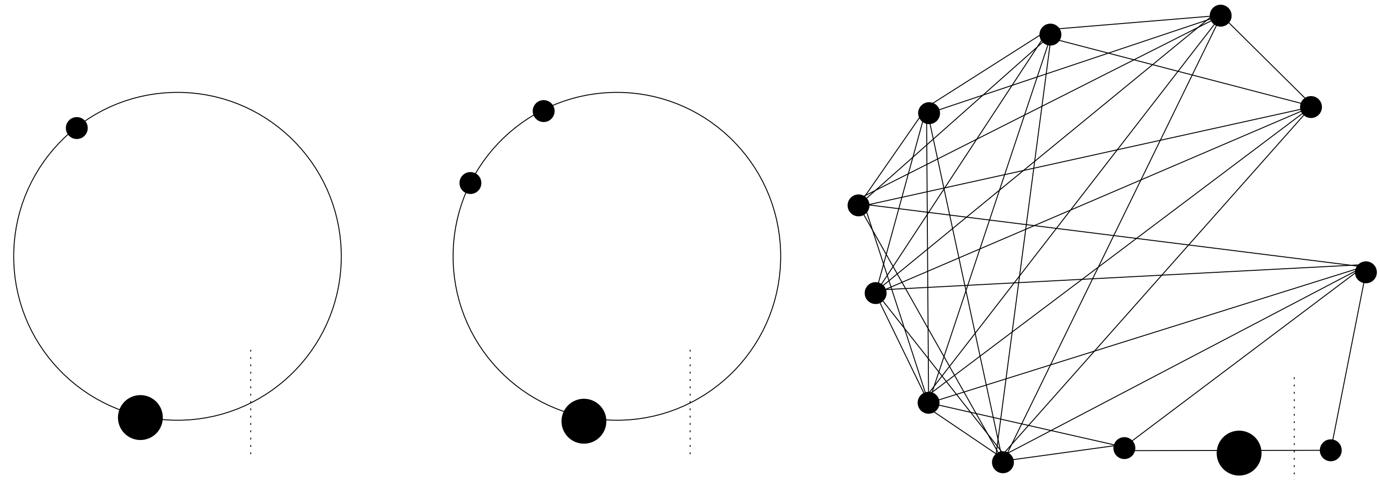
The "classical" distribution of Dirac chains

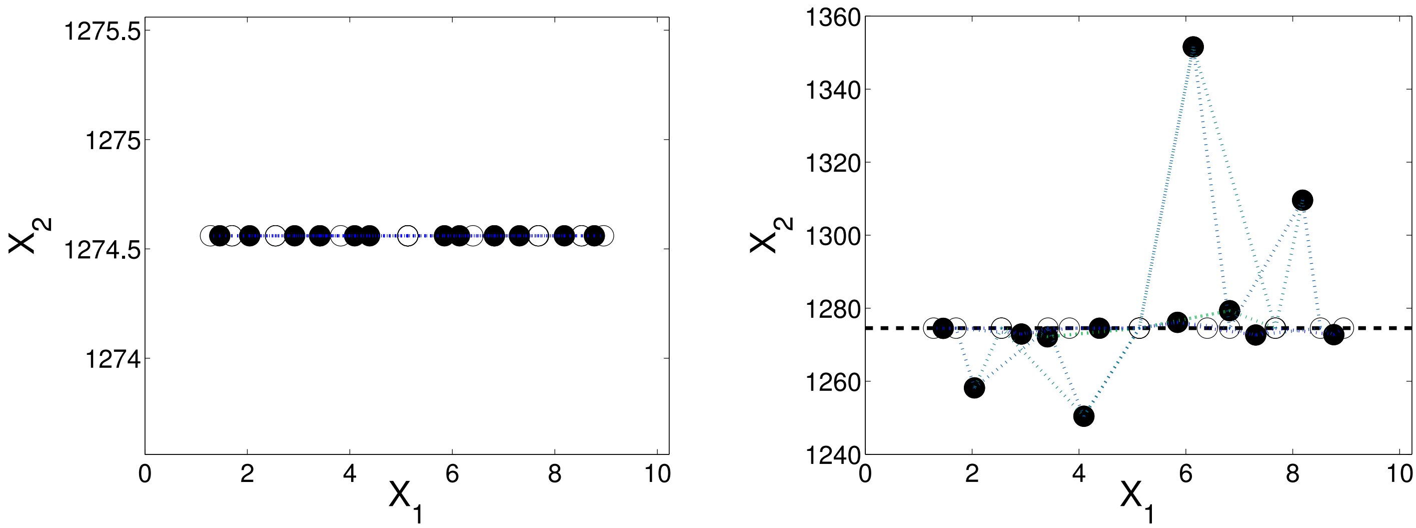


$$\sigma(X_1, X_2) = -\frac{e\mathrm{m}}{\pi\hbar^2}\frac{2(1-g_1^{cl})g_1^{cl}}{\left[1+\left(\left(\frac{\mathrm{m}}{\hbar^2 k_{\mathrm{F}}}X_2\right)^2-1\right)g_1^{cl}\right]^2}\left(\frac{\mathrm{m}}{\hbar^2 k_{\mathrm{F}}}X_2\right)$$



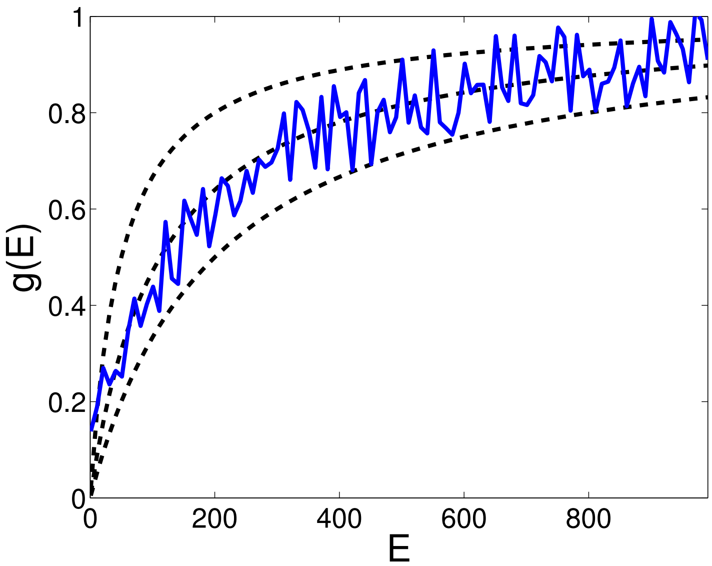
The quantum distribution of Dirac chains





$$Q \approx e\frac{\Delta X_1}{\lambda_E/2} = e\frac{k_E}{\pi}\times \Delta X_1$$

RMT analysis?



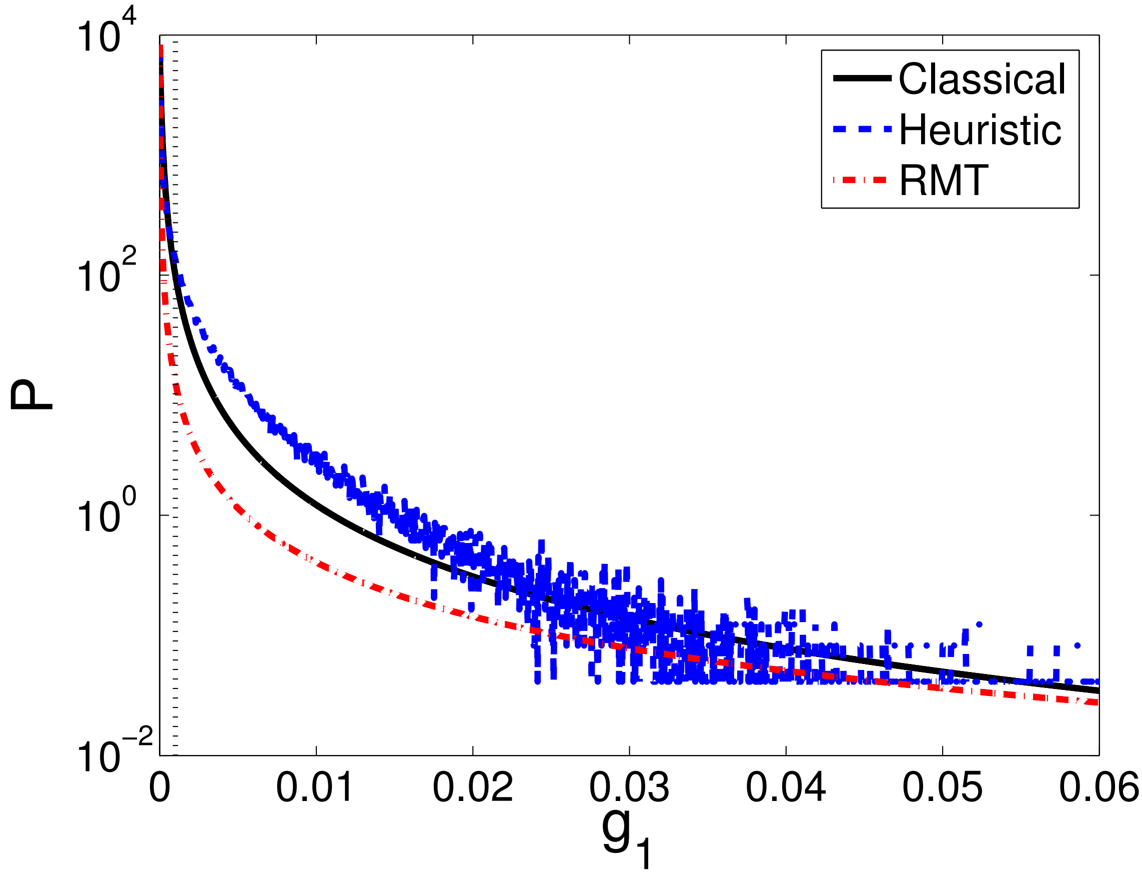
$$g_0(E;X_2) = \left[1 + \left(\frac{1}{\hbar v_{\mathrm{E}}} X_2\right)^2\right]^{-1}$$

$$\mathrm{Prob}\left[\text{degeneracy at } X_2\right] \;=\; \mathrm{Prob}\left[g_1 = g_0(X_2)\right]$$

Thus $\sigma(X_1, X_2)$ is related to $P(g_1; \bar{g}_1)$

$$\sigma(X_1, X_2) \;=\; \text{const} \times \frac{dg_0(X_2)}{dX_2}\, P(g_0(X_2); \bar{g}_1)$$

RMT?



$$\bar{g}_1 = 0.001$$

$$P_{\text{heuristic}}(g_1;\bar{g}_1) = \text{Prob}\left[\bar{g}_1 \eta_1^{\text{PT}} \eta_2^{\text{PT}} = g_1\right]$$

PT = Porter Thomas

$$P_{\text{RMT}}(g_1;\bar{g}_1) = \begin{cases} (2/\pi^2 \bar{g}_1)\, g_1^{-1/2} & \text{for } g_1 \ll (\bar{g}_1)^2 \ll 1 \\ (4\bar{g}_1/\pi^2)\, g_1^{-3/2} & \text{for } (\bar{g}_1)^2 \ll g_1 \ll 1 \end{cases}$$

Brouwer & Beenakker (1994), Kottos & Smilansky (2003)

$$P_{\text{Classical}}(g_1;\bar{g}_1) = \frac{(1 - g_1^{cl}) g_1^{cl}}{(g_1 + g_1^{cl} - 2 g_1 g_1^{cl})^2}$$

UCF?

$$\mathcal{H} = \frac{p^2}{2\mathrm{m}}[\text{network}] + \lambda\,\delta(x - X_0)$$

$$\mathcal{I} = \frac{e}{2\mathrm{m}}\left(\delta(x - X_1)p + p\delta(x - X_1)\right)$$

$$\mathcal{F} = -\frac{\partial \mathcal{H}}{\partial X_0} = \lambda\delta'(x - X_0)$$

$$G = 2\hbar \sum_n f(E_n) \sum_{m(\neq n)} \frac{\mathrm{Im}\,[\mathcal{I}_{nm}\mathcal{F}_{mn}]}{(E_m - E_n)^2 + (\Gamma/2)^2}$$