Quantum stirring of particles in closed devices [1]

Operating a quantum pump in a closed circuit [2]

[1] Gilad Rosenberg and Doron Cohen [JPA 2006]

[2] Itamar Sela and Doron Cohen [JPA 2006]

[3] DC, T. Kottos, H. Schanz [PRB(R) 2005]

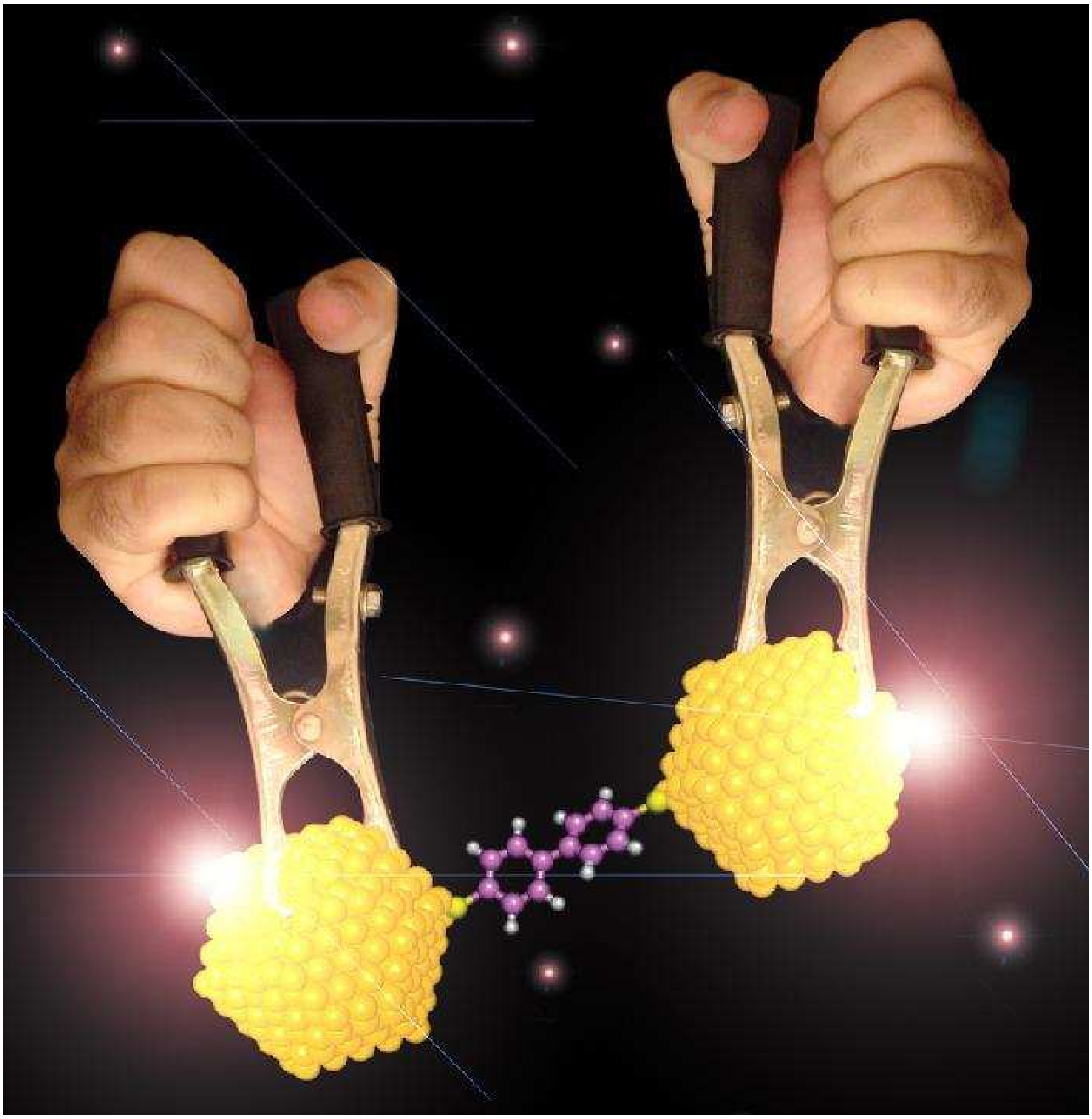
Discussions:

Tsampikos Kottos (Wesleyan / Gottingen)
Holger Schanz (Gottingen)
Nir Davidson (Weizmann)
Michael Wilkinson (UK)
Markus Buttiker (Geneva)
Misha Moskalets (Kharkov)

http://www.bgu.ac.il/~dcohen
LANL cond-mat archive

$GIF, $ISF

Let us go wireless



[Courtesy of Amir Yacoby]

We would like to induce current in a closed device (no leads), even if the the particles have no charge.

Driven Systems

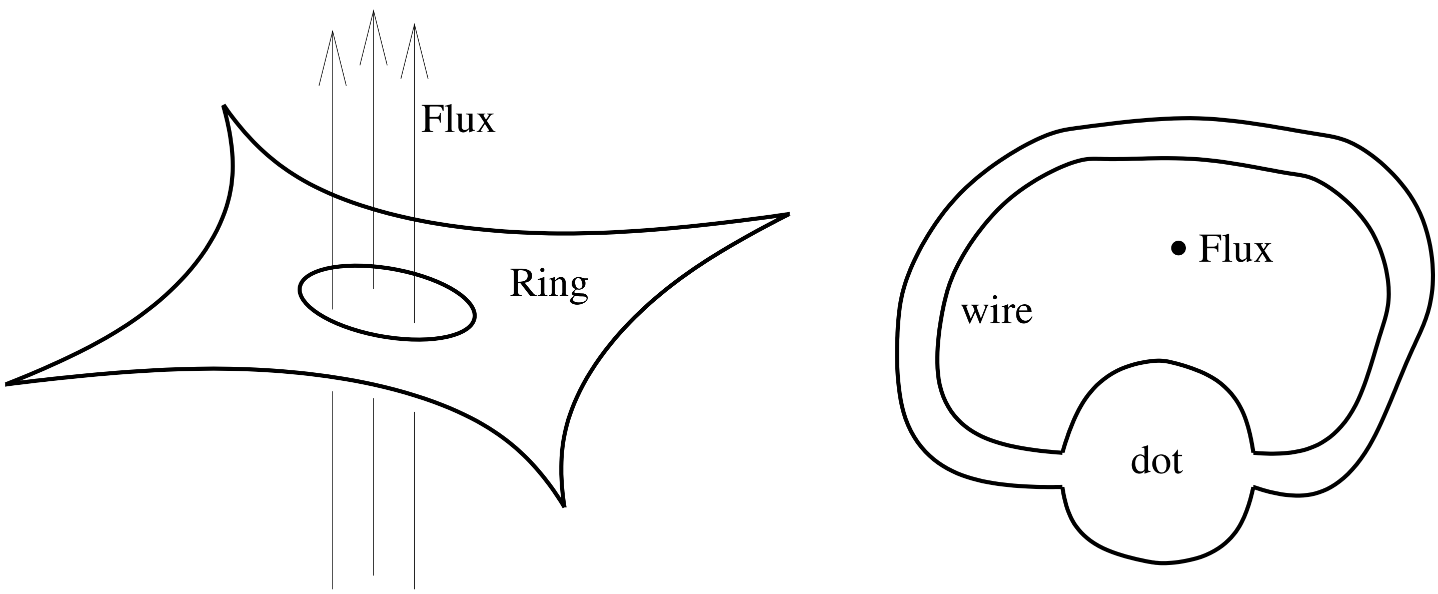
Non interacting "spinless" electrons.

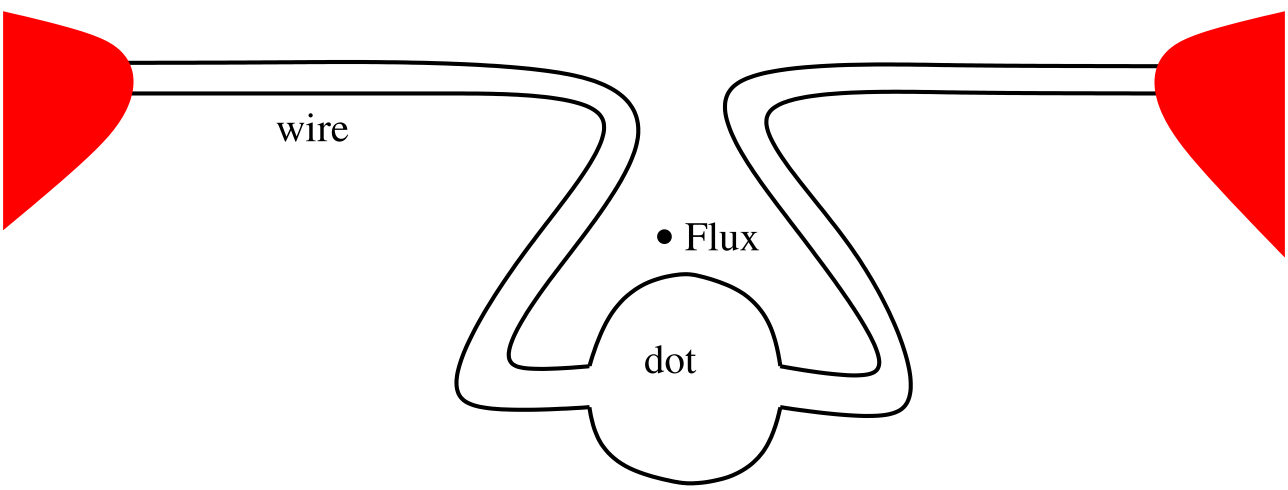
Held by a potential (e.g. AB ring geometry).

$$\mathcal{H} = \mathcal{H}(\boldsymbol{r}, \boldsymbol{p};\ X_1(t), X_2(t), X_3(t))$$

$X_1,\ X_2$ = shape parameters

$X_3 = \Phi = (\hbar/e)\phi$ = magnetic flux





"Ohm law"

For one parameter driving by EMF

$$I = G^{33} \times (-\dot{X}_3)$$

$$dQ = -G^{33}\, dX_3$$

For driving by changing another parameter

$$I = -G^{31}\, \dot{X}_1$$

$$dQ = -G^{31}\, dX_1$$

For two parameter driving

$$I = -G^{31}\dot{X}_1 - G^{32}\dot{X}_2$$

$$dQ = -G^{31}\, dX_1 - G^{32}\, dX_2$$

$$Q = -\oint \boldsymbol{G} \cdot dX$$

and in general

$$\langle F^k \rangle = -\sum_j G^{kj}\, \dot{X}_j$$

The Kubo formula approach [DC, PRB 2003]

$$\langle \mathcal{I} \rangle = -\boldsymbol{G}\dot{X}$$

$$\boldsymbol{G} = \int_0^\infty K(t)\,t\,dt$$

$$\boldsymbol{G} = \varrho(E_F) \int_0^\infty C(t)\,dt$$

$$\boldsymbol{G} = \sum_n f(E_n) \sum_{m(\neq n)} \frac{2\hbar \mathrm{Im}\,[\mathcal{I}_{nm}\mathcal{F}_{mn}]}{(E_m - E_n)^2 + (\Gamma/2)^2}$$

$$\boldsymbol{G} = \frac{e}{2\pi i}\mathrm{trace}\left(P_{\mathrm{A}} \frac{\partial S}{\partial x_j} S^\dagger\right)$$

$$K(t) = \frac{i}{\hbar}\langle [\mathcal{I}(t), \mathcal{F}(0)] \rangle_{\mathrm{F}}$$

$$C(\tau) = \frac{1}{2}\left(\langle \mathcal{I}(t)\mathcal{F}(0)\rangle_{\mathrm{E}} + cc\right)$$

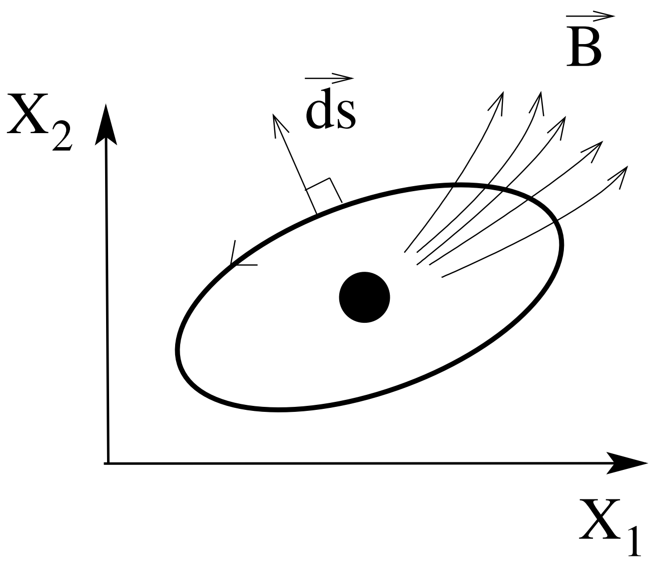
$$\mathcal{F} = -\frac{\partial \mathcal{H}}{\partial X}$$

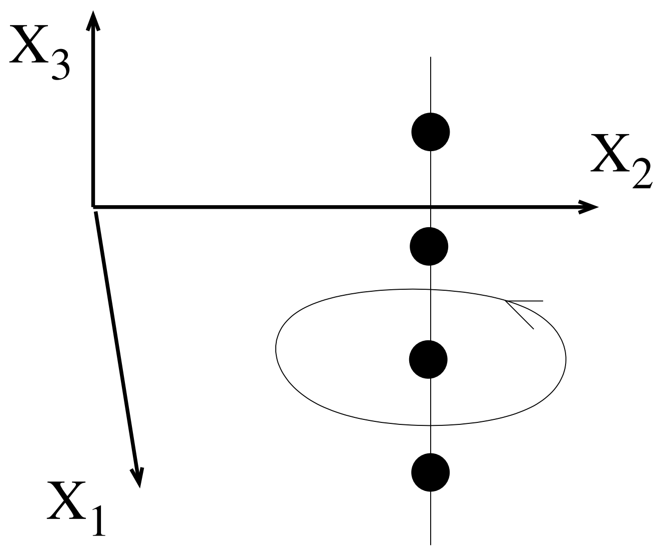
How do we calculate Q

$$Q = \oint_{\text{cycle}} I dt = - \oint (G^{31} dX_1 + G^{32} dX_2)$$

$$\begin{aligned} \vec{ds} &= (dX_2, -dX_1, 0) \\ \vec{B} &= (-G^{32}, G^{31}, 0) \end{aligned}$$

$$Q = \oint \boldsymbol{B} \cdot \vec{ds} = \iint \sigma(X_1, X_2) dX_1 dX_2$$





Note: BerryPhase $= \oint \boldsymbol{A} \cdot d\vec{X}$

Some references

Adiabatic transport and beyond

Thouless (PRL 1983) - Periodic arrays

Avron, Sadun, Raveh, Zur (1988) - Networks with fluxes

Berry, Robbins (JPA 1993) - Geometric magnetism

Wilkinson (1988-1995) - Dissipation in the adiabatic regime

DC (1999-2003) - Generalized Kubo approach to analyze closed driven systems

Open systems, S matrix formalism

The Landauer / Landauer-Buttiker formula (1970,1986)

The Buttiker Pretre Thomas [BPT] formula (1994)

Brouwer (1998)

Avron, Elgart, Graf, Sadun - ... the snow plow pump

Shutenko, Aleiner, Altshuler (PRB 2000) - quantization?

Levinson, Entin-Wohlman, Wolfle (2000) - the double barrier pump

Pumping / Stirring in closed systems

DC (2002-2003) - the Kubo approach + the double barrier pump

DC (PRB-Rapid, 2003) - from closed to open systems

Moskalets, Buttiker (PRB-Rapid, 2003) - the double barrier pump

Sela, DC (JPA 2006) - the double barrier pump

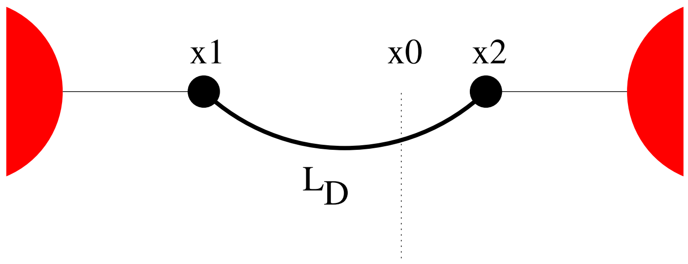
DC, Kottos, Schanz (PRE-Rapid, 2005) - the snow plow pump

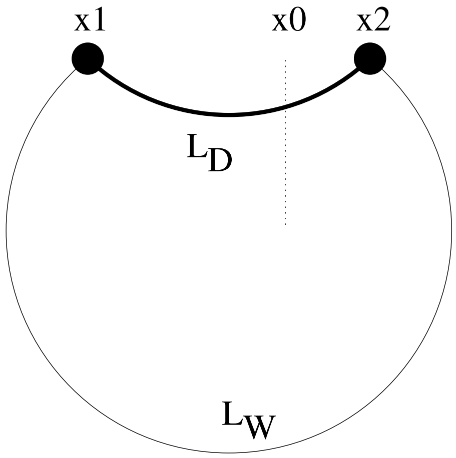
Rosenberg, DC (JPA 2006) - the snow plow pump, stirring

Aunola, Toppari (PRB, 2003) - Cooper pair pumping

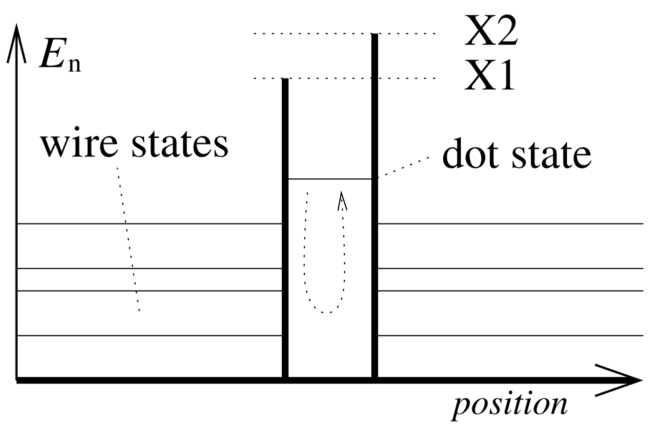
Mottonen, Pekola, Vartiainen, Brosco, Hekking (2006) - Cooper pair pumping

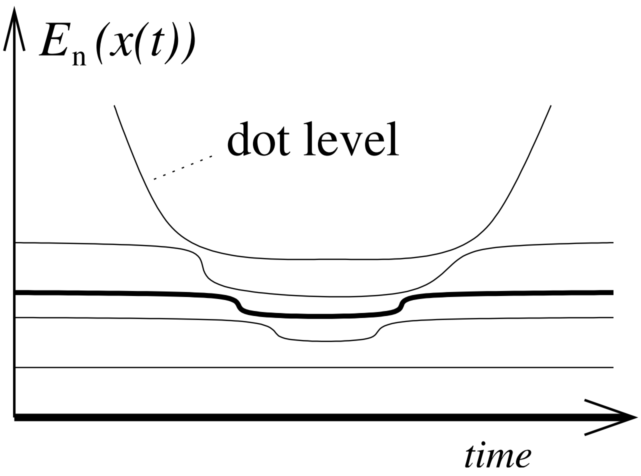
Operating a quantum pump in a closed circuit

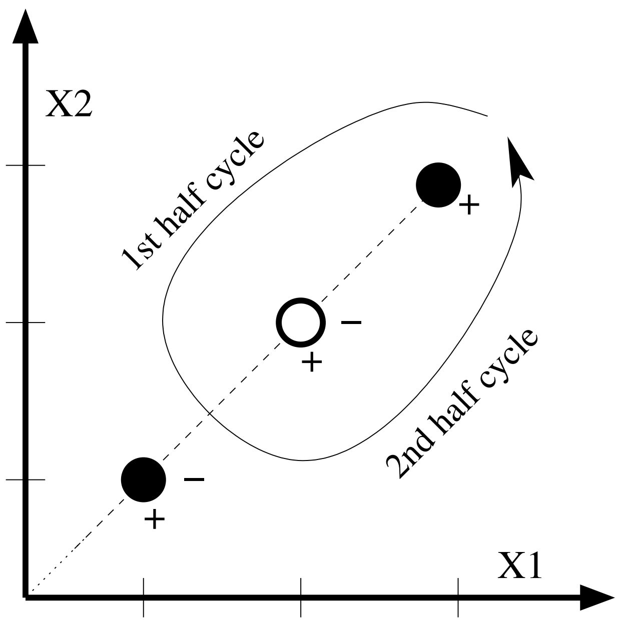


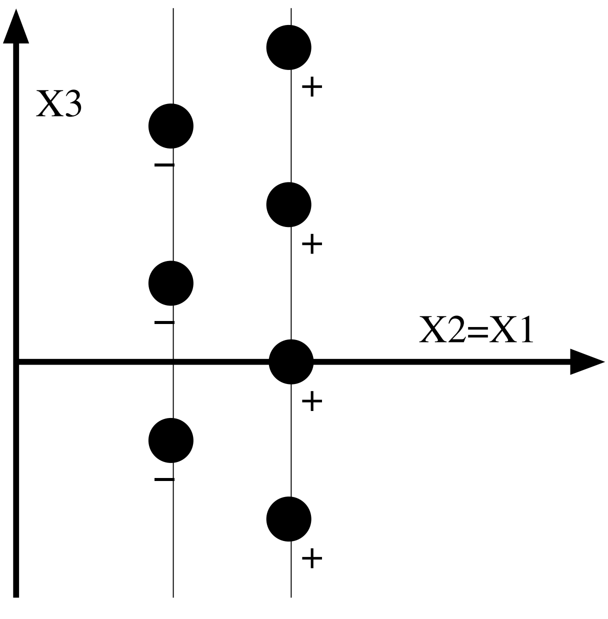


$$Q \;=\; (1 - g_T)e \qquad ???$$

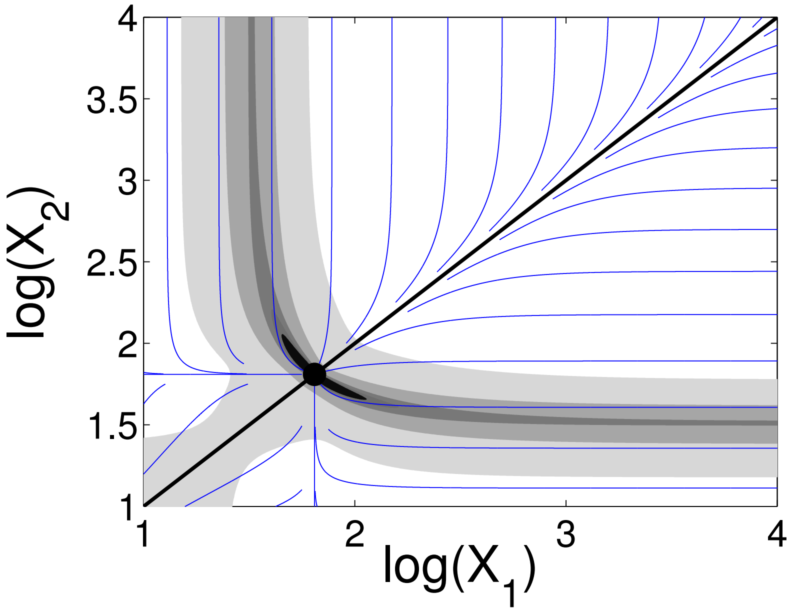




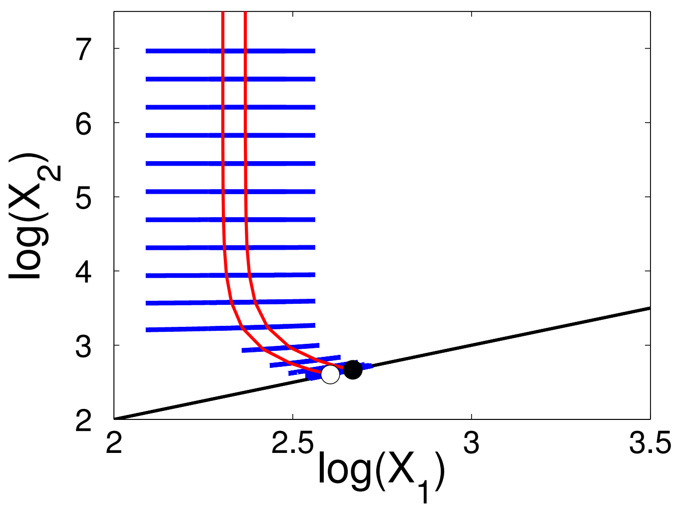


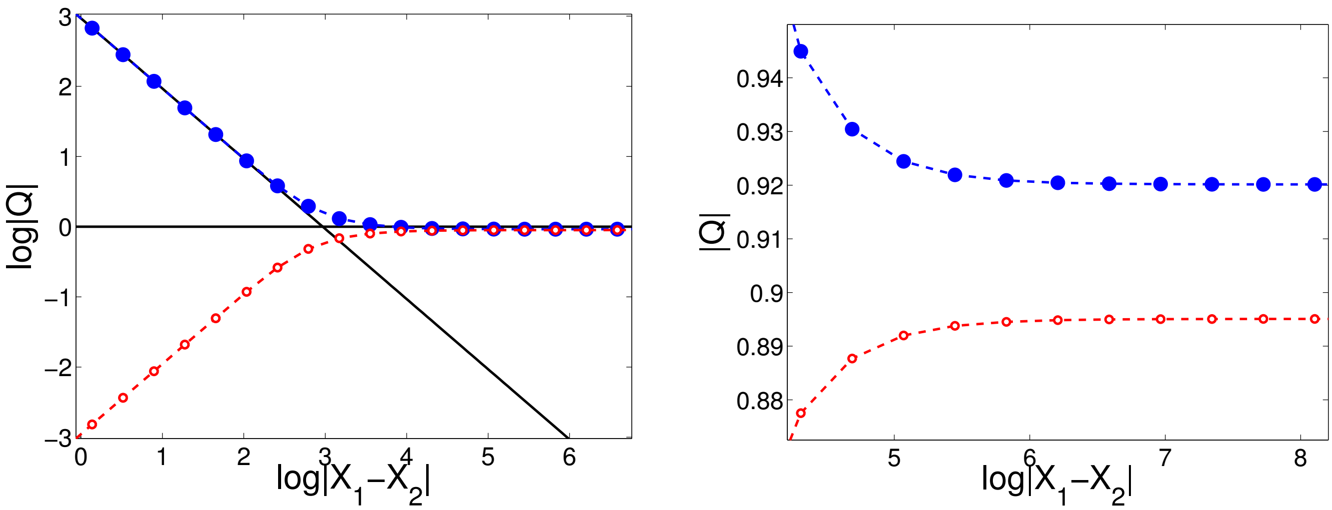


Results for Q



Two examples: $n = 2992$ and $n = 2993$ occupation:





Formulas

$$\mathcal{H} = \frac{1}{2\mathsf{m}}\hat{p}^2 + X_1\delta(\hat{x} - x_1) + X_2\delta(\hat{x} - x_2)$$

$$\mathcal{I} = \frac{e}{2\mathsf{m}}\left(\hat{p}\delta(\hat{x} - x_0) + \delta(\hat{x} - x_0)\hat{p}\right)$$

$$\mathcal{F} = -\frac{\partial\mathcal{H}}{\partial X_1} = \delta(\hat{x} - x_1)$$

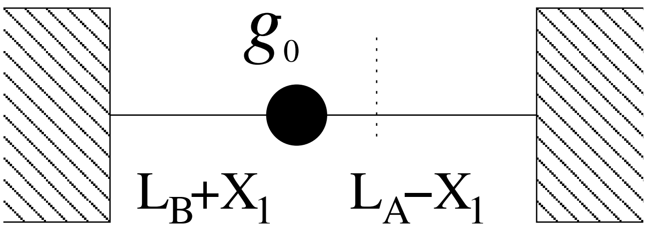
$$\boldsymbol{B}(X_1, X_2) = \frac{e\frac{2}{b}X^{(r)}x}{\left(\frac{4}{b}x^2 + y^2\right)^{3/2}}$$ in the radial (x) direction

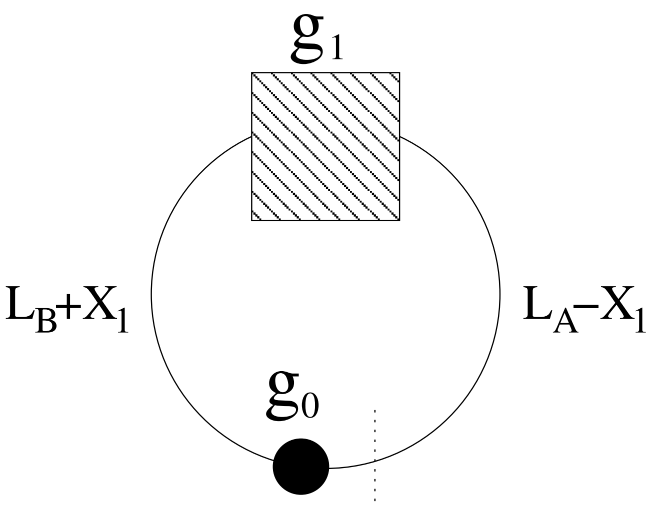
$$Q = \int \boldsymbol{B}dy = \frac{eX^{(r)}}{x}$$

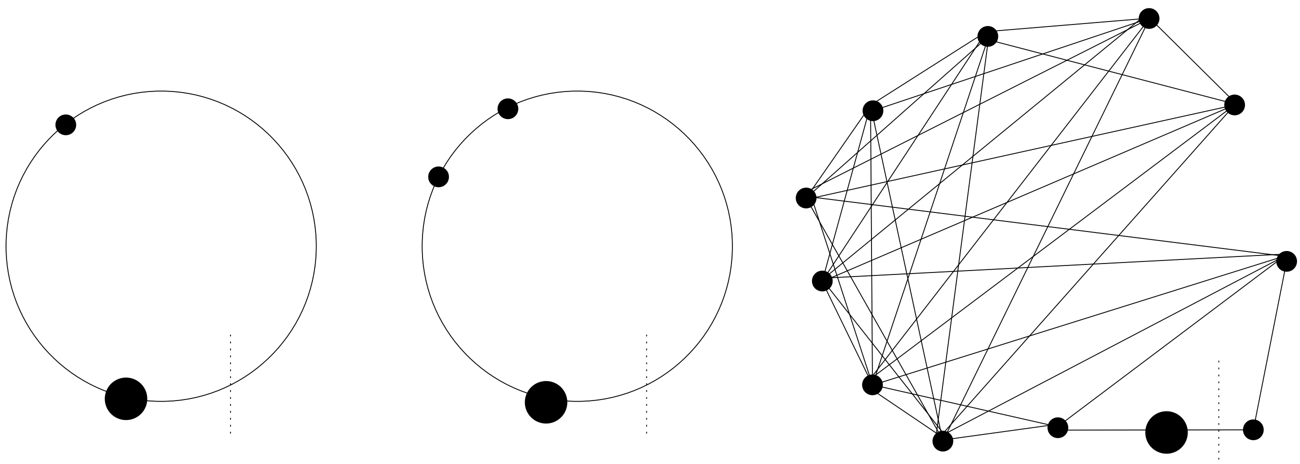
$$x \equiv \text{scaled}(|X_1 - X_2|) \leq X^{(r)}$$

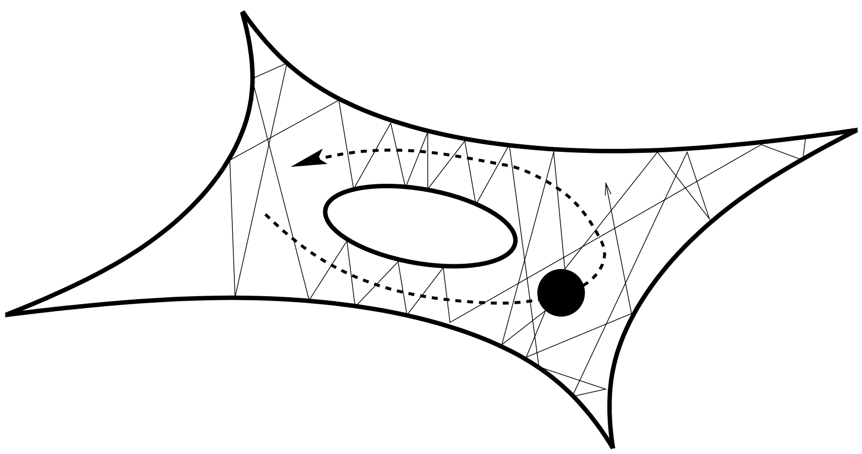
$$y \equiv \text{scaled}(X_1) + \text{scaled}(X_2) = \text{“gate voltage”}$$

Quantum stirring of particles in closed devices

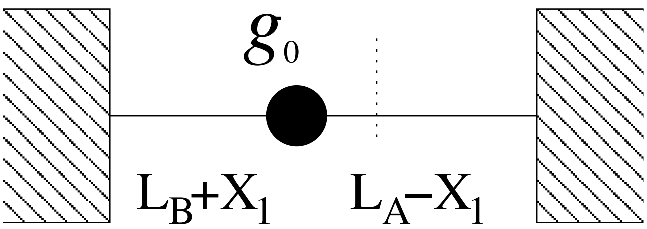




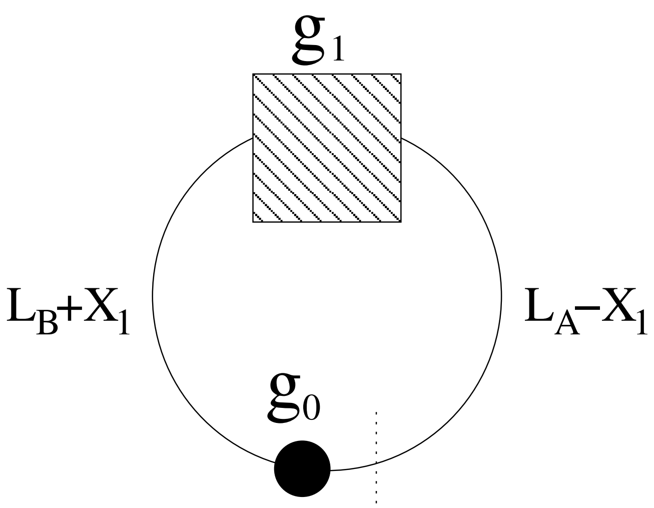




Stirring - "classical" reasoning

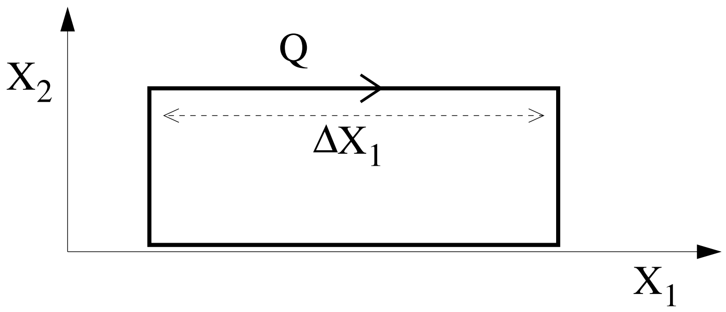


$$dQ = (1-g_0)\,\frac{e}{\pi}k_F\,dX$$



$$dQ = \left[\frac{(1-g_0)g_1^{cl}}{g_0+g_1^{cl}-2g_0g_1^{cl}}\right]\frac{e}{\pi}k_F\,dX$$

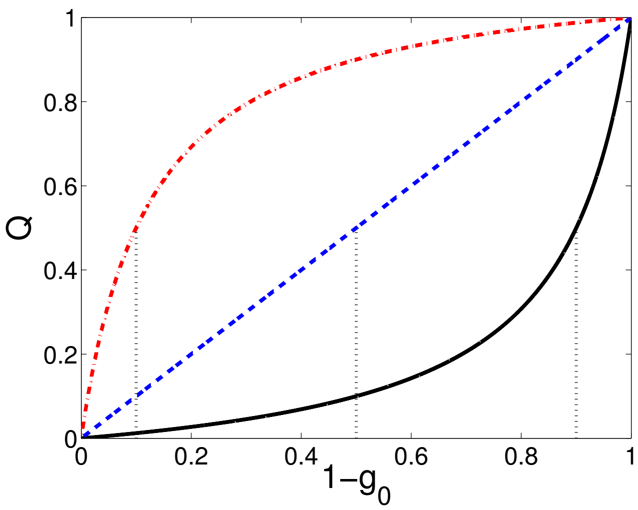
Quantum stirring and "pumping"

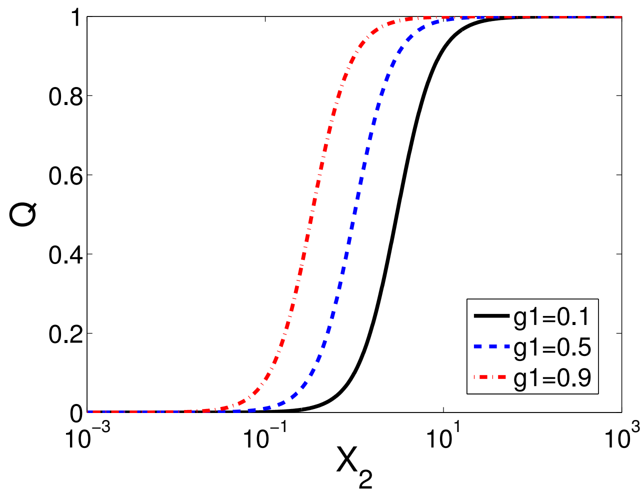


$$Q = \left[\frac{(1-g_0)g_1^{cl}}{g_0 + g_1^{cl} - 2g_0 g_1^{cl}}\right]\frac{e}{\pi}k_F\,\Delta X_1$$

where

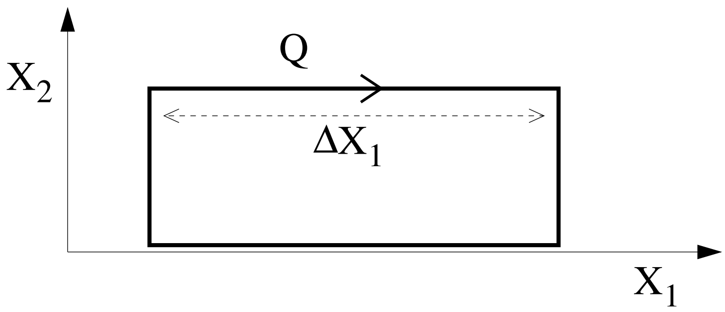
$$g_0(X_2) = \left[1+\left(\frac{\mathrm{m}}{\hbar^2 k_F}X_2\right)^2\right]^{-1}$$



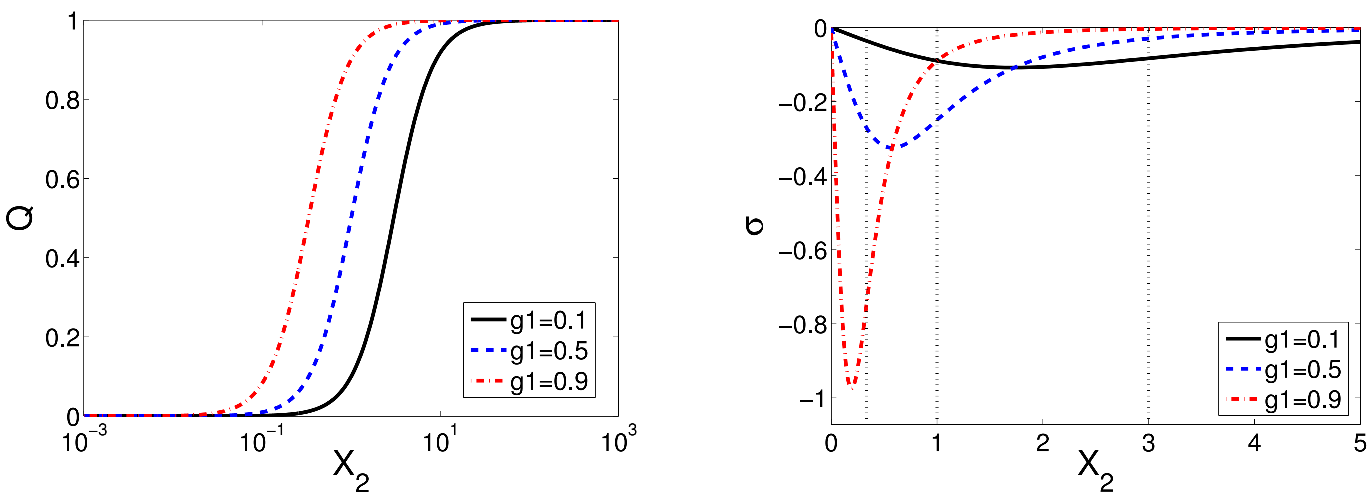


$$Q = (1-g_0)\,\frac{e}{\pi}k_F\,\Delta X_1 \qquad \text{for } g_1^{cl} = 1/2$$

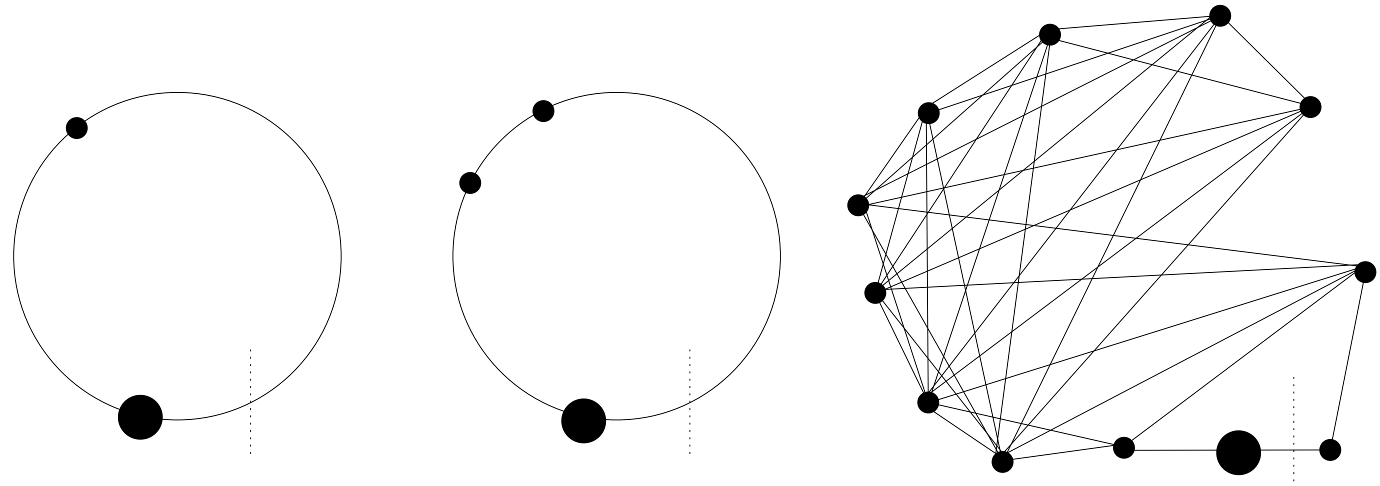
The "classical" distribution of Dirac chains

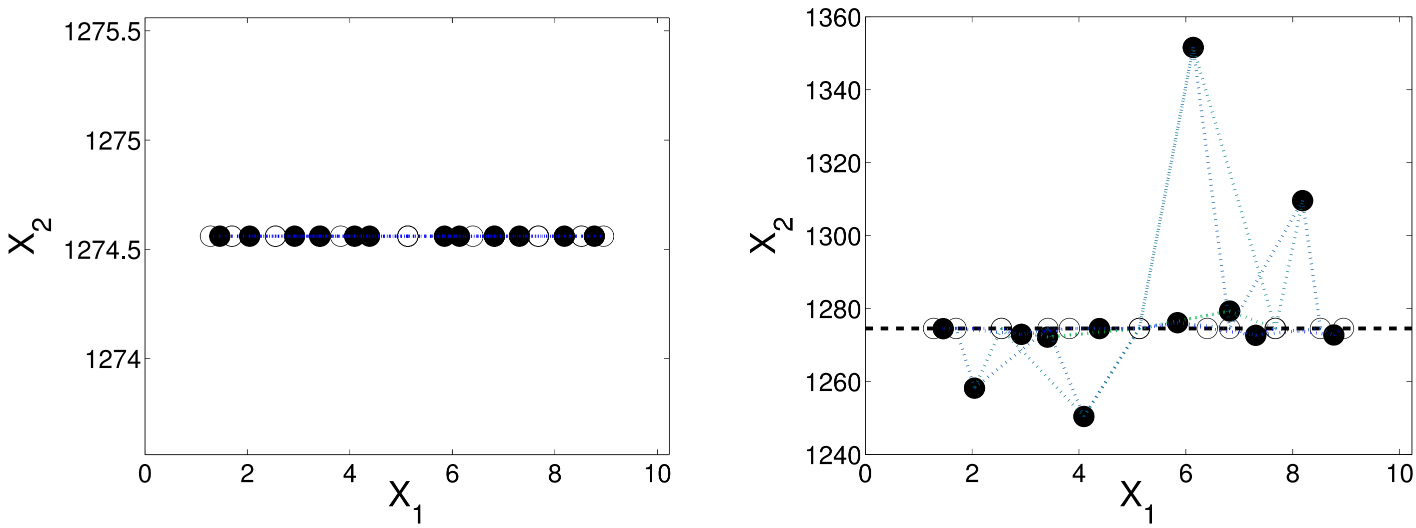


$$\sigma(X_1, X_2) = -\frac{e\mathrm{m}}{\pi\hbar^2}\frac{2(1-g_1^{cl})g_1^{cl}}{\left[1+\left(\left(\frac{\mathrm{m}}{\hbar^2 k_\mathrm{F}}X_2\right)^2-1\right)g_1^{cl}\right]^2}\left(\frac{\mathrm{m}}{\hbar^2 k_\mathrm{F}}X_2\right)$$



The quantum distribution of Dirac chains

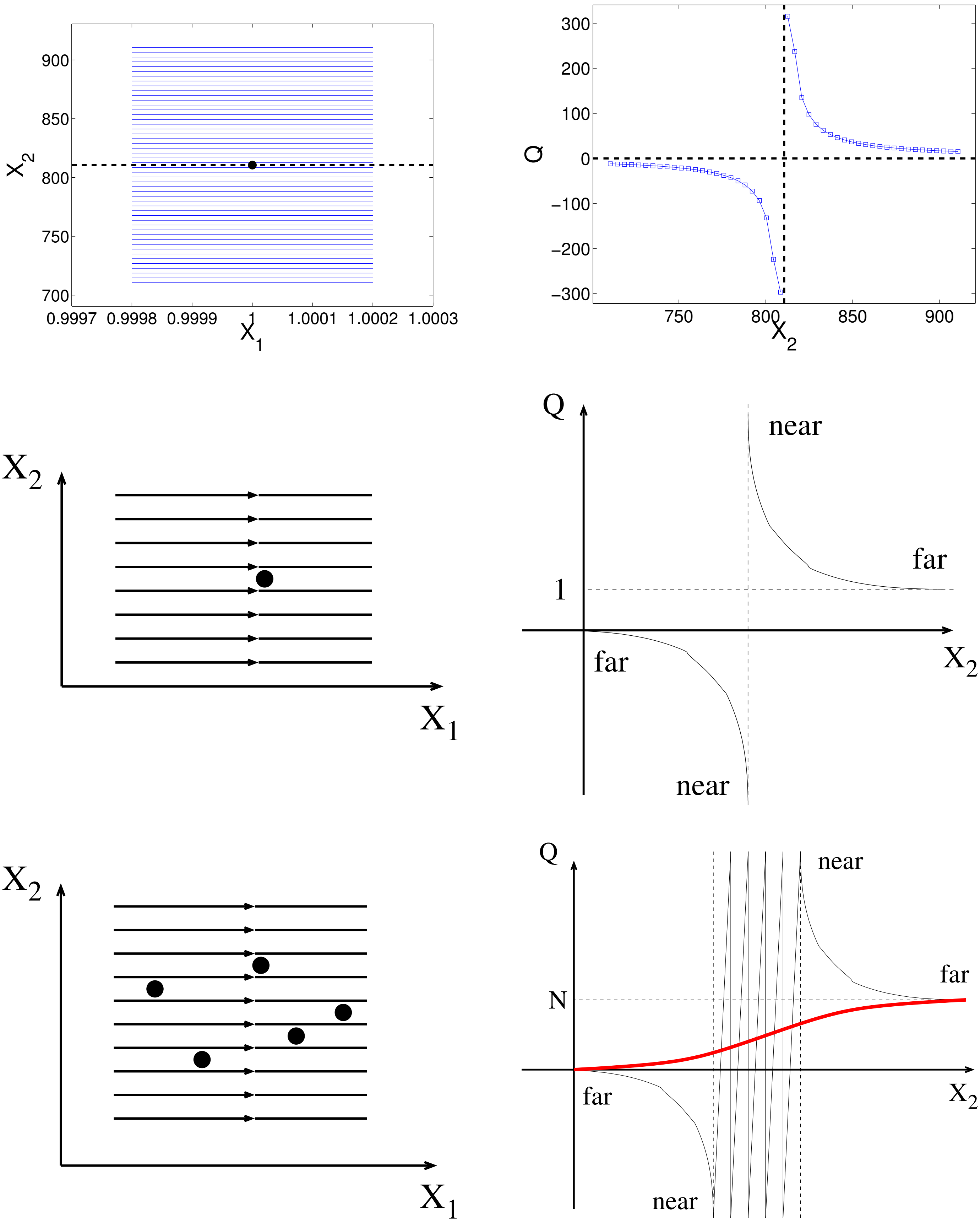




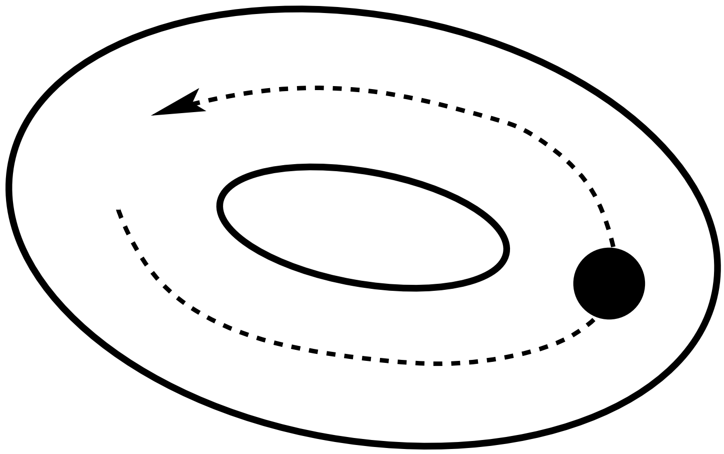
$$Q \approx e\frac{\Delta X_1}{\lambda_E/2} = e\frac{k_E}{\pi} \times \Delta X_1$$

Related: M. Hiller, T. Kottos and DC,
quantum pumping / stirring of BEC (2007).

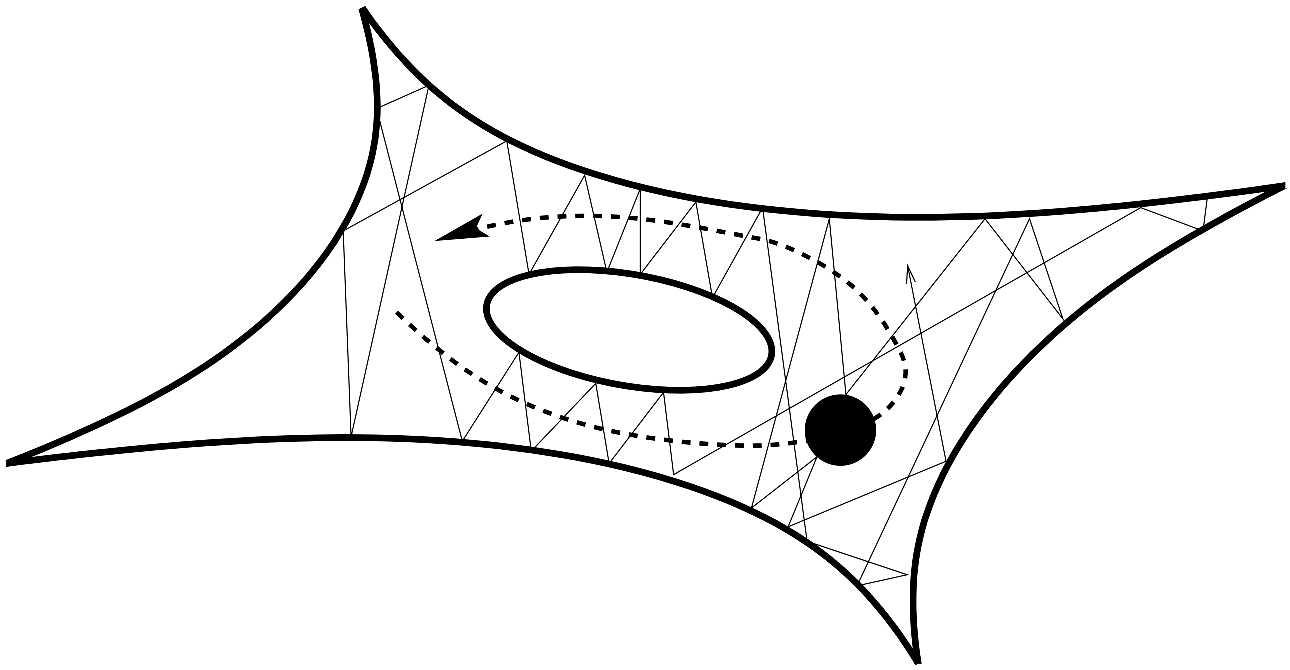
The dependence of Q on X_2



Remark on dissipation



- There is a stationary solution.
- There is no dissipation.
- Pumping: g_0 does not matter



- There is no stationary solution.
- There is dissipation.
- Pumping: g_0 does matter

Summary

- Near-field versus far field pumping cycles around "Dirac chains".
- The analysis of deviations from "quantized" pumping.
- In the far field peristaltic mechanism leads to $Q \sim 1$.
- In the near field the induced circulating current may lead to $Q \gg 1$.
- Detailed analysis of the prototype 2-barrier pump.

- Quantum stirring is more complicated in simpler systems.
- Currents can be induced in the opposite sense to the translated scatterer.
- Larger scatterer does not necessarily imply larger current.
- The semiclassical limit cannot be derived from RMT.

- The Kubo approach gives a unified framework for the theory of pumping.
- Distinction between adiabatic, non-adiabatic and non-prtrb regimes.
- The emergence / relevance of dissipation.
- "Quantum chaos" considerations are essential.