Semi linear response, the absorption of radiation by small metallic grains, and the conductance of mesoscopic devices

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References

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Google "Doron Cohen"

http://www.bgu.ac.il/~dcohen
cond-mat archive

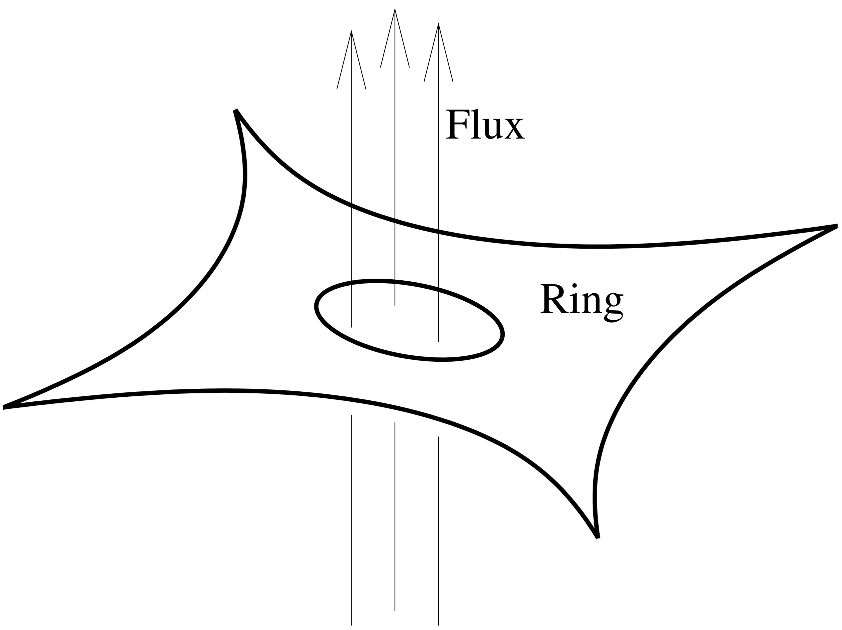
Driven Systems

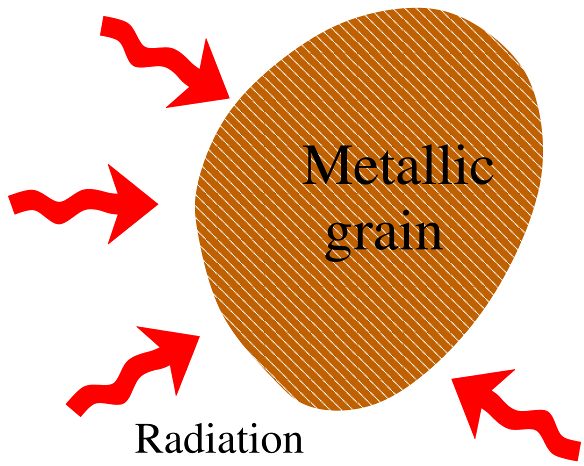
Non interacting "spinless" electrons.

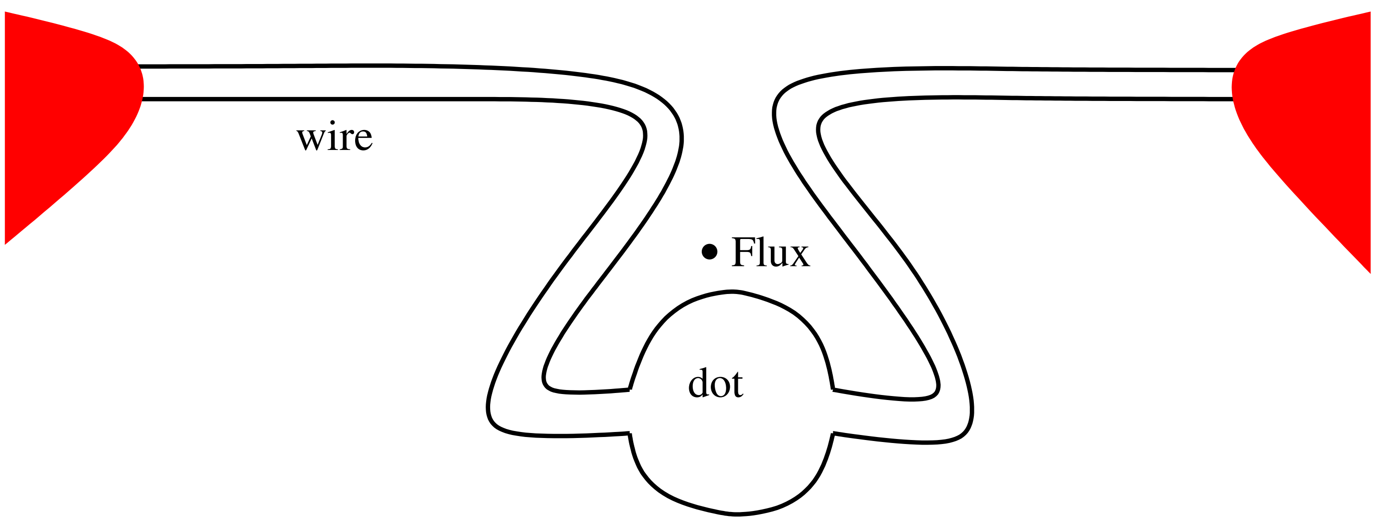
Held by a potential (e.g. AB ring geometry).

$\mathcal{H}(Q, P; X(t))$ = quantized chaotic system

X = some parameter in the Hamiltonian







"Quantum Chaos"

$\mathcal{H}(Q, P; X(t))$ = quantized chaotic system

X = some parameter in the Hamiltonian

Universality on small energy scales ($\propto \hbar^d$)

Fingerprints on larger energy scales ($\propto \hbar$)

Questions:

How is the $\hbar^d$ scale reflected in the response???

How is the $\hbar$ scale reflected in the response???

The message:

The Kubo formalism should be revised!

The main idea

There are circumstance in which the rate of energy absorption depends on the possibility to make long sequences of transitions.

The possibility to make a connected sequence of transitions between energy levels is greatly affected by structures in the energy landscape of the device.

Even if from statistical point of view the matrix elements are very large, still a small finite probability for a bottleneck may lead to suppression of the absorption process.

The Kubo formalism should be revised!

“Response”

$$\mathcal{H}(Q, P; X(t)) \; = \; \text{quantized chaotic system}$$

$$X \; = \; \text{some parameter in the Hamiltonian}$$

$$\dot{X} \; = \; \text{rate of the driving}$$

$$E \; = \; \langle \mathcal{H} \rangle \; = \; \text{the energy of the system}$$

$$\dot{E} \; = \; \text{rate of energy absorption}$$

$$\mathcal{I} \; = \; \mathcal{F} \; = \; -\frac{\partial \mathcal{H}}{\partial X} \; = \; \text{generalized force or current}$$

$$\dot{E} = \frac{d}{dt}\langle \mathcal{H} \rangle = \left\langle \frac{\partial \mathcal{H}}{\partial t} \right\rangle = -\langle \mathcal{F} \rangle \dot{X}$$

“Ohm law” and ”Joule law”

For one parameter driving by EMF

$$\langle \mathcal{I} \rangle \quad = \quad \boldsymbol{G} \times (-\dot{X})$$

More generally

$$\langle \mathcal{F} \rangle \quad = \quad -\boldsymbol{G}\ \dot{X}$$

leading to

$$\dot{E} \ = \ -\langle \mathcal{F} \rangle \dot{X} \ = \ \boldsymbol{G}\ \dot{X}^2$$

The dissipation coefficient $\boldsymbol{G}$ reflects the stochastic-like diffusion D_E in energy space.

more generally

$$\dot{E} \ = \ \int \frac{d\omega}{2\pi} G(\omega) |\dot{X}_\omega|^2 \ = \ \int \alpha(\omega) \Phi(\omega) d\omega$$

Semi-linear Response

Within the framework of linear response

$$\dot{E} \;=\; \int \alpha(\omega)\Phi(\omega)d\omega$$

leading to

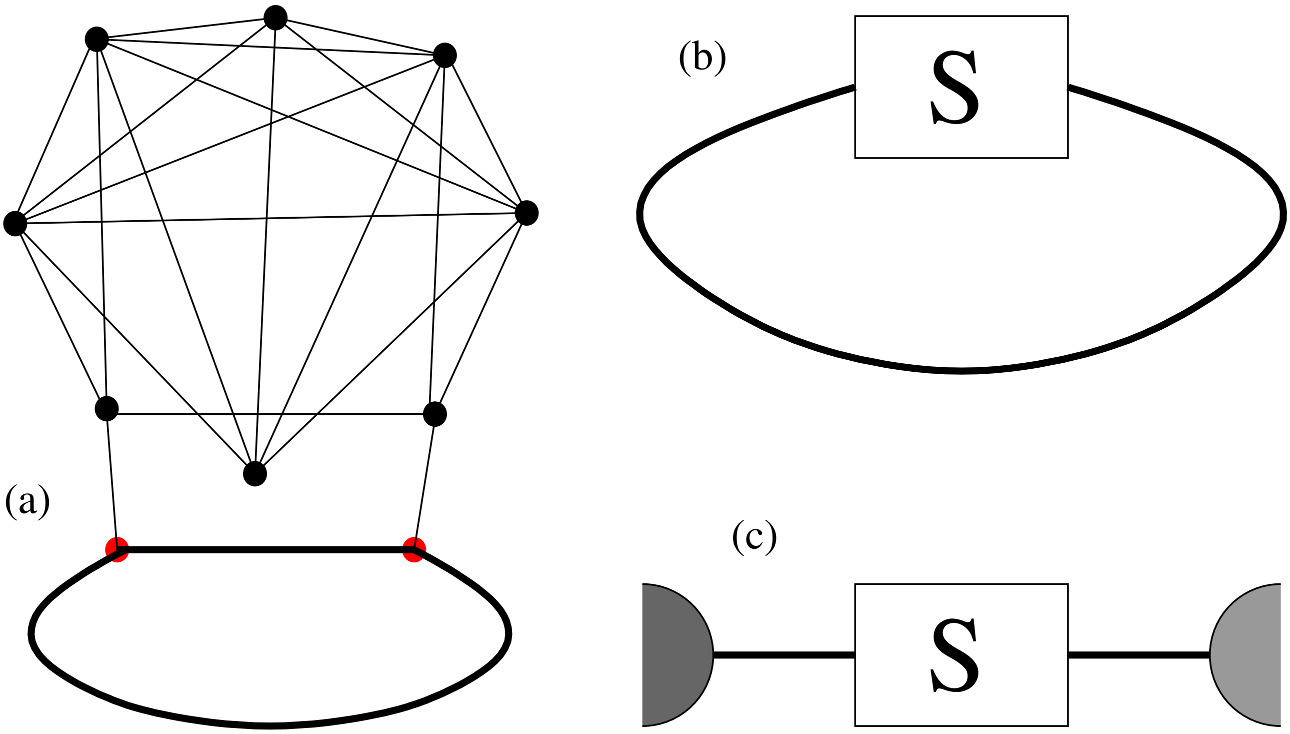
$$\Phi(\omega) \mapsto \lambda\Phi(\omega) \qquad \Longrightarrow \qquad \dot{E} \mapsto \lambda\dot{E}$$

$$\Phi(\omega) \mapsto \sum_i \Phi_i(\omega) \qquad \Longrightarrow \qquad \dot{E} \mapsto \sum_i \dot{E}_i \ .$$

But we shall find circumstance such that

$$\dot{E} \;=\; \left[\int \mu(\omega)\,[\Phi(\omega)]^{-1}\,d\omega\right]^{-1}$$

Problem No.1 - driving by EMF



$$G_{\text{Landauer}} = \frac{e^2}{2\pi\hbar}\; g_{cl}$$

$$G = \frac{e^2}{2\pi\hbar}\left(\frac{g_{cl}}{1-g_{cl}}\right)$$

[classical]

The "classical" conductance formula

Single mode versions:

$$G_{\text{Landauer}} = \frac{e^2}{2\pi\hbar} g_{cl}$$

$$G = \frac{e^2}{2\pi\hbar} \left(\frac{g_{cl}}{1 - g_{cl}} \right)$$

Multimode versions:

$$\boldsymbol{g} = \begin{pmatrix} g^R & g^T \\ g^T & g^R \end{pmatrix}$$

$$G_{\text{Landauer}} = \frac{e^2}{2\pi\hbar} \sum_{n,m} g^T_{nm}$$

$$G = \frac{e^2}{2\pi\hbar} \sum_{nm} \left[2g^T/(1 - g^T + g^R) \right]_{nm}$$

[D.C. and Etzioni, JPA 2005]

Can we trust the classical result?

We are interested in the case $g_{cl} \sim 1$

The velocity-velocity correlaion time

$$\tau_{cl} \approx \left(\frac{1}{1-g_{cl}}\right) \times \frac{L_0}{v_F}$$

Quantum mechanics introduces

$$t_{\text{Heisenberg}} \approx \mathcal{M}_{\text{modes}} \times \frac{L_0}{v_F}$$

Necessary condition for QCC:

$$\mathcal{M}_{\text{modes}} \quad \gg \quad \left(\frac{1}{1-g_{cl}}\right)$$

Single mode ring??? Ring-Tree model???

Pure quantum results!

New ingredient in the theoy of mesoscopic condctance!

Quantum results

$g_{cl} \quad = \quad$ the average Landauer conductance.

The linear response result:

$$G_{\text{Kubo}} = \frac{e^2}{2\pi\hbar}\left(\frac{g_{cl}}{1-g_{cl}}\right)$$

in general level statistics is important.

The mesoscopic result:

In the limit $g_{cl} \to 1$

$G \quad \to \quad \infty \qquad$ [classical]

$G \quad \to \quad 0 \qquad$ [quantal]

For the ring-tree model:

$$G \quad = \quad \frac{e^2}{2\pi\hbar}\,(1-g_{cl})^2\, g_{cl}$$

The Kubo formula

$$\boldsymbol{G} = \pi\hbar \sum_{n,m} |\mathcal{I}_{mn}|^2 \; \delta_T(E_n - E_F) \; \delta_\Gamma(E_m - E_n)$$

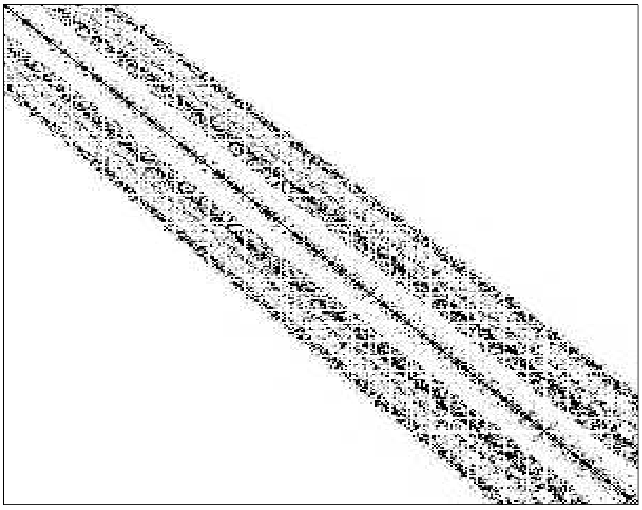
$$\boldsymbol{G} \;=\; \pi\hbar(\varrho(E_F))^2 \; \langle\langle|\mathcal{I}_{mn}|^2\rangle\rangle$$

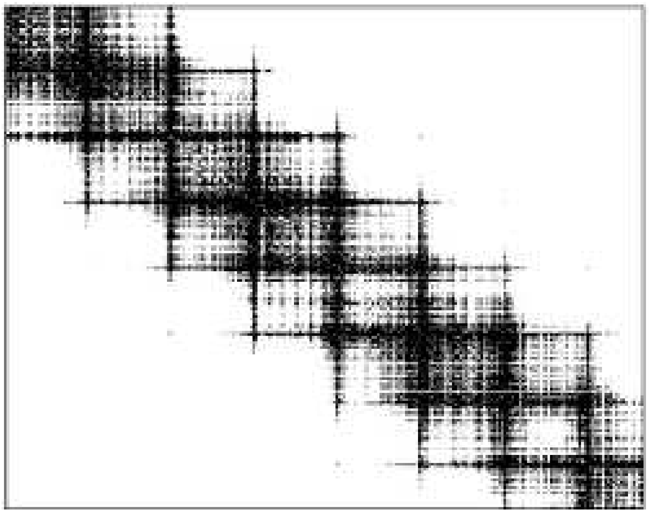
$$\dot{E} \;=\; \pi\hbar(\varrho(E_F))^2 \; \langle\langle|\mathcal{I}_{mn}|^2\rangle\rangle \times \dot{X}^2$$

$$D_E \;=\; \pi\hbar\varrho(E) \; \langle\langle|\mathcal{I}_{mn}|^2\rangle\rangle \times \dot{X}^2$$

Let us look on $\mathcal{I}_{nm}$

The implication of a structured energy landscape



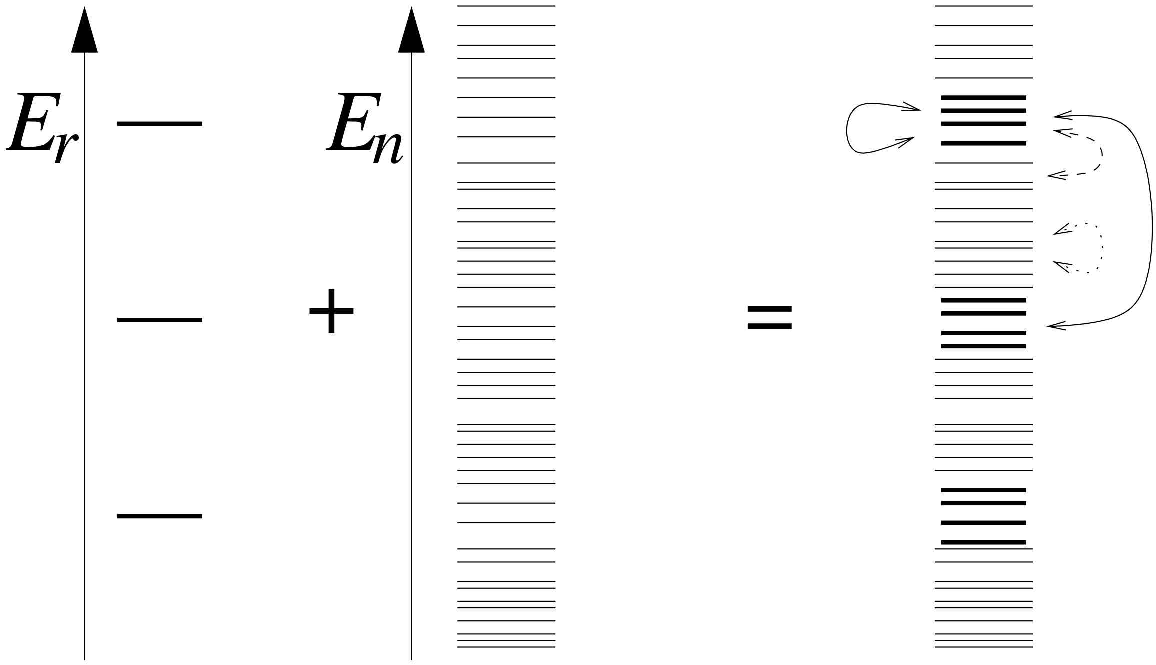


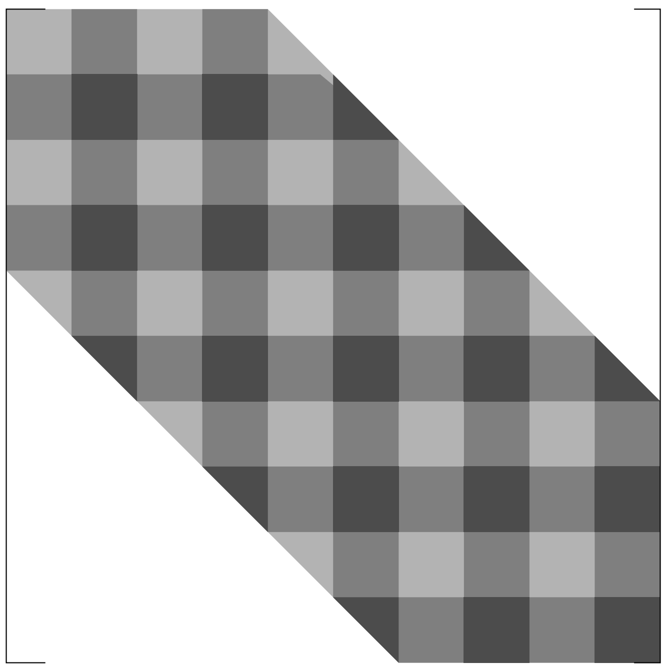
For structured matrix algebraic average is wrong!
Therefore the "classical" result is not obtained.

How is the coarse grained diffusion determined?

$$\langle\langle D_E \rangle\rangle \quad = \quad \left[\ \overline{1/D_E} \ \right]^{-1}$$

Why do we have structures?





Large scale $\mathcal{O}(\hbar)$ structures are the fingerprints of the "non-universal" classical limit.

What about the implications of the "universal" (RMT) statistics?

Problem No.2 - driving by noise

The Hamiltonian:

$$\mathcal{H} = \mathcal{H}_0 - X(t)\mathcal{F}$$

$$\mathcal{H}_0 \quad \mapsto \quad \{E_n\}$$

$$\mathcal{F} \quad \mapsto \quad \{\mathcal{F}_{nm}\}$$

The driving:

$$\langle X(t)X(t')\rangle = \phi(t-t')$$

$$\Phi(\omega) = \int_{-\infty}^{\infty} \phi(\tau)\exp(\mathrm{i}\omega t)\mathrm{d}\tau$$

For example:

$$\Phi(\omega) \;=\; \frac{\varepsilon^2}{\omega_0}\exp\left(-\frac{|\omega|}{\omega_0}\right)$$

For example $\omega_0 = k_{\mathrm{B}}T/\hbar$

We assume $k_{\mathrm{B}}T \ll \Delta$

The diffusion picture

Fermi Golden rule

$$\Gamma_{nm} \quad = \quad \frac{1}{\hbar^2} \, \Phi \left(\frac{E_n - E_m}{\hbar} \right) \; |\mathcal{F}_{nm}|^2$$

Master equation

$$\frac{\mathrm{d}p_n}{\mathrm{d}t} = \sum_m \Gamma_{nm}(p_m - p_n)$$

Diffusion in energy space

$$\frac{\partial}{\partial t} p = \frac{\partial}{\partial n} \left[D \frac{\partial}{\partial n} p \right]$$

If the the transitions are only between neighboring levels:

$$D = \left\langle \Gamma_{\mathrm{n.n.}}^{-1} \right\rangle^{-1}$$

leading to semi-linear response:

$$D_E \;=\; \frac{\sigma^2}{(\varrho\hbar)^3} \left[\int \frac{\mathrm{d}\mathbf{x} \, \mathrm{e}^{-\mathbf{x}^2/2}}{(2\pi)^{N/2} \mathbf{x}^2} \right]^{-1} \left[\int_0^\infty \mathrm{d}\omega \, \frac{P_2(\varrho\hbar\omega)}{\Phi(\omega)} \right]^{-1}$$

Results of LRT and SLR

Linear response theory (LRT):

$$D_E = \sigma^2 \hbar \varrho \int_0^\infty \mathrm{d}\omega\, \omega^2\, R_2(\hbar\omega)\Phi(\omega)$$

Semi-linear response (SLR):

$$D_E \;=\; \frac{\sigma^2}{(\varrho\hbar)^3}\left[\int \frac{\mathrm{d}\mathbf{x}\, \mathrm{e}^{-\mathbf{x}^2/2}}{(2\pi)^{N/2}\mathbf{x}^2}\right]^{-1}\left[\int_0^\infty \mathrm{d}\omega \frac{P_2(\varrho\hbar\omega)}{\Phi(\omega)}\right]^{-1}$$

Level spacing statistics:

$$P_2(S) \approx a_\beta S^\beta \exp(-c_\beta S^2) \qquad \text{with } \beta = 1, 2, 4$$

The LRT result of Gorkov and Eliashberg:

$$D_E \;=\; C_\beta \sigma^2 \varepsilon^2 (\hbar\varrho)^{\beta+1}\, \omega_0^{\beta+2}$$

Our SLR result (large S statistics!):

$$D_E \;=\; \frac{\varepsilon^2\sigma^2}{2\hbar\varrho}\frac{1}{(\hbar\varrho\omega_0)^{\beta-1}}\; \exp\left[-\frac{1}{\pi(\hbar\varrho\omega_0)^2}\right]$$

The SLR result - details

For $\mathcal{H} = \mathcal{H}_0 - X(t)\mathcal{F}$ we would get $D_E = 0$ because RMT implies a non-zero probability to have a vanishingly small matrix element.

It is only in 3D that we get absorption:

$$\mathcal{H} = \mathcal{H}_0 - \sum_j X_j(t)\mathcal{F}^j$$

with

$$\langle X_i(t) X_j(t') \rangle = \delta_{ij}\phi(t-t')$$

$$\Phi(\omega) = \int_{-\infty}^{\infty} \phi(\tau) \exp(\mathrm{i}\omega t) \mathrm{d}\tau$$

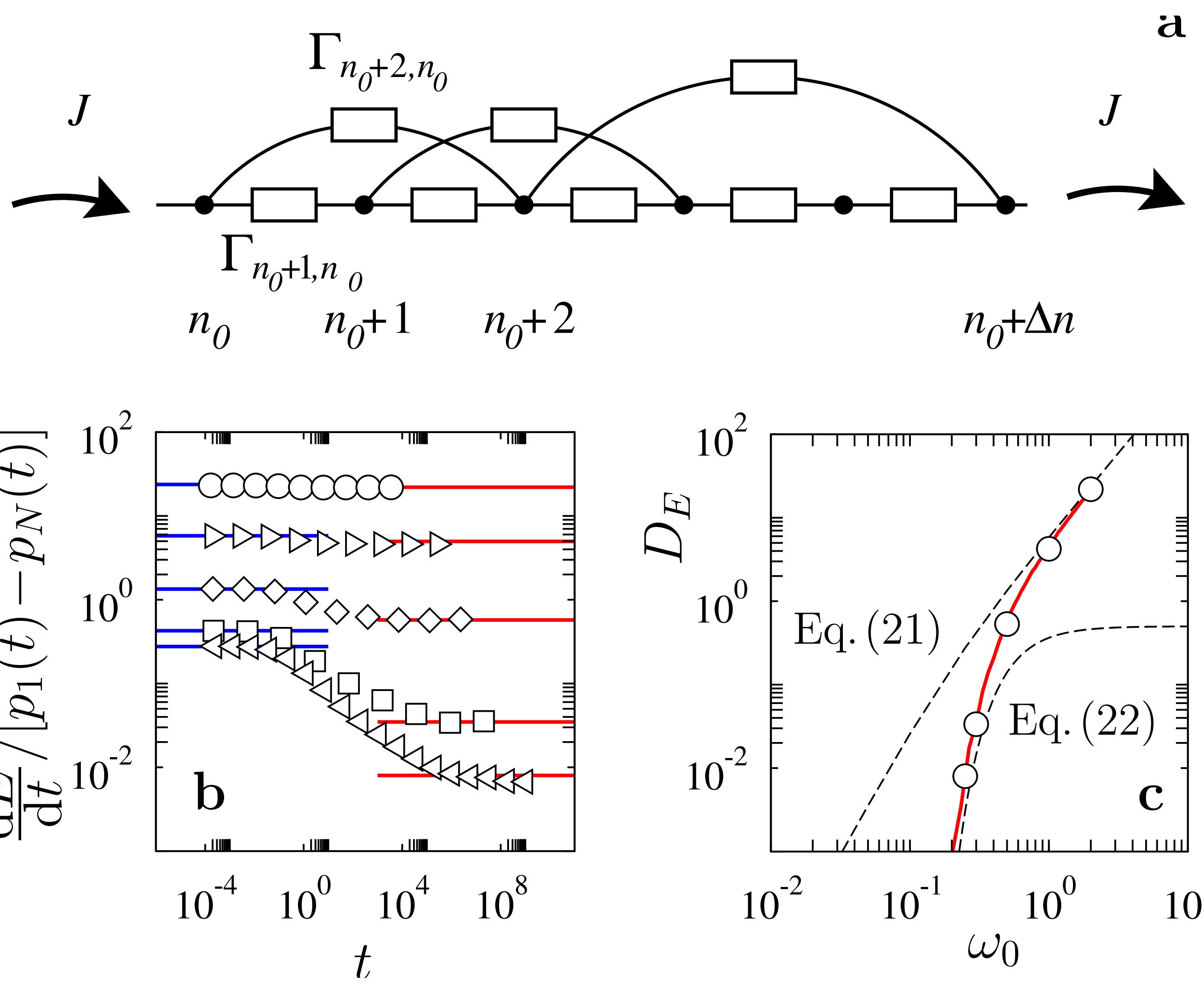
We have used:

$$\Phi(\omega) \;=\; \frac{\varepsilon^2}{\omega_0} \exp\left(-\frac{|\omega|}{\omega_0}\right)$$

For example $\omega_0 = k_{\mathrm{B}}T/\hbar$

We assume $k_{\mathrm{B}}T \ll \Delta$

Numerics



a
J
Γ_{n_0+2,n_0}
J
Γ_{n_0+1,n_0}
n_0
n_0+1
n_0+2
$n_0+\Delta n$
$\frac{dE}{dt}/[p_1(t)-p_N(t)]$
b
t
D_E
Eq. (21)
Eq. (22)
c
ω_0