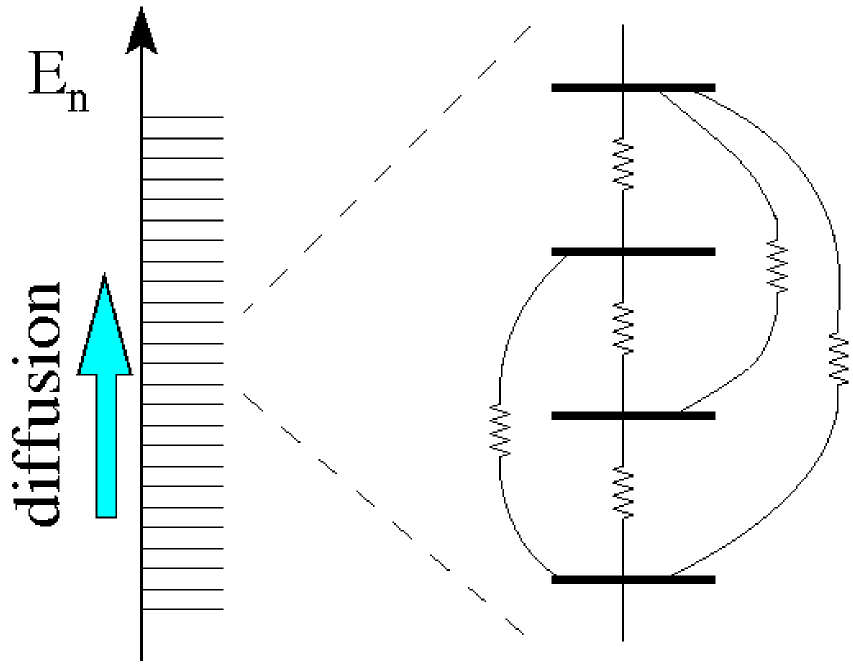
Semilinear response theory: non-Gaussian random ensembles for the study of conductance and energy absorption by vibrating billiards

Doron Cohen

Ben-Gurion University

Collaborations:

Tsampikos Kottos (Wesleyan)
Holger Schanz (Gottingen)
Michael Wilkinson (UK)
Bernhard Mehlig (Goteborg)
Swarnali Bandopadhyay (BGU)
Yoav Etzioni (BGU)
Alex Stotland (BGU)
Rangga Budoyo (Wesleyan)
Tal Peer (BGU)
Nir Davidson (Weizmann)



$ISF, $GIF, $DIP, $BSF

Diffusion and Energy absorption

Driven chaotic system with Hamiltonian $\mathcal{H}(X(t))$

$$X \quad = \quad \text{some control parameter}$$

$$\dot{X} \quad = \quad \text{rate of the (noisy) driving}$$

$\rightsquigarrow$ diffusion in energy space:

$$\boldsymbol{D} \quad = \quad \boldsymbol{G}_{\text{diffusion}} \ \overline{\dot{X}^2}$$

$\rightsquigarrow$ energy absorption:

$$\dot{\boldsymbol{E}} \quad = \quad \boldsymbol{G}_{\text{absorption}} \ \overline{\dot{X}^2}$$

There is a dissipation-diffusion relation.
In the canonical case $\dot{E} = D/T$.

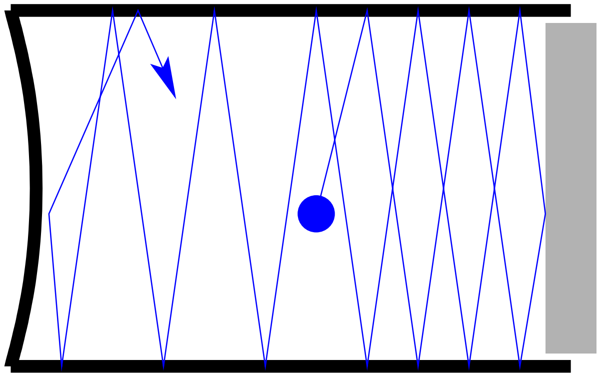
Below we use for G scaled units.

[Ott, Brown, Grebogi, Wilkinson, Jarzynski, Cohen]

Models

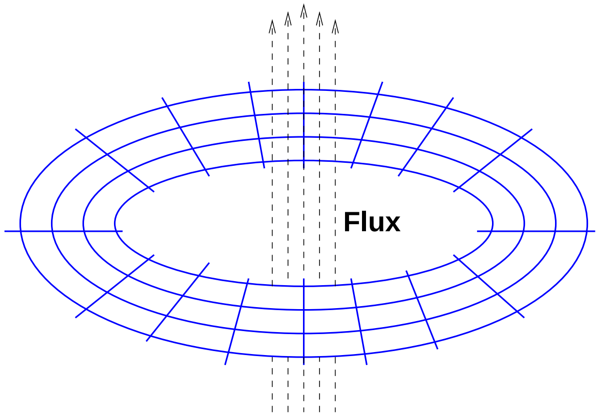
$$\mathcal{H} = \{E_n\} - f(t)\{V_{nm}\},$$

$f(t) \equiv X(t) - X_0$ = low freq noisy driving



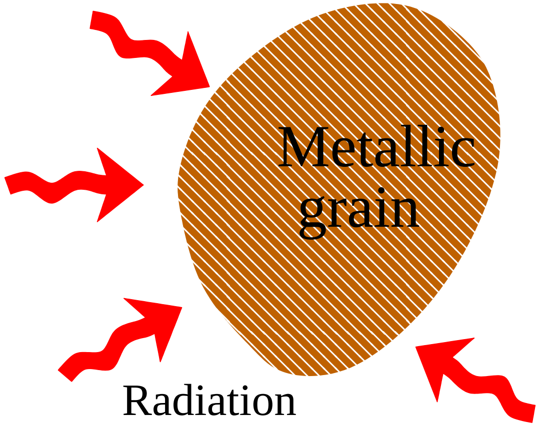
2-D box with a vibrating wall.

[6][7]



Disordered ring driven by EMF.

[1][2][4][5]



Irradiated metallic grains.

[3]

[1] D. Cohen, T. Kottos, H. Schanz (JPA 2006)

[2] S. Bandopadhyay, Y. Etzioni, D. Cohen (EPL 2006)

[3] M. Wilkinson, B. Mehlig, D. Cohen (EPL 2006)

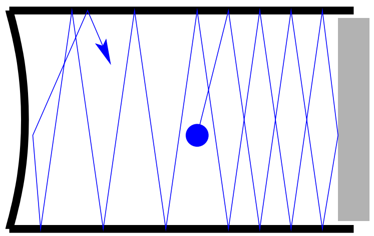
[4] D. Cohen (PRB 2007)

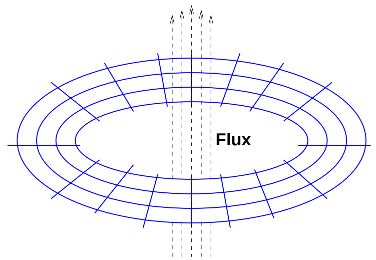
[5] A. Stotland, R. Budoyo, T. Peer, T. Kottos, D. Cohen (JPA/FTC 2008)

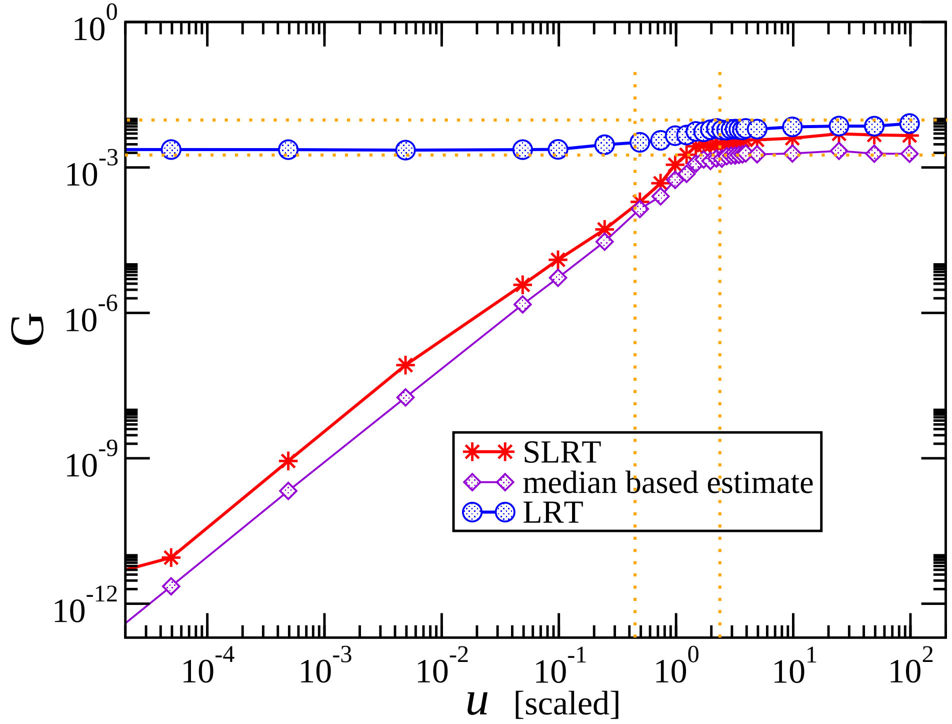
[6] A. Stotland, D. Cohen, N. Davidson (EPL 2009)

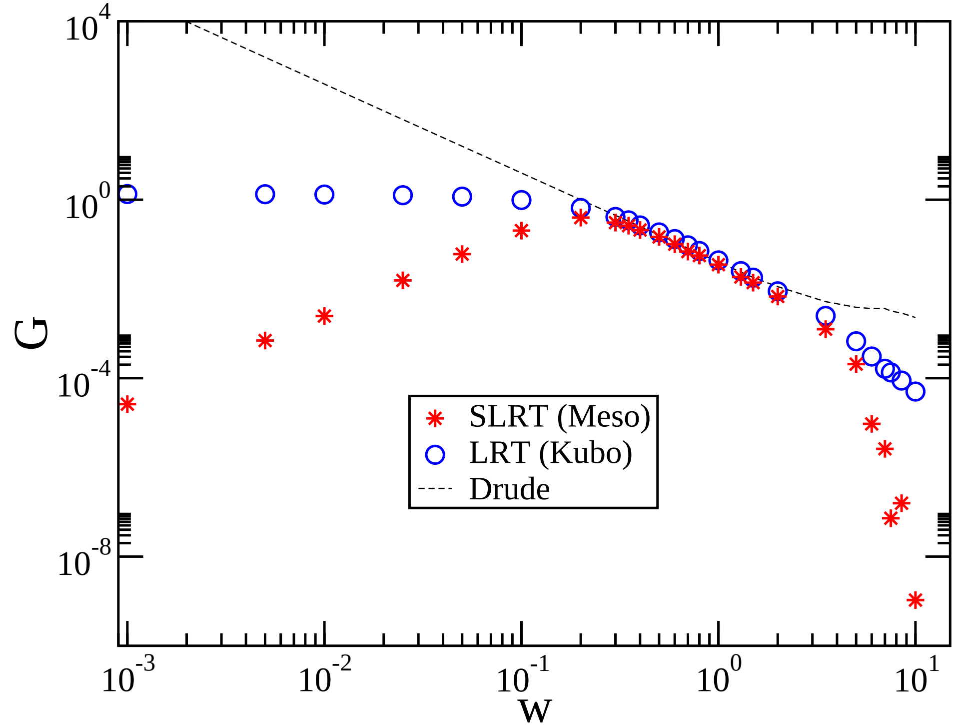
[7] A. Stotland, T. Kottos, D. Cohen (arXiv 2009)

Some results









u - strength of the deformation

W - strength of the disorder

The resistor network calculation

$$\mathcal{H} = \{E_n\} - f(t)\{V_{nm}\}$$

$$g_{\text{SLRT}} \equiv \frac{\langle\langle |V_{mn}|^2 \rangle\rangle_{\text{SLRT}}}{\langle\langle |V_{mn}|^2 \rangle\rangle_{\text{algebraic}}}$$

$$\boldsymbol{G}_{\text{LRT}} = \pi \varrho_{\text{E}} \langle\langle |V_{mn}|^2 \rangle\rangle_{\text{algebraic}}$$

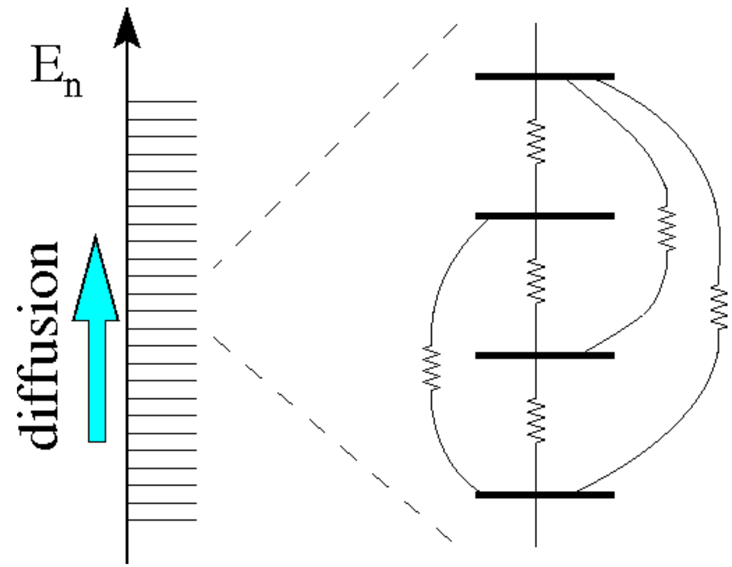
$$\boldsymbol{G}_{\text{SLRT}} = \pi \varrho_{\text{E}} \langle\langle |V_{mn}|^2 \rangle\rangle_{\text{SLRT}}$$

LRT applies if the driven transitions are slower than the environmental relaxation

$$\mathrm{g}_{nm} = 2\varrho_{\text{E}}^{-3} \frac{|V_{nm}|^2}{(E_n - E_m)^2} \delta_0(E_n - E_m)$$

$$\langle\langle |V_{mn}|^2 \rangle\rangle_{\text{SLRT}} \equiv \text{inverse resistivity}$$

$$\langle\langle |V_{mn}|^2 \rangle\rangle_{\text{harmonic}} \ll \langle\langle |V_{mn}|^2 \rangle\rangle_{\text{SLRT}} \ll \langle\langle |V_{mn}|^2 \rangle\rangle_{\text{algebraic}}$$



Digression: the Fermi golden rule picture

Master equation:

$$\frac{dp_n}{dt} = -\sum_m w_{nm}(p_n - p_m)$$

The Hamiltonian in the standard representation:

$$\mathcal{H} = \{E_n\} - f(t)\{V_{nm}\}$$

The transformed Hamiltonian:

$$\tilde{\mathcal{H}} = \{E_n\} - \dot{f}(t)\left\{\frac{iV_{nm}}{E_n - E_m}\right\} \qquad \tilde{S}(\omega) \equiv \text{FT}\,\langle \dot{f}(t)\dot{f}(0)\rangle = \overline{|\dot{f}|^2}\,\delta_0(E_n - E_m)$$

The FGR transition rate due to the low frequency noisy driving:

$$w_{nm} = 2\pi \left|\frac{V_{nm}}{E_n - E_m}\right|^2 \tilde{S}(E_n - E_m) = \text{const} \times \mathbf{g}_{nm} \times \overline{\dot{f}^2}$$

The LRT / SLRT formula

$$\boldsymbol{D} = \pi \varrho_{\text{E}} \,\langle\langle |V_{mn}|^2 \rangle\rangle \times \overline{\dot{f}^2} \equiv \boldsymbol{G}\,\overline{\dot{f}^2}$$

Digression: random walk and the calculation of the diffusion coefficient

w_{nm} = probability to hop from m to n per step.

$$\text{Var}(n) = \sum_n [w_{nm} t]\,(n-m)^2 \equiv 2Dt$$

For n.n. hopping with rate w we get $D = w$.

The continuity equation:

$$\frac{\partial p_n}{\partial t} = -\frac{\partial}{\partial n} J_n$$

Fick's law:

$$J_n = -D \frac{\partial}{\partial n} p_n$$

The diffusion equation:

$$\frac{\partial p_n}{\partial t} = D \frac{\partial^2}{\partial n^2} p_n$$

If we have a sample of length N then

$$J = -\frac{D}{N} \times [p_N - p_0]$$

$$D/N = \text{inverse resistance of the chain}$$

If the w are not the same:

$$\frac{D}{N} = \left[\sum_{n=1}^{N} \frac{1}{w_{n,n-1}}\right]^{-1}$$

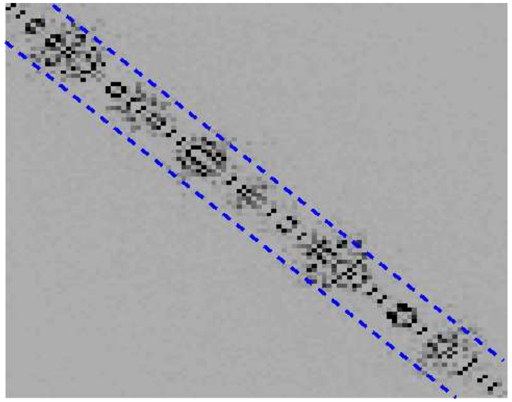
Hence, for n.n. hopping

$$D = \langle\langle w \rangle\rangle_{\text{harmonic}}$$

In general, $w_{nm} = W_{nm} f(n-m)$

$$D = \langle\langle (n-m)^2 W_{nm} \rangle\rangle$$

$\{|V_{nm}|^2\}$ **as a random matrix** $\{x\}$

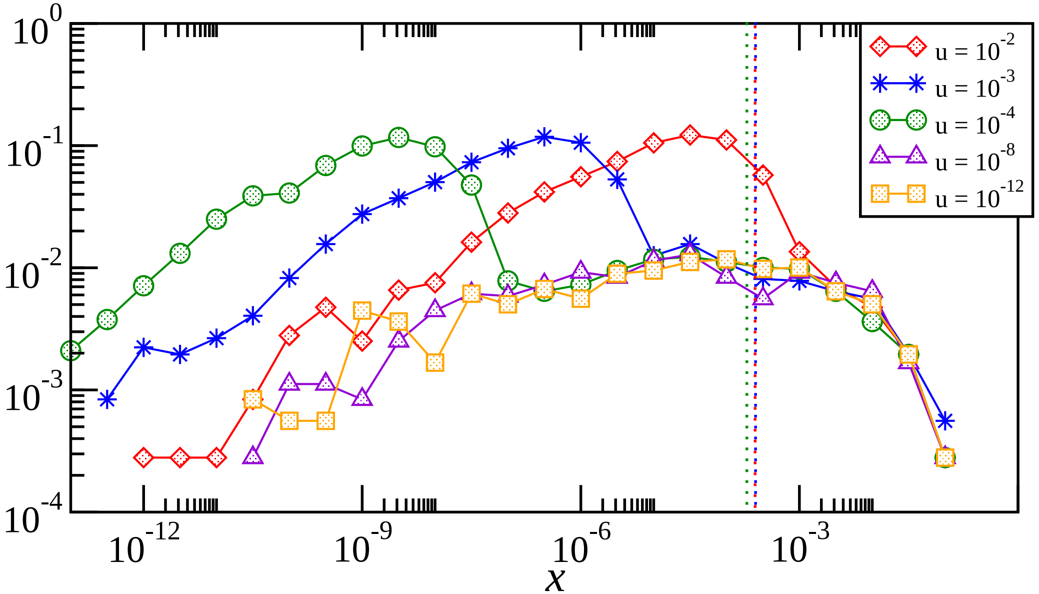


- bandwidth
- texture
- sparsity

$$q \equiv \frac{\langle\langle x\rangle\rangle_g}{\langle\langle x\rangle\rangle_a}$$

$$p \equiv \text{Prob}\Big(x > \langle x\rangle\Big)$$

Histogram of x :



$x \sim$ LogNormal

Averages:

Algebraic: $\langle\langle x\rangle\rangle_a = \langle x\rangle$

Harmonic: $\langle\langle x\rangle\rangle_h = [\langle 1/x\rangle]^{-1}$

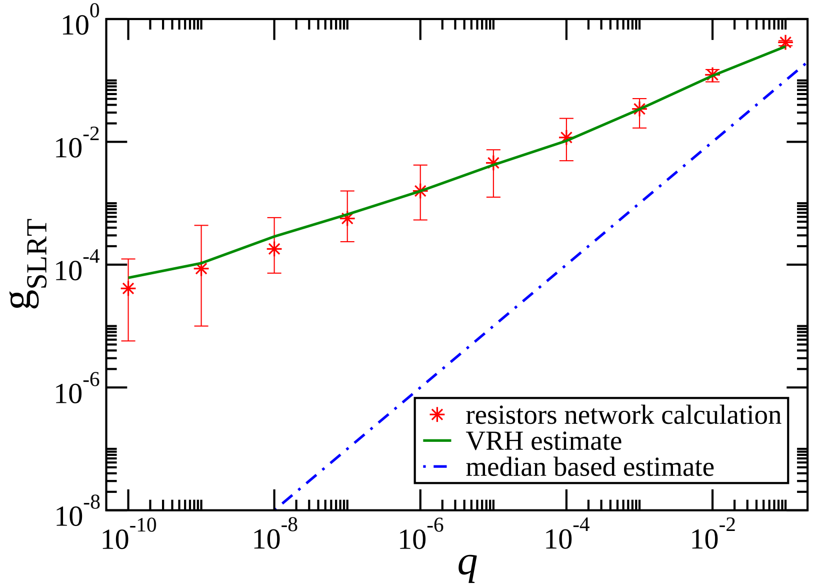
Geometric: $\langle\langle x\rangle\rangle_g = \exp[\langle \log x\rangle]$

For "log - wide" distributions:

$$\langle\langle x\rangle\rangle_h \ll \langle\langle x\rangle\rangle_g \ll \langle\langle x\rangle\rangle_a$$

RMT modeling, generalized VRH approx scheme

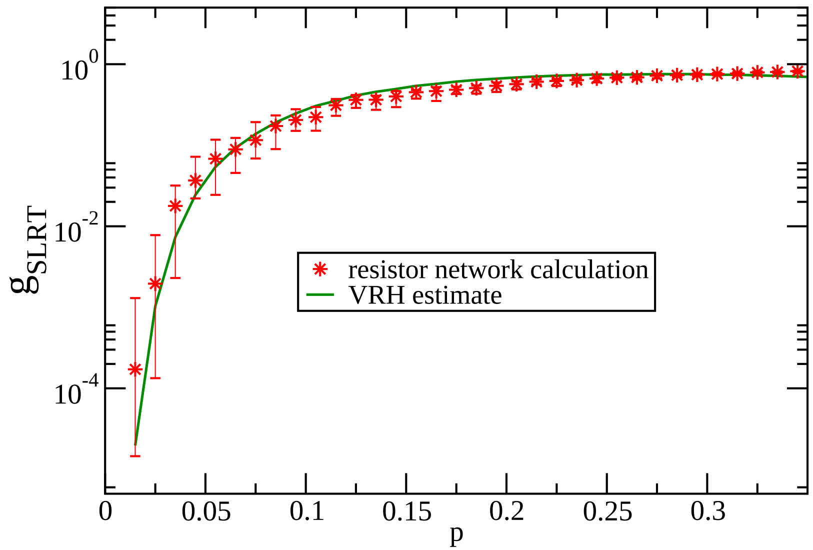
- Log-normal RMT modeling



For the rectangular $\tilde{S}(\omega)$ of width ω_c

$$g_{\text{SLRT}} \approx q \, \exp\left[2\sqrt{-\ln q \, \ln(\omega_c/\Delta)}\right]$$

- Log-box RMT modeling



For the exponential $\tilde{S}(\omega)$ of width ω_c

$$g_{\text{SLRT}} \approx \frac{1}{p} \exp\left[-2\sqrt{\frac{\Delta}{p\,\omega_c}}\right]$$

Digression: Generalized VRH

Definition of the typical matrix element for a range ω transition:

$$\left(\frac{\omega}{\Delta}\right) \mathrm{Prob}\Big(x > x_{\omega}\Big) \;\sim\; 1$$

In the standard-like case (ring with strong disorder):

$$x_{\omega} \;\approx\; v_{\mathrm{F}}^2 \;\exp\left(\frac{\Delta_l}{|\omega|}\right)$$

[corresponding to a log-box distribution]

An example for the power spectrum of the driving:

$$\tilde{S}(\omega) \;\propto\; \exp\left(-\frac{|\omega|}{T}\right)$$

[here the temparature $T \Longleftrightarrow \omega_c$]

Generalized VRH estimate:

$$D_{\mathrm{SLRT}} \;\approx\; \int x_{\omega} \; \tilde{S}(\omega) d\omega$$

[should be contrasted with]

$$D_{\mathrm{LRT}} = \int \tilde{C}(\omega) \; \tilde{S}(\omega) d\omega$$

In the standard-like case (ring with strong disorder):

$$D_{\mathrm{SLRT}} \;\approx\; \int \exp\left(\frac{\Delta_l}{|\omega|}\right) \exp\left(-\frac{|\omega|}{T}\right) \, d\omega$$

SLRT *vs* LRT

$$\mathcal{H}(X(t)) \approx \mathcal{H}_0 + f(t)V$$
$$f(t) = X(t) - X_0$$

Kubo formula:

$$\boldsymbol{D} = \int \tilde{C}(\omega)\tilde{S}(\omega)d\omega$$

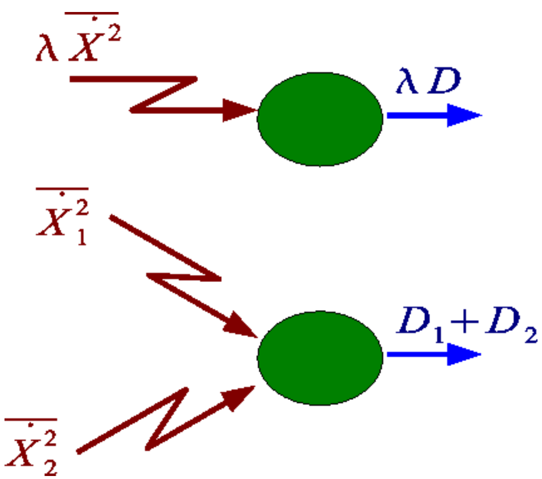
$$\tilde{C}(\omega) = \text{FT}\ \langle V(t)V(0)\rangle$$
$$\tilde{S}(\omega) = \text{FT}\ \langle \dot{f}(t)\dot{f}(0)\rangle$$

SLRT example:

$$\boldsymbol{D} = \left[\int R(\omega)\left[\tilde{S}(\omega)\right]^{-1} d\omega\right]^{-1}$$

Linear response implies

$$\tilde{S}(\omega) \mapsto \lambda\tilde{S}(\omega) \quad \Longrightarrow \quad \boldsymbol{D} \mapsto \lambda\boldsymbol{D}$$
$$\tilde{S}(\omega) \mapsto \sum_i \tilde{S}_i(\omega) \quad \Longrightarrow \quad \boldsymbol{D} \mapsto \sum_i \boldsymbol{D}_i$$



The quest for quantum anomalies, beyond LRT (1995-2000)

• Anomaly means that LRT applies classically,
but at the same circumstances - not upon quantization.

• The obvious quantum anomaly [1]:
adiabatic driving, Landau-Zener mechanism.

• An anomaly in the FGR regime? [2].
Turns out to be a non-generic artifact of their RMT model [3].

• An anomaly due to breakdown of FGR picture [3]:
perturbative/FGR/Wigner *vs* non-perturbative/semiclassical regimes [4].
Demonstration for the standard Wigner (banded) RMT model [5].

Classical correlation time:	t_{cl}	(determines the bandwidth $= \hbar/t_{cl}$)
Generalized Wigner time:	$t_{\varepsilon} = (\hbar^2/D_E)^{1/3}$	(by Kubo $D_E \propto \varepsilon^2$)
FGR/Wigner regime:	$t_{cl} \ll t_{\varepsilon}$	(violated if ε is too large)

[**1**] Wilkinson (JPA 1988)
[**2**] Wilkinson and Austin (JPA 1995)
[**3**] D.C. (PRL 1999)
[**4**] D.C. and Heller (PRL 2000)
[**5**] D.C. and Kottos (PRL 2000)

The quest for quantum anomalies, beyond LRT (2001-2004)

- New challenge in the theory of wavepaket dynamics - *The study of fidelity* [1]
- Regimes in the theory of fidelity [2,3]
- New capabilities arise - *The study of driven billiards* [4,5]
- Absence of the conjectured anomaly [6,7,8] - unlike in the theory of fidelity

RMT fails to describe Semiclassics in the non-perturbative regime.

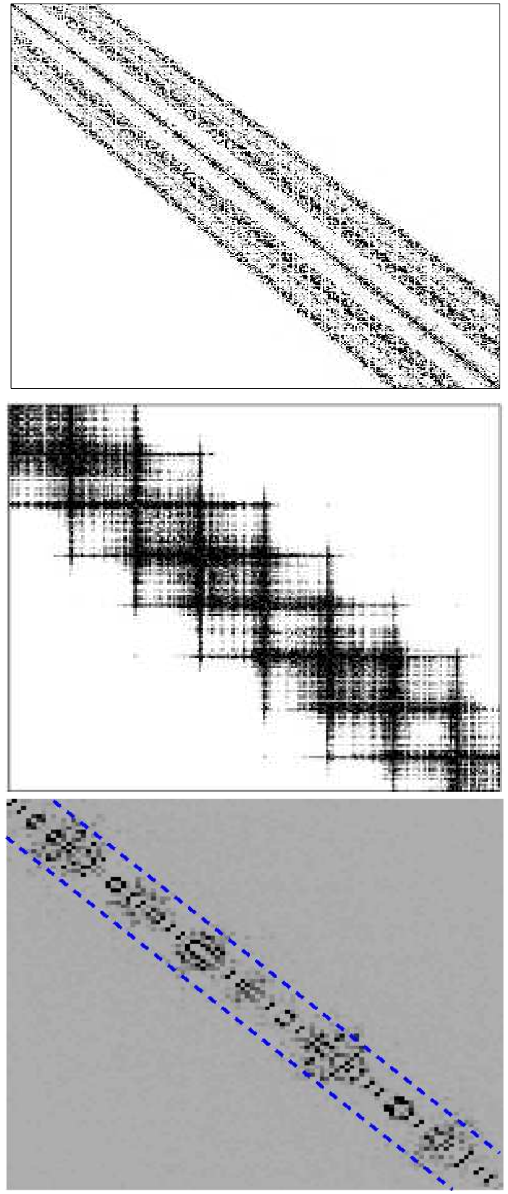
LRT survives due to detailed QCC, in-spite of the breakdown of FGR.

[1] Cucchietti, Pastawski, Jalabert, Wisniacki, Lewenkopf, Mucciolo, Vallejos (2000-2002)
[2] Jacquod, Silvestrov, Beenakker (PRE 2001)
[3] Wisniacki and D.C. (PRE 2002)
[4] Wisniacki and Vergini (PRE 1999)
[5] Barnett, D.C., Heller (2000-2001)
[6] Kottos and D.C. (PRE 2001) - Failure of RMT to correctly describe quantum dynamics
[7] D.C. and Wisniacki (PRE 2003) - Stadium Billiard with Moving Walls
[8] Silva and Kravtsov (PRB 2007)

The quest for quantum anomalies, beyond LRT (2005-2009)

$$\tilde{C}(\omega) \equiv \langle\langle |V_{nm}|^2 \rangle\rangle = \overline{\sum_n |V_{nn_0}|^2 \, \delta(\omega - (E_n - E_{n_0}))}$$

$$\boldsymbol{G} = \pi \varrho_{\mathrm{E}} \, \tilde{C}(\omega{\sim}0) = \pi \varrho_{\mathrm{E}} \, \langle\langle |V_{mn}|^2 \rangle\rangle$$



Sparsity and texture are not reflected in Kubo!
LRT can fail even if there is no breakdown of FGR.
FGR implies Semilinear response.

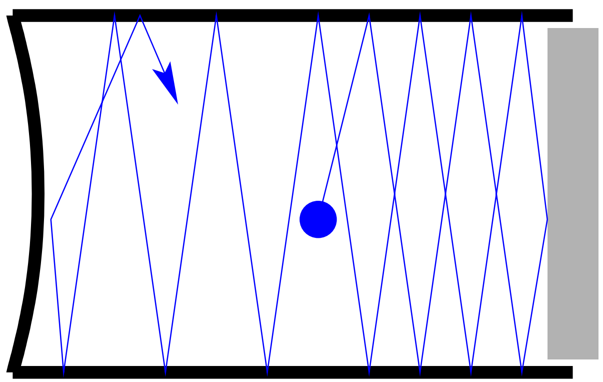
Single-channel ballistic ring coupled to a big network [1]
Multi-channel ballistic ring with a scatterer [2]
Metallic grains [3]
Multichannel channel ring with disorder, tight binding model [4,5]
Billiard with a vibrating wall [6]

[1] D.C, Kottos, Schanz (2005..., JPA 2006)
[2] Bandopadhyay, Etzioni, D.C. (EPL 2006)
[3] Wilkinson, Mehlig, D.C, (EPL 2006)
[4] D.C. (PRB 2007)
[5] Stotland, Budoyo, Peer, Kottos, D.C. (JPA/FTC 2008)
[6] Stotland, D.C., Davidson (EPL 2009)

Billiard with a vibrating wall

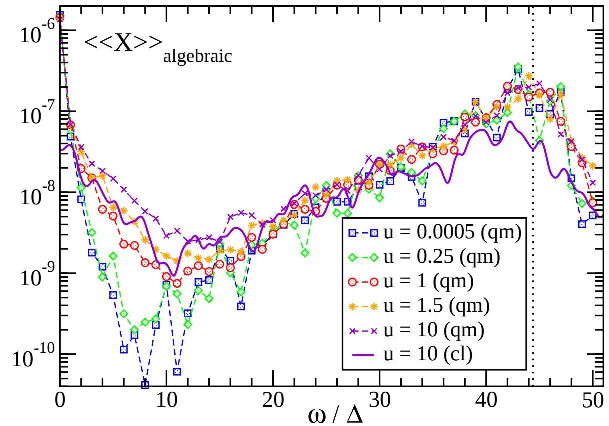
The Hamiltonian in the $\boldsymbol{n} = (n_x, n_y)$ basis:

$$\mathcal{H} = \mathrm{diag}\{E_{\boldsymbol{n}}\} + u\{U_{\boldsymbol{nm}}\} + f(t)\{V_{\boldsymbol{nm}}\}$$



The matrix elements for the wall displacement:

$$V_{\boldsymbol{nm}} = -\delta_{n_y,m_y} \times \frac{\pi^2}{\mathsf{M} L_x^3} n_x m_x \qquad \text{[sparse]}$$



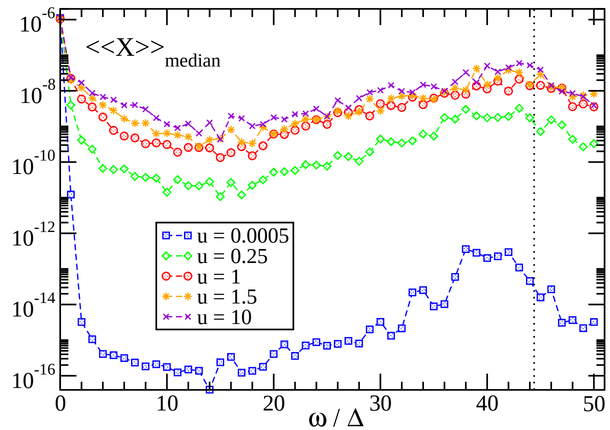
The Hamiltonian in the E_n basis:

$$\mathcal{H} = \mathrm{diag}\{E_n\} + f(t)\{V_{nm}\}$$

Statsistics along the diagonal $E_n - E_m = \omega$

$$\langle\langle |V_{nm}|^2 \rangle\rangle_{\text{algebraic}} = \text{average value}$$

$$\langle\langle |V_{nm}|^2 \rangle\rangle_{\text{median}} = \text{median value}$$



Conclusions

(*) Wigner ($\sim$ 1955):
The perturbation is represented by a random matrix whose elements are taken from a Gaussian distribution.

Not always...

1. No "strong quantum chaos" $\Longrightarrow$ log-normal distribution.
2. The heating process $\sim$ a percolation problem.
3. Resistors network calculation to get G_{SLRT}.
4. Generalization of the VRH estimate
5. SLRT is essential whenever the distribution of matrix elements is wide ("sparsity") or if the matrix has "texture".