Triangular Bose-Hubbard trimer as a minimal model for a superfluid circuit

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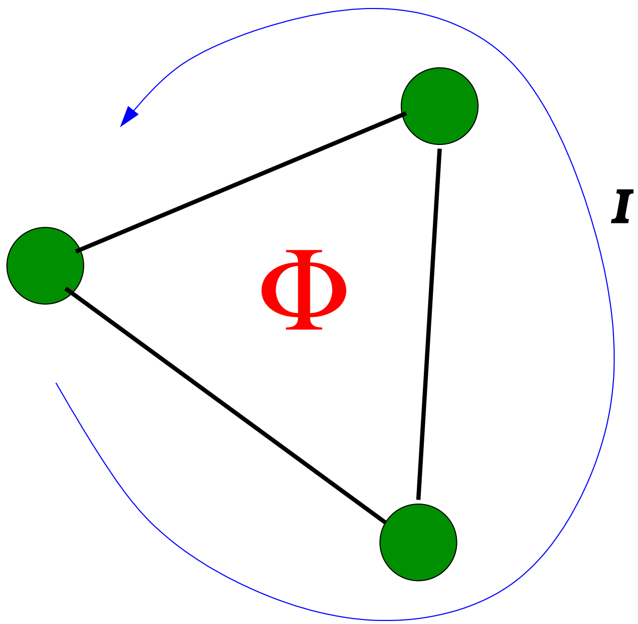


The Model

A rotating 3 site system with N bosons.

$$\mathcal{H} \;=\; \sum_{j=1}^{M} \left[\frac{U}{2} a_j^{\dagger} a_j^{\dagger} a_j a_j - \frac{K}{2}\left(\mathrm{e}^{i(\Phi/M)} a_{j+1}^{\dagger} a_j + \mathrm{e}^{-i(\Phi/M)} a_j^{\dagger} a_{j+1}\right)\right] \quad , \quad M{=}3$$

$$N \;=\; \sum_{j=1}^{M} n_j \quad , \quad n_j \;=\; a_j^{\dagger} a_j$$

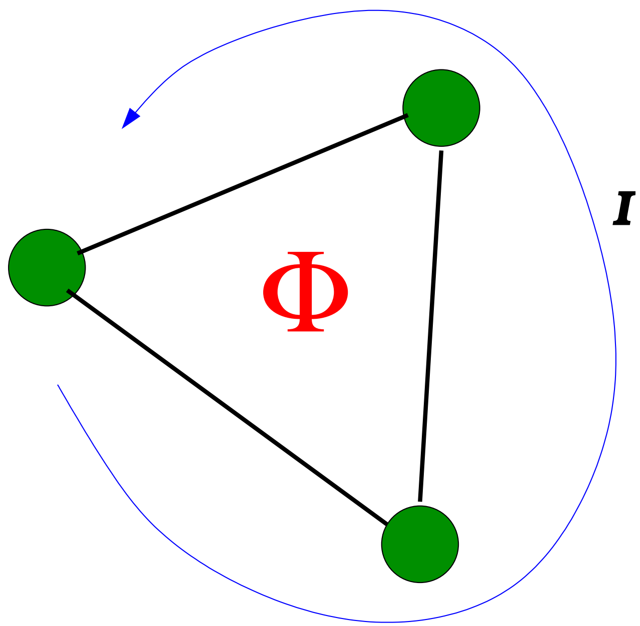
like M "coupled oscillators".

Dimensionless parameters (Φ, u):

$$u \;=\; \frac{NU}{K}$$

$$\Phi \;=\; \frac{M^2}{2\pi}\left(\frac{\mathsf{m}}{\mathsf{m}_{\mathrm{eff}}}\right)\frac{\Omega}{K}$$

Upon quantization we have $\hbar = 1/N$.



Eigenstates

For a fixed particle number $N = n_1 + n_2 + n_3$ the BH trimer can be regarded as a 2 degrees of freedom system with coordinates: $(n_1 - n_2, n_3)$.

$$|\psi(\mathbf{r})|^2 = |\langle \mathbf{r}|E_\alpha\rangle|^2 \quad , \quad \mathbf{r} = \frac{1}{N}(n_1 - n_2, n_3)$$

For each eigenstate $|E_\alpha\rangle$ we calculate:

one-body reduced probability matrix:

$$\rho_{ij} = \langle a_j^\dagger a_i \rangle_\alpha$$

$$S_\alpha \equiv \mathrm{trace}(\rho^2) \quad , \quad 1/S \in [1,3]$$

$1/S = \#$ of participating orbitals.

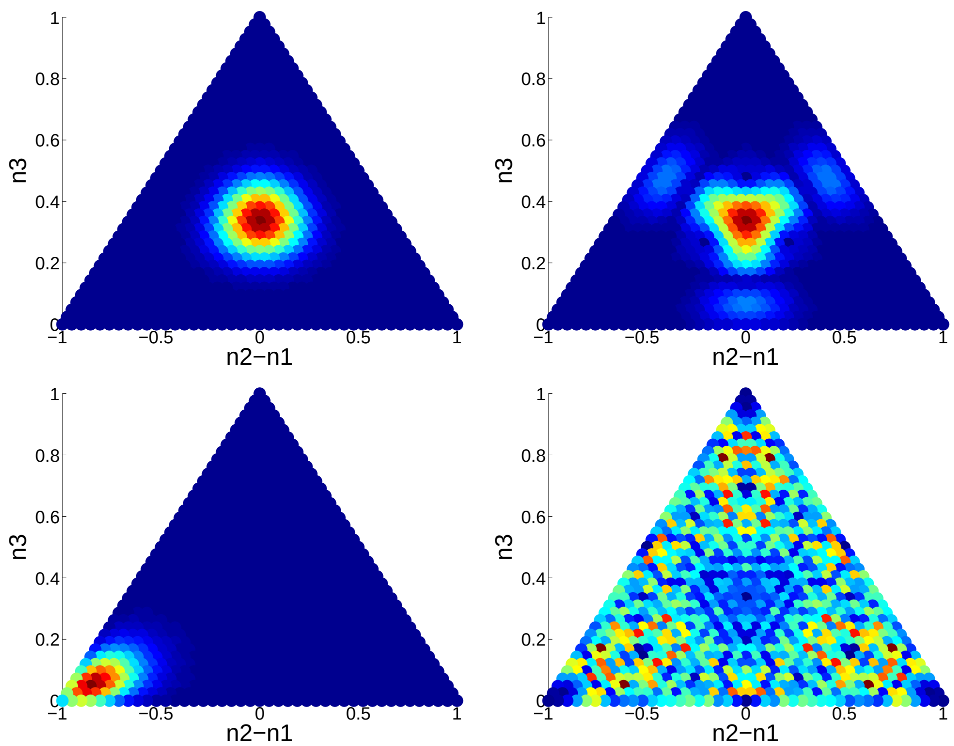
$1/S = 1$ means a coherent state.

$1/S = 3$ is a maximum fragmentation.

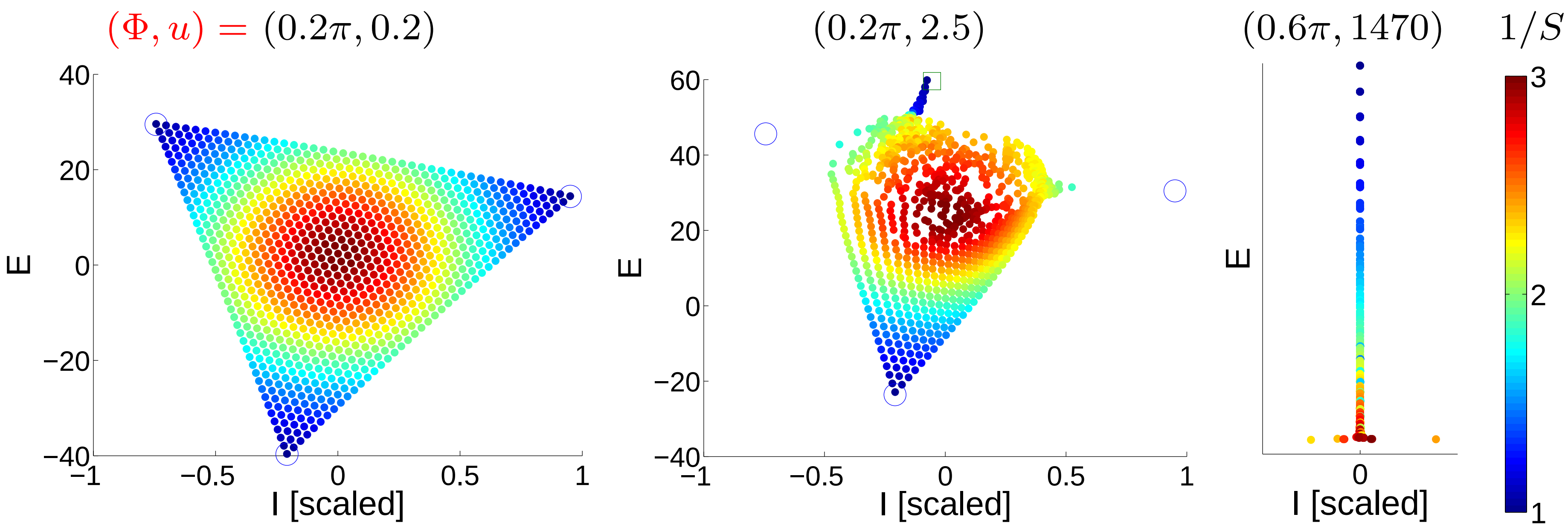
Bond averaged current:

$$\mathcal{I}_\alpha \equiv -\left\langle \frac{\partial \mathcal{H}}{\partial \Phi} \right\rangle_\alpha$$

Representative Eigenstates:



The energy spectrum of the trimer ($N = 42$)



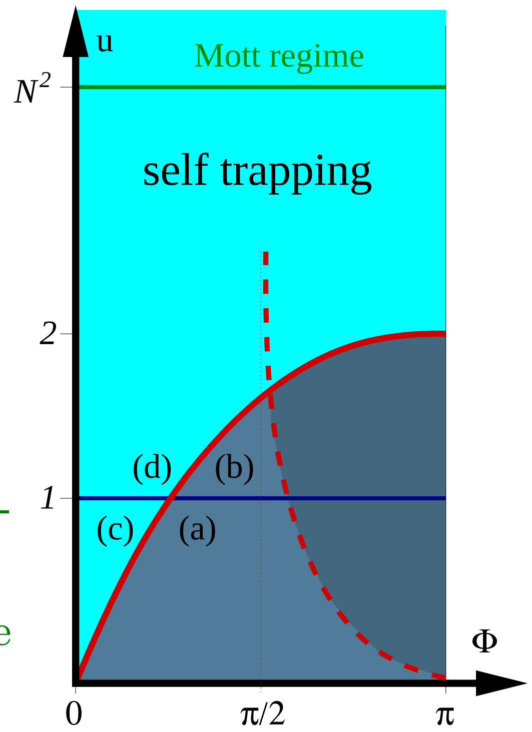
- Vortex States $\qquad m = 0, \pm 1$

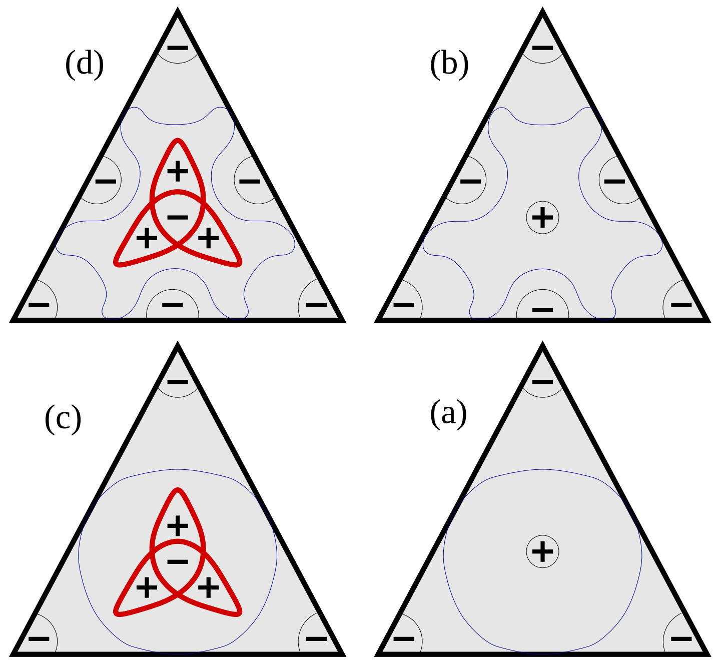
$$I_m \;=\; \frac{N}{M} K \sin\left(\frac{1}{M}(2\pi m - \Phi)\right)$$

- Mott Transition
- Self-Trapping (Bright Solitons)
- Metastable Vortex state

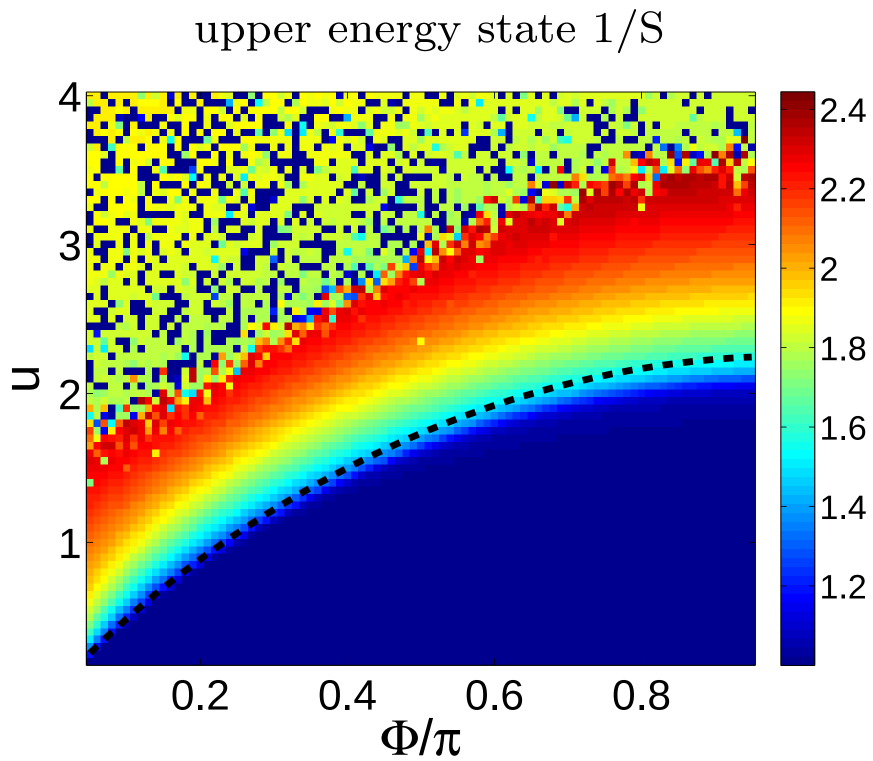
Vortex state = Condensation in momentum orbital.

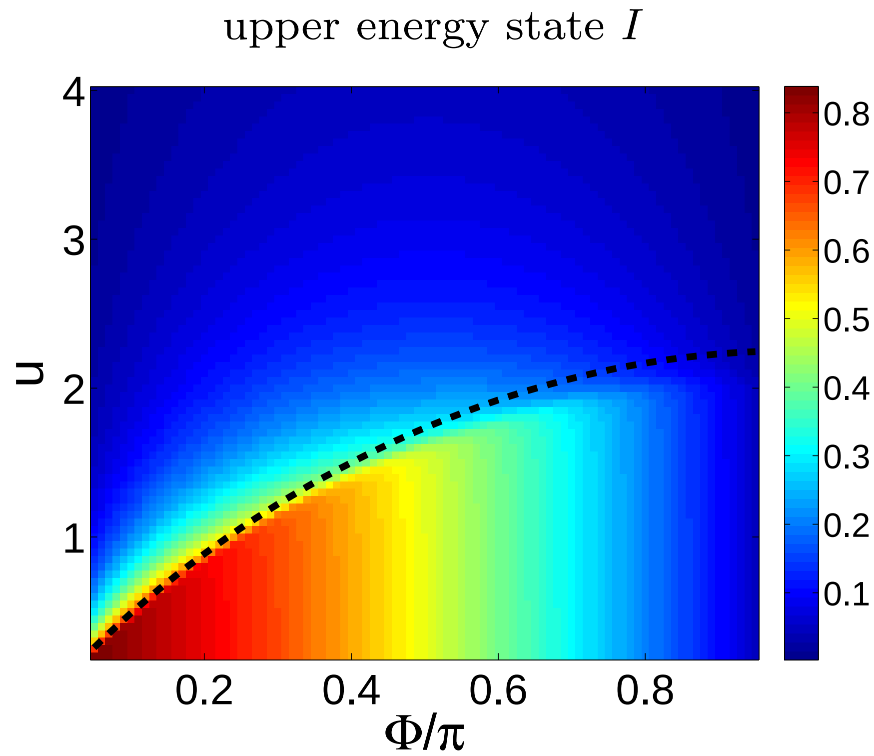
Self-trapped state = Condensation in site orbital.





Self Trapping





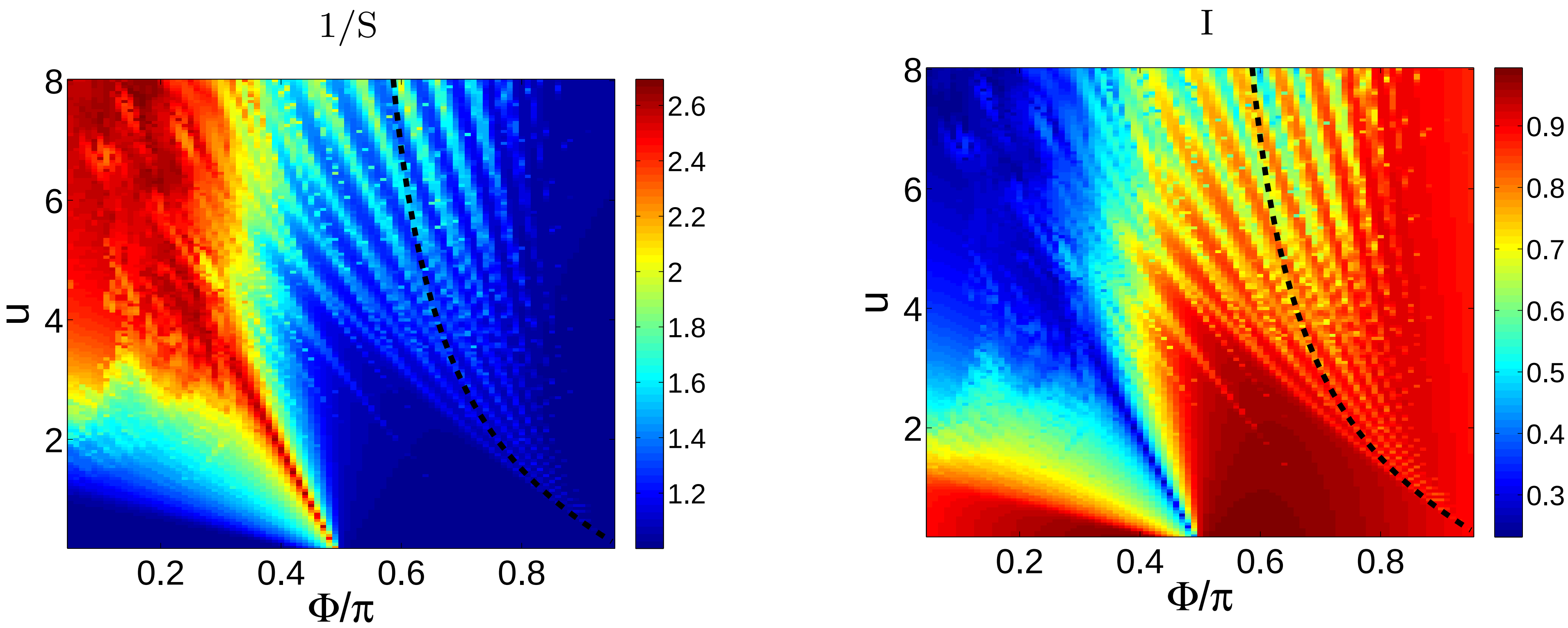
$$u = \frac{6 - 9\cos\left(\frac{\pi+2\Phi}{3}\right) - 3\cos\left(\frac{\pi-4\Phi}{3}\right)}{6\cos\left(\frac{\pi-\Phi}{3}\right) - 2\cos(\Phi)}$$

$$I_m = \frac{N}{M} K \sin\left(\frac{1}{M}(2\pi m - \Phi)\right)$$

$$m = -1$$

- Self-trapping can occur for arbitrarily small interaction.

Quasi-Stability



$$u = \frac{6 - 9\cos\left(\frac{\pi - 2\Phi}{3}\right) - 3\cos\left(\frac{\pi + 4\Phi}{3}\right)}{6\cos\left(\frac{\pi + \Phi}{3}\right) - 2\cos(\Phi)}$$

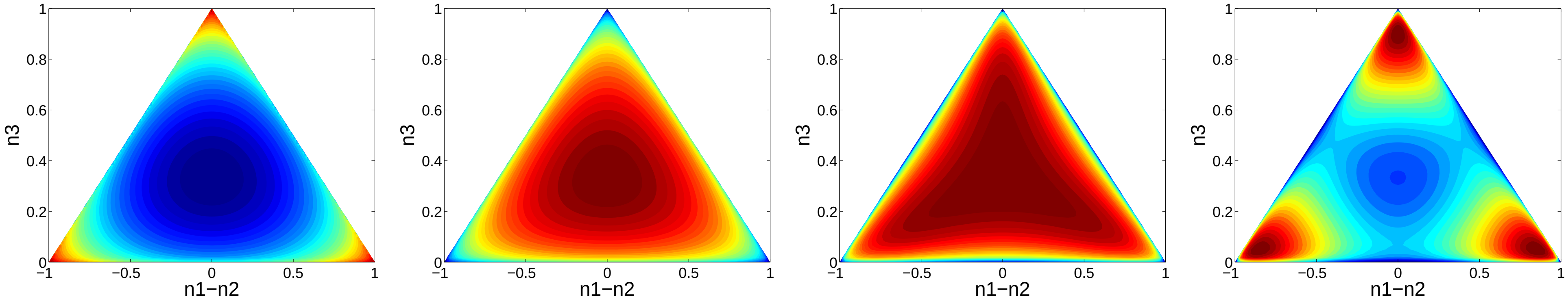
- The quantum metastability regime extends beyond the classically expected.
- Quantum scarring related effect(?).

The energy landscape

$$\mathcal{H} = \sum_{j=1}^{M} \left[\frac{U}{2} \boldsymbol{n}_j^2 - K \sqrt{\boldsymbol{n}_{j+1} \boldsymbol{n}_j} \cos\left((\varphi_{j+1} - \varphi_j) - \frac{\Phi}{M} \right) \right] \quad , \quad a_j = \sqrt{\boldsymbol{n}_j} \mathrm{e}^{i\varphi_j}$$

$$\mathcal{H} = \frac{U}{2} \left(\boldsymbol{n}_1^2 + \boldsymbol{n}_2^2 + \boldsymbol{n}_3^2 \right) - K \left(\sqrt{\boldsymbol{n}_2 \boldsymbol{n}_3} \cos(q_1) + \sqrt{\boldsymbol{n}_3 \boldsymbol{n}_1} \cos(q_2) + \sqrt{\boldsymbol{n}_1 \boldsymbol{n}_2} \cos(\Phi + q_1 + q_2) \right)$$

$$V(r) = \min_{\varphi}[\mathcal{H}(r, \varphi)] \quad \text{or} \quad \max_{\varphi}[\mathcal{H}(r, \varphi)]$$

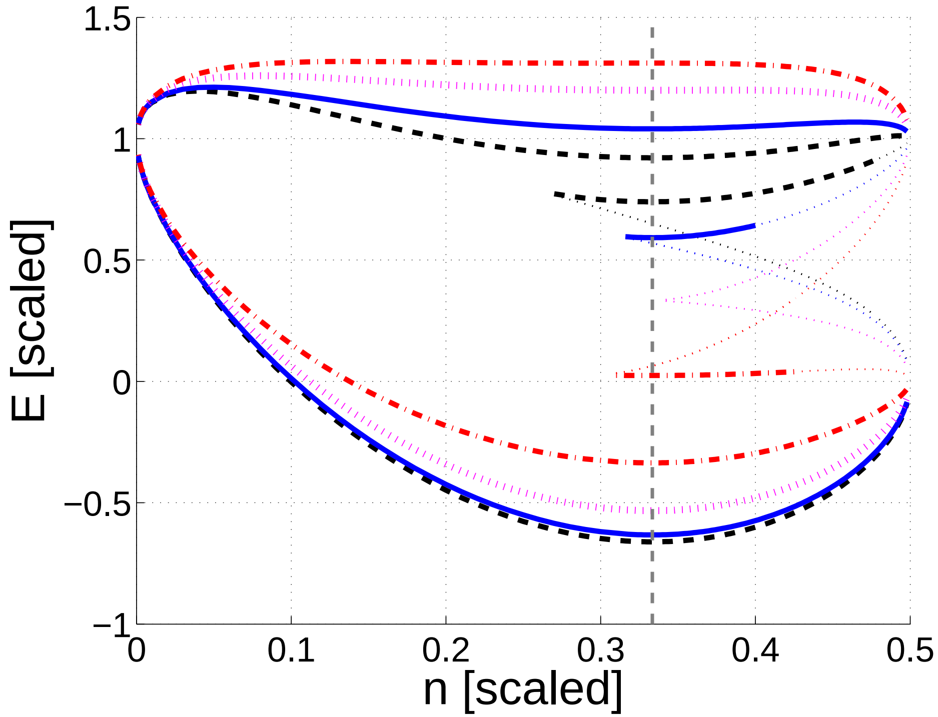


$$E_m = V_m(r_0) = \frac{1}{6} N^2 U - NK \cos\left(\frac{2\pi m - \Phi}{3} \right), \quad m = 0, \pm 1$$

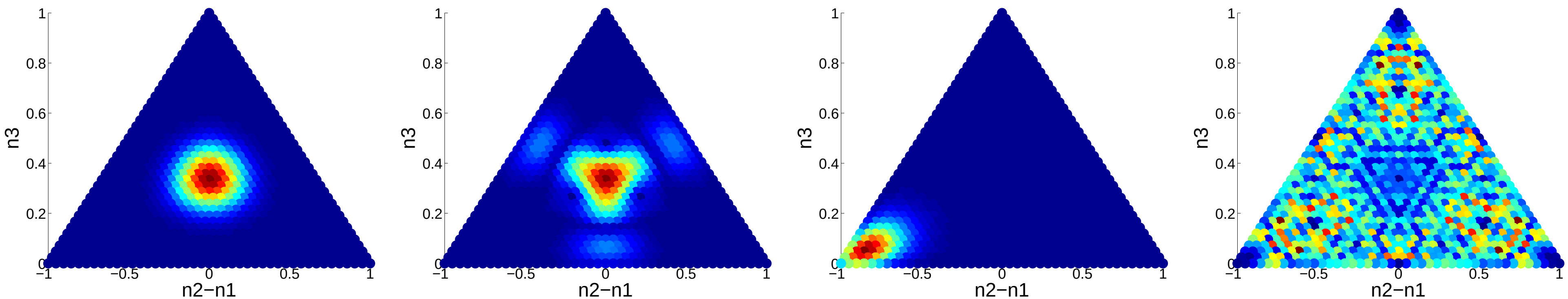
$$I_m = \frac{N}{M} K \sin\left(\frac{1}{M} (2\pi m - \Phi) \right)$$

The energy landscape (cont.)

$$V''_{\pm}(r_0) \;=\; \left.\frac{d^2H(q_{\pm}(n),n)}{dn^2}\right|_{N/3} \;=\; 6U \;-\; 9\frac{K}{N}\left[\frac{2-3\cos\left(\frac{\pi\pm2\Phi}{3}\right)-\cos\left(\frac{\pi\mp4\Phi}{3}\right)}{3\cos\left(\frac{\pi\mp\Phi}{3}\right)-\cos\left(\Phi\right)}\right]$$



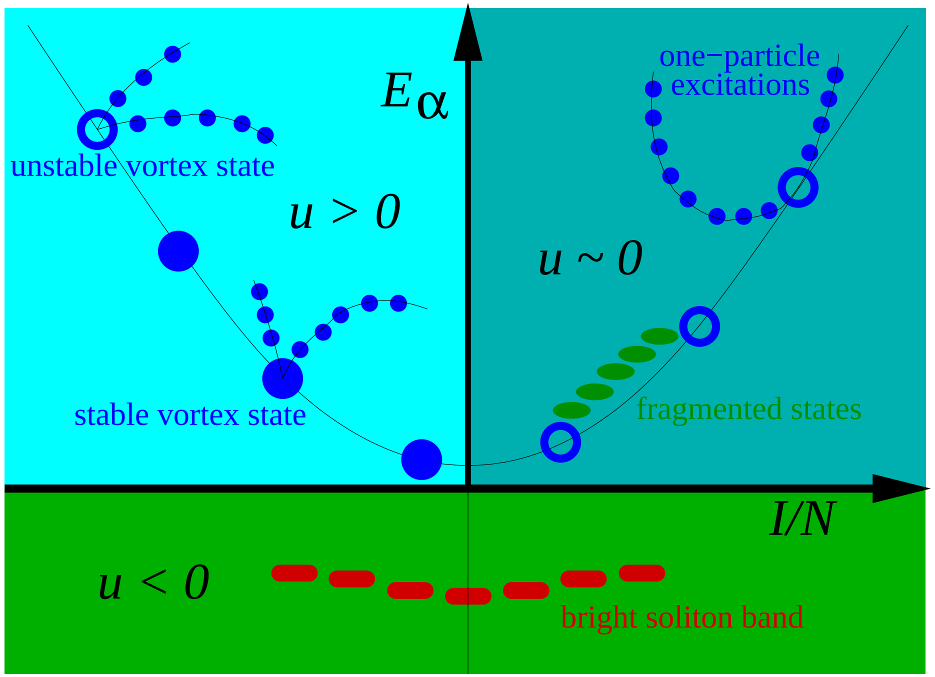
Some eigenstates



Concluding Remarks

- The essence of superfluidity is the possibility to witness metastable vortex states
- In the standard classical stability analysis one finds that vortex states whose rotation velocity is less than a critical velocity are metastable ("Landau criterion")
- For a non-rotating 3-site model the same type of classical analysis implies that there are no metastable vortex states [1,2]
- We have explored the full regime diagram. In the presence of rotation we find (Ω, u) regimes where metastable vortex states exists.

- In the quantum analysis we find that the metastable vortex state is quasi-stable in a much wider regime, even for a small rotation frequency, contrary to the classical expectation.



[1] PC with J. Anglin, T. Leggett, P. Ghosh

[2] P. Ghosh, F. Sols, Phys. Rev. A 77, 033609 (2008)