Triangular Bose-Hubbard trimer as a minimal model for a superfluid circuit

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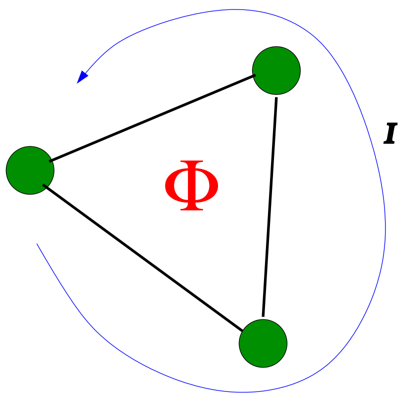
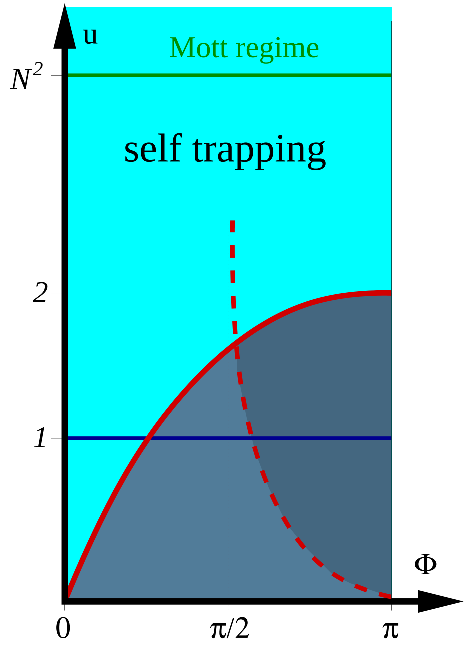
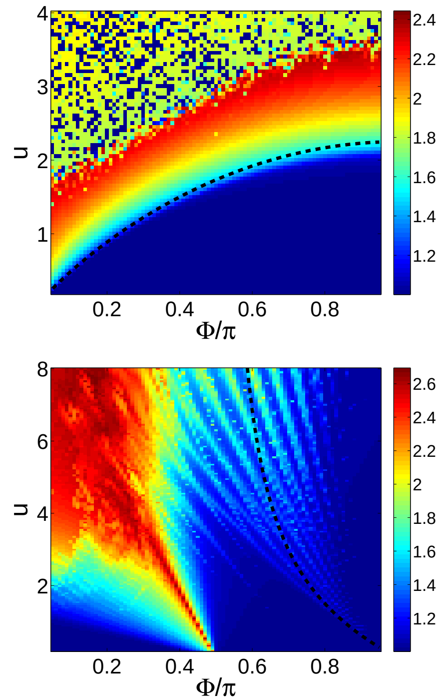
+ additional collaborations (see next page)







Ref: Arwas, Vardi, Cohen (PRA 2014)

http://www.bgu.ac.il/~dcohen

$ISF

BHH - dimers and trimers

The Bose-Hubbard Hamiltonian (BHH):

$$\mathcal{H}_{\text{BHH}} \;=\; \frac{U}{2}\sum_{j=1}^{M} a_j^\dagger a_j^\dagger a_j a_j \;-\; \frac{K}{2}\sum_{j=1}^{M}\left(a_{j+1}^\dagger a_j + a_j^\dagger a_{j+1}\right) \qquad\qquad u \equiv \frac{NU}{K}$$

Dimer (M=2): minimal BHH; Bosonic Josephson junction; Pendulum physics [1,5].

Driven dimer: Landau-Zener dynamics [2], Kapitza effect [3], Zeno effect [4], Standard-map physics [5].

Linear trimer: minimal model for chaos; Coupled pendula physics.

Triangular trimer: minimal model with topology, Superfluidity [6], Stirring [7].

Coupled trimers: minimal model for mesoscopic thermalization [8].

[1] Maya Chuchem, Smith-Mannschott, Hiller, Kottos, Vardi, Cohen (PRA 2010).

[2] Smith-Mannschott, Chuchem, Hiller, Kottos, Cohen (PRL 2009).

[3] Boukobza, Moore, Cohen, Vardi (PRL 2010).

[4] Khripkov, Vardi, Cohen (PRA 2012)

[5] Christine Khripkov, Cohen, Vardi (JPA 2013, PRE 2013).

[6] Geva Arwas, Vardi, Cohen (PRA 2014).

[7] Hiller, Kottos, Cohen (EPL 2008, PRA 2008).

[8] Tikhonenkov, Vardi, Anglin, Cohen, PRL (2013).

The BHH for a dimer is like a pendulum

$$\mathcal{H} \;=\; \frac{U}{2}\sum_{i=1,2}\hat{n}_i(\hat{n}_i-1) \;-\; \frac{K}{2}(\hat{a}_2^\dagger\hat{a}_1+\hat{a}_1^\dagger\hat{a}_2), \qquad \hat{n}_i \equiv \hat{a}_i^\dagger\hat{a}_i, \qquad \hat{n} \equiv \frac{1}{2}\left(\hat{n}_1-\hat{n}_2\right), \qquad u \equiv \frac{NU}{K}$$

N particles in a double well
is like spin $j = N/2$ system

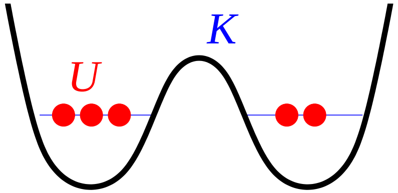
$$\mathcal{H} \;=\; U\hat{J}_z^2 \;-\; K\hat{J}_x$$

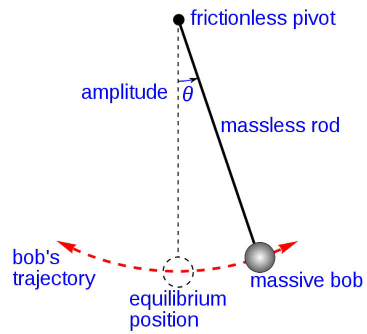
$\hat{J}_z =$ occupation difference

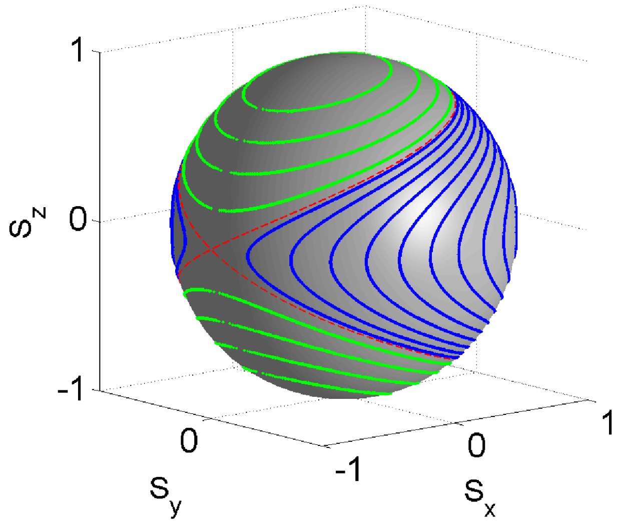
Analogous to Josephson junction
if the occupation difference $\ll N/2$

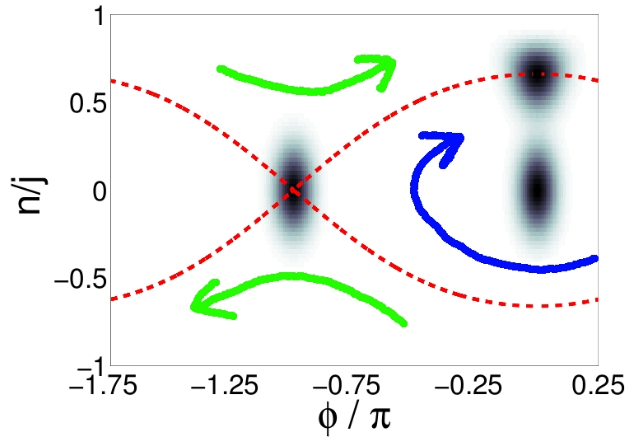
$$\mathcal{H}(n,\varphi) \;=\; Un^2 - \frac{NK}{2}\cos(\varphi)$$

$\hat{n} =$ occupation difference









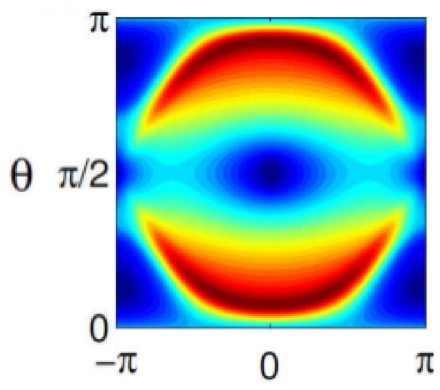
Quasi stability

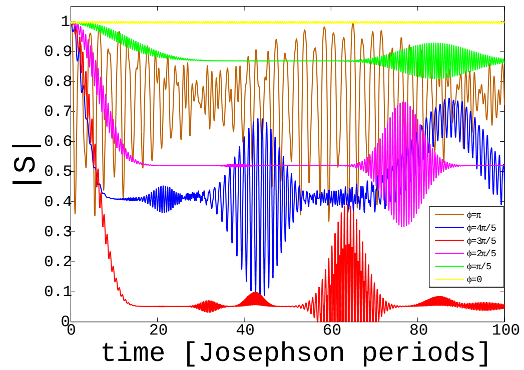
"0" preparation - all bosons are condensed in the lower orbital

"π" preparation - all bosons are condensed in the upper orbital

φ preparation - all bosons are condensed in $|1\rangle + \mathrm{e}^{i\varphi}|2\rangle$

$$\mathrm{PN} \approx \begin{cases} \sqrt{u} & \text{"0" state} \\ \sqrt{u}\log(N/u) & \text{"}\pi\text{" state} \\ \sqrt{N} & \text{generic} \end{cases}$$





- The "π" preparation is classically unstable: irreversible decay
- Quantum mechanically the π preparation is quasi-periodic $t_{\text{recurrances}} \sim \log(N)$
- Generic φ preparations that are quasi-irreversible $t_{\text{recurrances}} \sim \sqrt{N}$

Notes:

- Similar, but not the same: Scars at hyperbolic points [Heller, Kaplan]; Anderson localization
- You can stabilize the π preparation using high frequency periodic driving (Kapitza)
- You can stabilize the π preparation using noisy driving (Zeno) - it is a classical effect!

The Trimer

A rotating $M{=}3$ site system with N bosons:

$$\mathcal{H} \;=\; \sum_{j=1}^{M}\left[\frac{U}{2}a_j^\dagger a_j^\dagger a_j a_j - \frac{K}{2}\left(\mathrm{e}^{i(\Phi/M)}a_{j+1}^\dagger a_j + \mathrm{e}^{-i(\Phi/M)}a_j^\dagger a_{j+1}\right)\right]$$

This is like M coupled oscillators with an additional constant of motion:

$$N \;=\; \sum_{j=1}^{M} a_j^\dagger a_j \;=\; \sum_{j=1}^{M} n_j$$

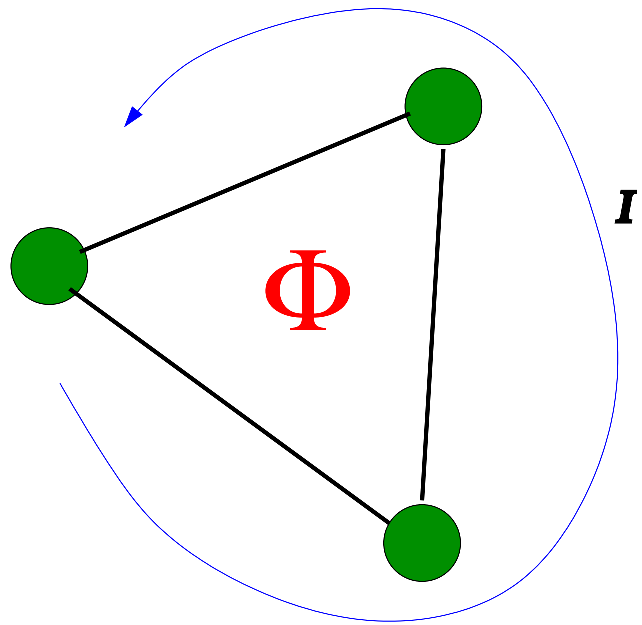
Dimensionless parameters (Φ, u):

$$u \;=\; \frac{NU}{K}$$

$$\Phi \;=\; \frac{M^2}{2\pi}\left(\frac{\mathsf{m}}{\mathsf{m}_{\text{eff}}}\right)\frac{\Omega}{K}$$

Upon quantization we have:

$$\hbar \;=\; \frac{1}{N}$$



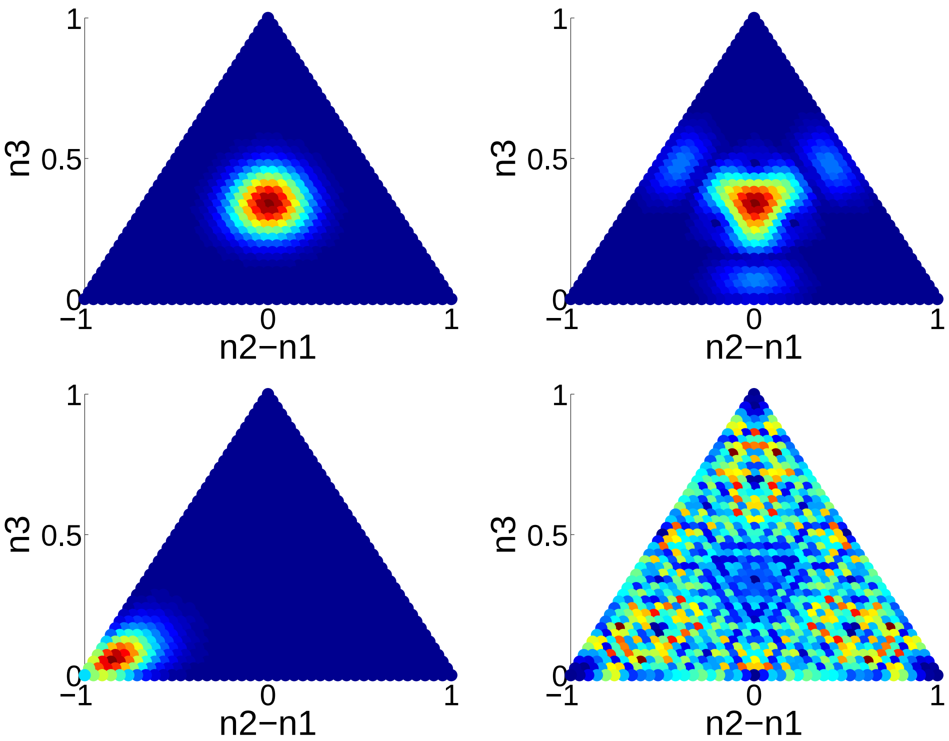
Eigenstates

For $N = n_1+n_2+n_3$ the standard basis is $|\boldsymbol{r}\rangle$ where $r_\parallel = (n_2 - n_1)/N$ and $r_\perp = n_3/N$

$$|\psi(\mathbf{r})|^2 \;=\; |\langle\mathbf{r}|E_\alpha\rangle|^2 \;=\; \text{occupation probability distribution}$$

For each eigenstate $|E_\alpha\rangle$ we calculate:

Representative Eigenstates:



One-body reduced probability matrix –

$$\rho_{ij} \;=\; (1/N)\langle a_j^\dagger a_i\rangle_\alpha$$

The number of participating orbitals –

$1 < (1/S_\alpha) < M$

$S_\alpha \;\equiv\; \text{trace}(\rho^2)$

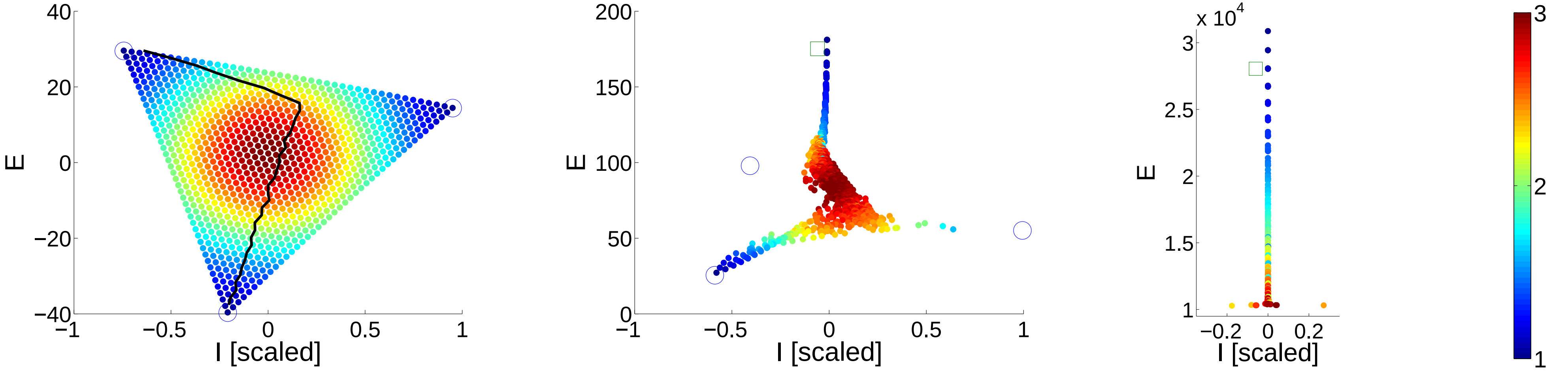
$1/S{=}1$ means a coherent state.

$1/S{=}3$ means maximum fragmentation.

Bond averaged current –

$$\mathcal{I}_\alpha \;=\; -\left\langle \frac{\partial\mathcal{H}}{\partial\Phi}\right\rangle_\alpha$$

The energy spectrum of the trimer ($N = 42$)



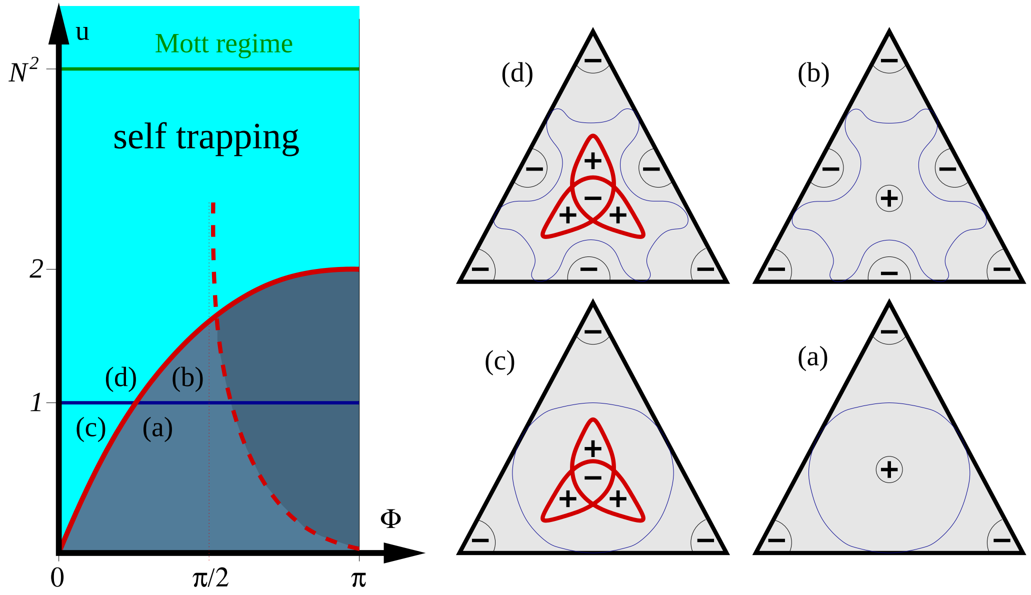
- Mott Transition (ground state)
- Self-Trapping (upper state)
- Metastable vortex state?

Self-trapped state = condensation in site orbital.

Vortex state = condensation in momentum orbital.

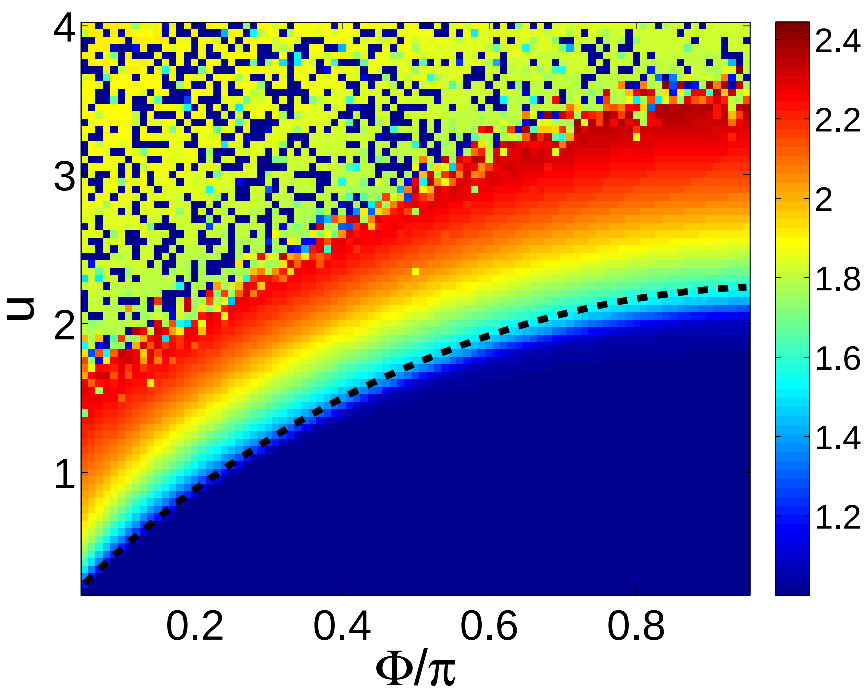
$m = 0, \pm 1$

$$I_m = \frac{N}{M} K \sin\left(\frac{1}{M}(2\pi m - \Phi)\right)$$

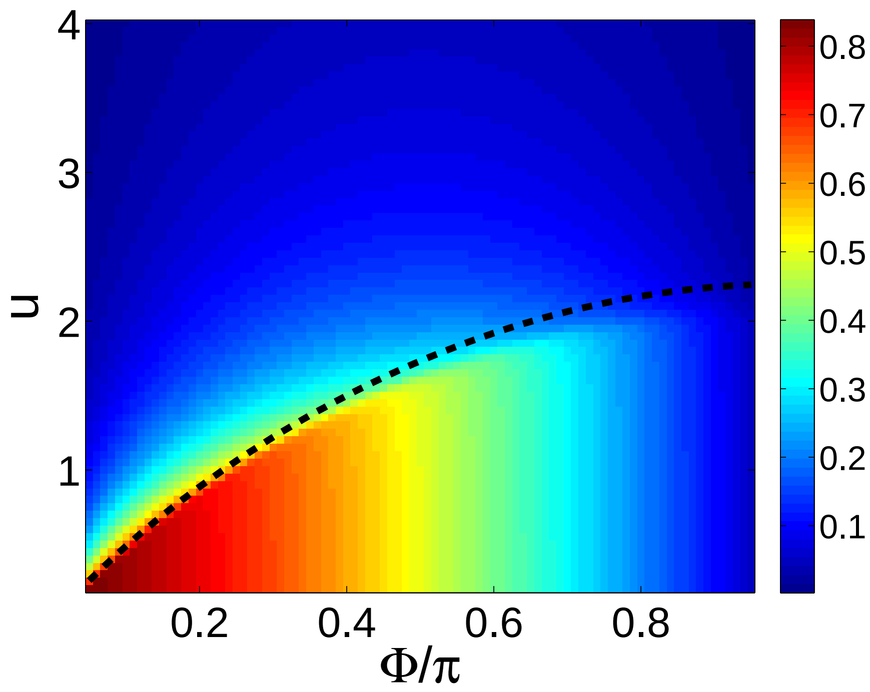


Self Trapping

$1/S$ of upper energy state



I of upper energy state



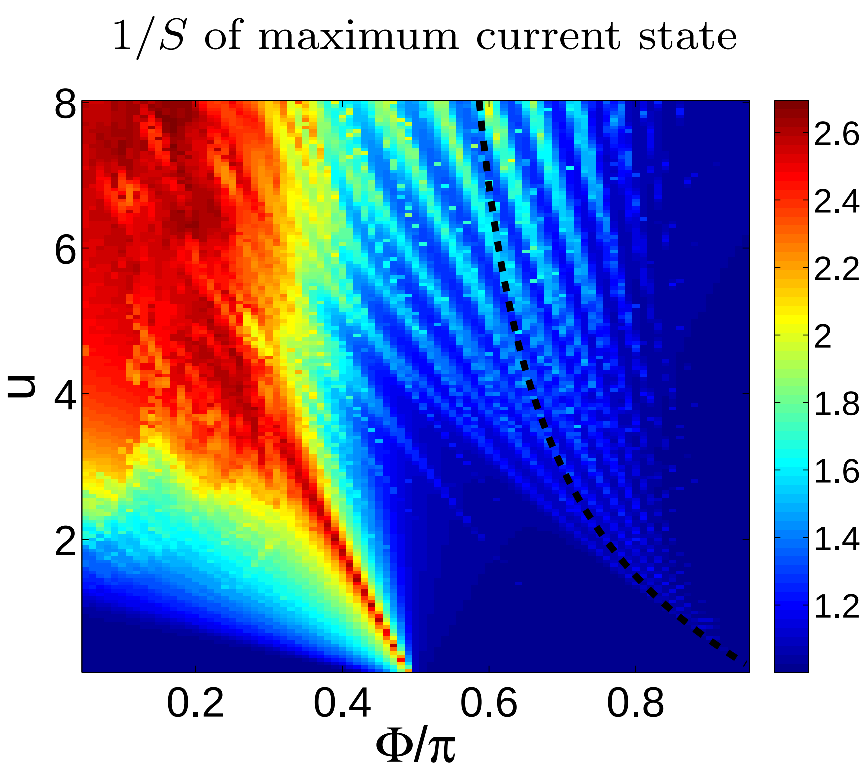
$$u = \frac{6 - 9\cos\left(\frac{\pi+2\Phi}{3}\right) - 3\cos\left(\frac{\pi-4\Phi}{3}\right)}{6\cos\left(\frac{\pi-\Phi}{3}\right) - 2\cos(\Phi)}$$

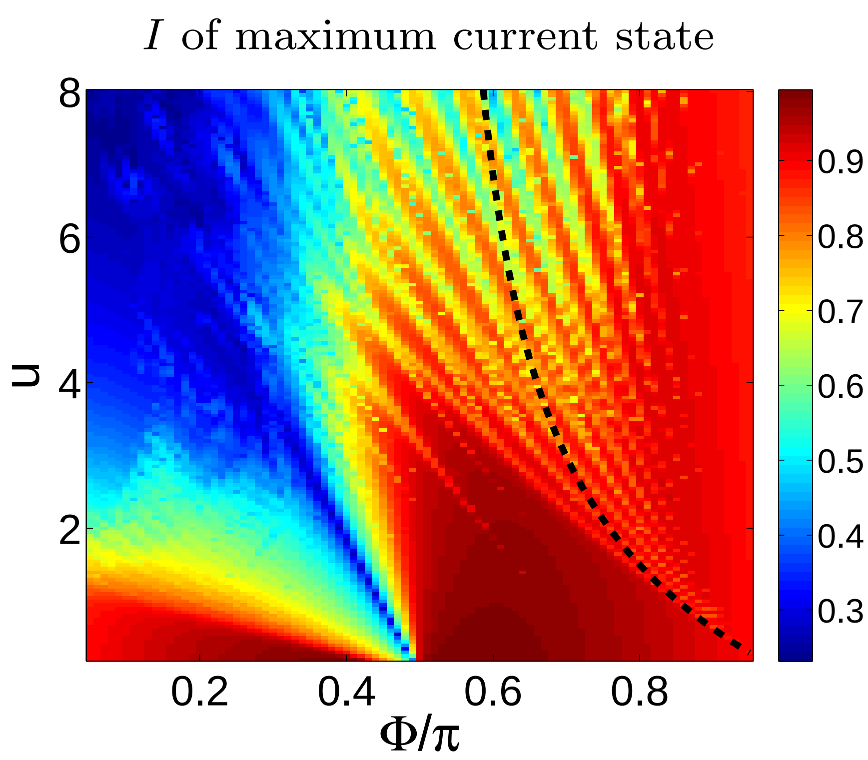
$$I_m = \frac{N}{M} K \sin\left(\frac{1}{M}(2\pi m - \Phi)\right)$$

$m = -1$

- Self-trapping can occur for arbitrarily small interaction.

Quasi-stability of the intermediate vortex state





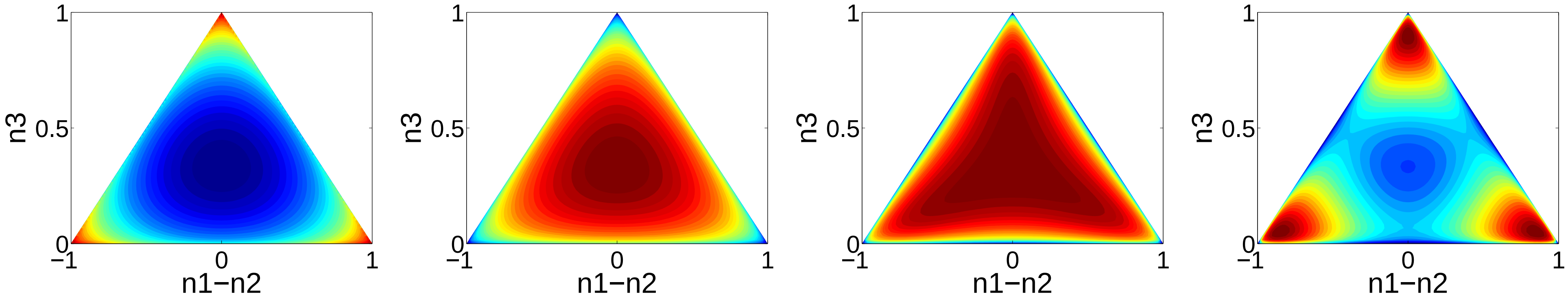
$$u = \frac{6 - 9\cos\left(\frac{\pi - 2\Phi}{3}\right) - 3\cos\left(\frac{\pi + 4\Phi}{3}\right)}{6\cos\left(\frac{\pi + \Phi}{3}\right) - 2\cos(\Phi)}$$

- The quantum metastability regime extends beyond the classically expected.
- Quantum scarring related effect(?).

The energy landscape

$$\mathcal{H} \;=\; \sum_{j=1}^{M}\left[\frac{U}{2}\boldsymbol{n}_j^2 - K\sqrt{\boldsymbol{n}_{j+1}\boldsymbol{n}_j}\cos\left((\varphi_{j+1}-\varphi_j)-\frac{\Phi}{M}\right)\right] \qquad a_j = \sqrt{\boldsymbol{n}_j}\,\mathrm{e}^{i\varphi_j}$$

$$V(r) \;=\; \min_{\varphi}[\mathcal{H}(r,\varphi)] \quad \text{or} \quad \max_{\varphi}[\mathcal{H}(r,\varphi)] \qquad \text{[Peierls-Nabarro surfaces]}$$

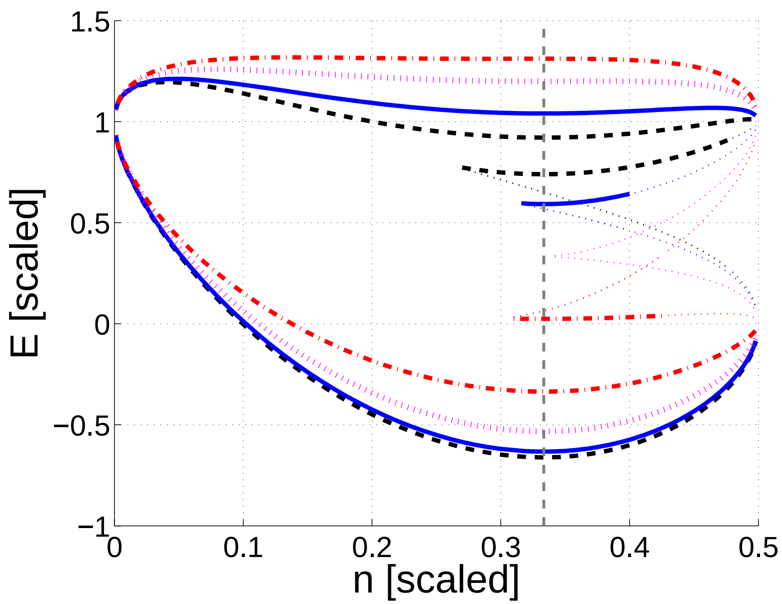


$V_+ =$ top surface

$V_- =$ intermediate surface

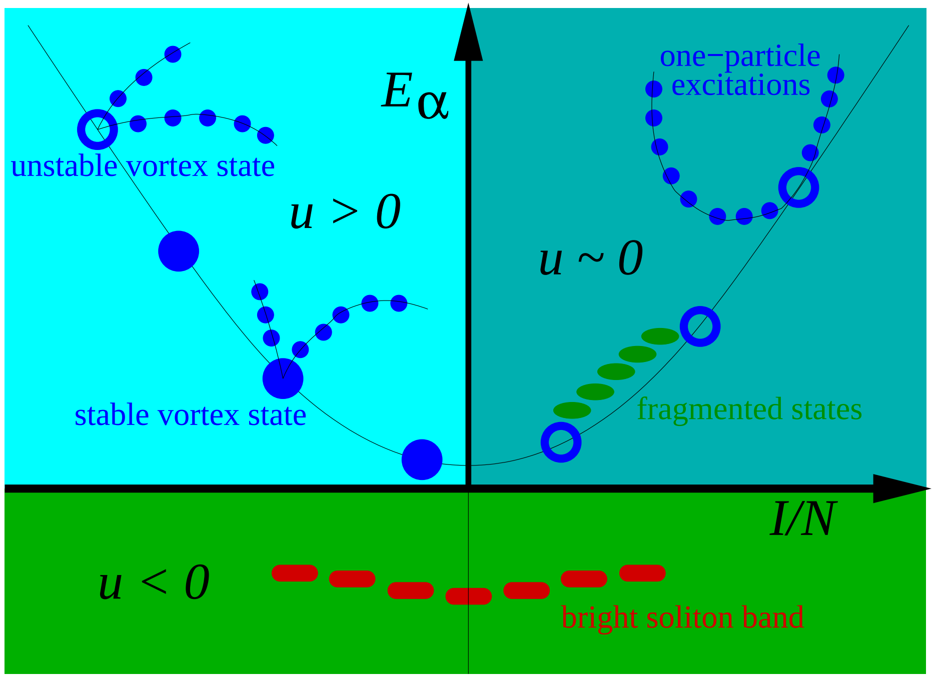
$V_0 =$ bottom surface

$$V''_{\pm}(r_0) \;=\; 6U - 9\,\frac{K}{N}\left[\frac{2-3\cos\left(\frac{\pi\pm2\Phi}{3}\right)-\cos\left(\frac{\pi\mp4\Phi}{3}\right)}{3\cos\left(\frac{\pi\mp\Phi}{3}\right)-\cos\left(\Phi\right)}\right]$$



Concluding Remarks

- The essence of superfluidity is the possibility to witness metastable vortex states ("dissipationless current")
- In the standard classical stability analysis one finds that vortex states whose rotation velocity is less than a critical velocity are metastable ("Landau criterion")
- For a non-rotating 3-site model the same type of classical analysis implies that there are no metastable vortex states [1,2]



- We have explored the full regime diagram. In the presence of rotation we find (Ω, u) regimes where metastable vortex states exists.
- In the quantum analysis we find that the intermediate vortex state is quasi-stable beyond the classical expectation, even for non-rotating device.

[1] PC with J. Anglin, T. Leggett, P. Ghosh
[2] P. Ghosh, F. Sols, Phys. Rev. A 77, 033609 (2008)