Superfluidity and Chaos in low dimensional circuits

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Scope

- Consider N bosons in an M site ring, that are condensed into a single plane-wave orbital. This is called a "vortex state". It has a macroscopically large current.
- The conventional paradigm associates vortex states with a stationary stable fixed-points in phase space. Consequently the Landau criterion, and more generally the Bogoliubov de Gennes stability analysis, are normally used to determine the viability of superfluidity.
- We challenge the application of the traditional paradigm to low-dimensional circuits and highlight the role of chaos in their analysis.

The model

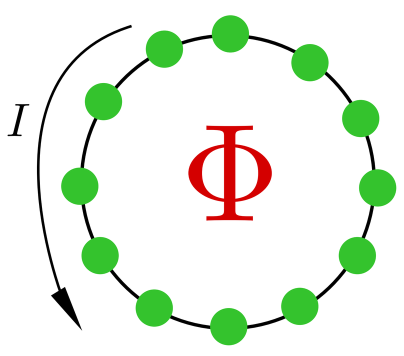
A rotating Bose-Hubbard system with M sites and N bosons.

$$\mathcal{H} = \sum_{j=1}^{M}\left[\frac{U}{2}a_j^\dagger a_j^\dagger a_j a_j - \frac{K}{2}\left(e^{i(\Phi/M)}a_{j+1}^\dagger a_j + e^{-i(\Phi/M)}a_j^\dagger a_{j+1}\right)\right]$$

Dimensionless parameters (Φ, u):

$$u = \frac{NU}{K}$$

$$\Phi = \frac{M^2}{2\pi}\left(\frac{\mathrm{m}}{\mathrm{m}_{eff}}\right)\frac{\Omega}{K}$$



Upon quantization we have:

$$\hbar = \frac{1}{N}$$

The number of particles N is a constant of motion:

$$N = \sum_{j=1}^{M} a_j^\dagger a_j$$

hence the model has effectively $d = M-1$ degrees of freedom.
$M = 2$ Bosonic Josephson junction, Integrable (d=1).
$M = 3$ Minimal circuit, Chaotic (mixed) phase-space
$M \geq 4$: High dimensional chaos (Arnold diffusion)
$M \to \infty$: Continuous ring, Integrable.

Fixed-points and Stability

In a semi-classical context one define phase-space action-angle coordinates as follows:

$$a_j = \sqrt{\boldsymbol{n}_j}e^{i\varphi_j} \quad , \quad z = (\varphi_1, \cdots, \varphi_M, \boldsymbol{n}_1, \cdots, \boldsymbol{n}_M)$$

The dynamics is generated by: (equivalent to DNLS)

$$\dot{z} = \mathbb{J}\partial H \quad , \quad \mathbb{J} = \begin{pmatrix} 0 & \mathbb{I} \\ -\mathbb{I} & 0 \end{pmatrix}$$

Coherent states are supported by stable fixed-points ($\partial H = 0$) of the classical Hamiltonian.
Linear stability analysis (BdG):

$$\dot{z} = \mathbb{J}\mathcal{A}z \quad , \quad \mathcal{A}_{\nu,\mu} = \partial_\nu\partial_\mu H$$

Spectral stability: Energy minima (Landau criterion)
Dynamical stability: Zero Lyapunov exponents (real Bogoliubov frequencies)

Beyond the traditional view

- Dynamical instability of a vortex state does not necessarily mean that superfluidity is diminished, due to KAM structures. Chaotic and irregular vortex states.
- Dynamical stability of a vortex state does not always imply actual stability. For $M \geq 4$ KAM tori do not block transport (Arnold diffusion).
- Due to the quantum uncertainty width of a vortex-state, stability is required within a Plank cell around the fixed-point. Phase-diagram should be $\hbar$ dependent

Launching trajectories at the vicinity of the vortex fixed-point we encounter the following possibilities:
the trajectories are:

1. locked at the vortex fixed point (regular vortex state)
2. quasi-periodic in phase-space (breathing vortex)
3. chaotic but unidirectional (chotic vortex)

Spectrum

For each eigenstate $|E_\alpha\rangle$ we calculate the bond averaged current and the one-body reduced probability matrix:

$$\mathcal{I}_\alpha \equiv -\left\langle \frac{\partial \mathcal{H}}{\partial \Phi} \right\rangle_\alpha$$

$$\rho_{ij} = \frac{1}{N}\langle a_j^\dagger a_i \rangle_\alpha$$

$$S_\alpha \equiv \mathrm{trace}(\rho^2)$$

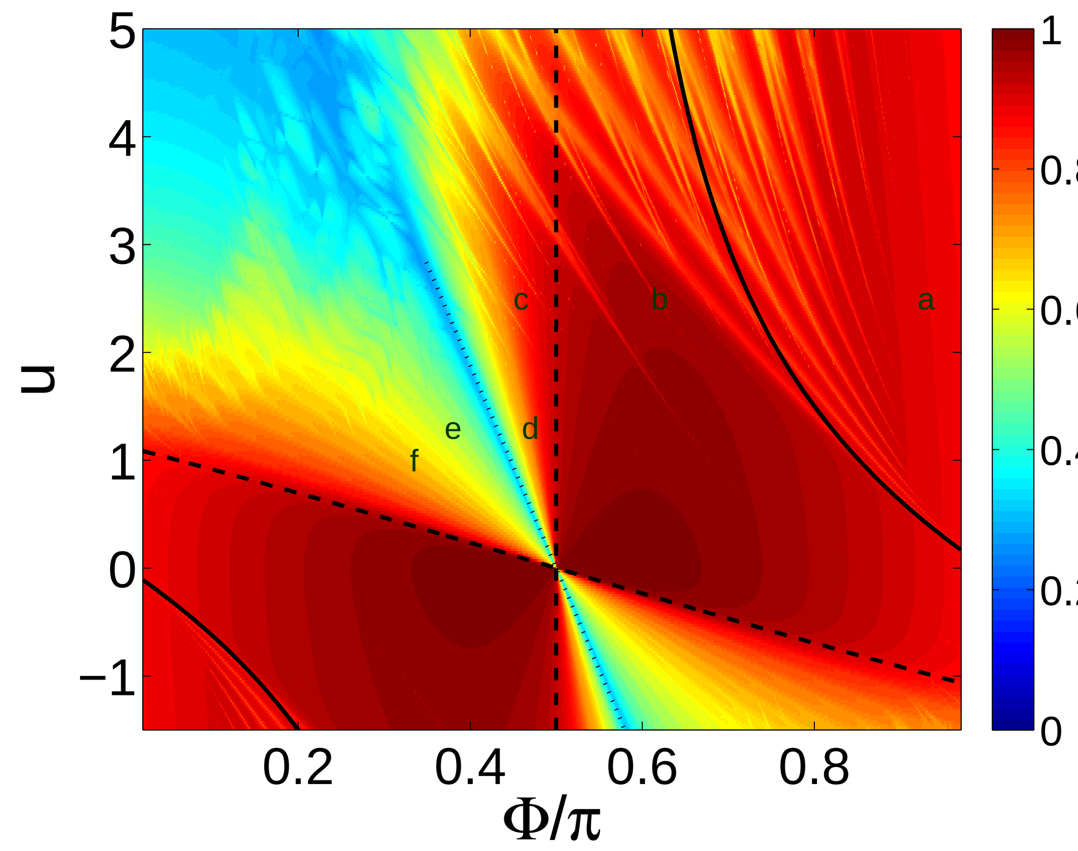
$$1/S \in [1, M]$$

$1/S = $ # of participating orbitals.
$1/S = 1$ means a coherent state.
$1/S = M$ is a maximum fragmentation.

Vortex state = Condensation in momentum orbital.
Self-trapped state = Condensation in site orbital.

Regime Diagram for $M = 3$

The I of a maximal current state



A stable vortex state carries current:

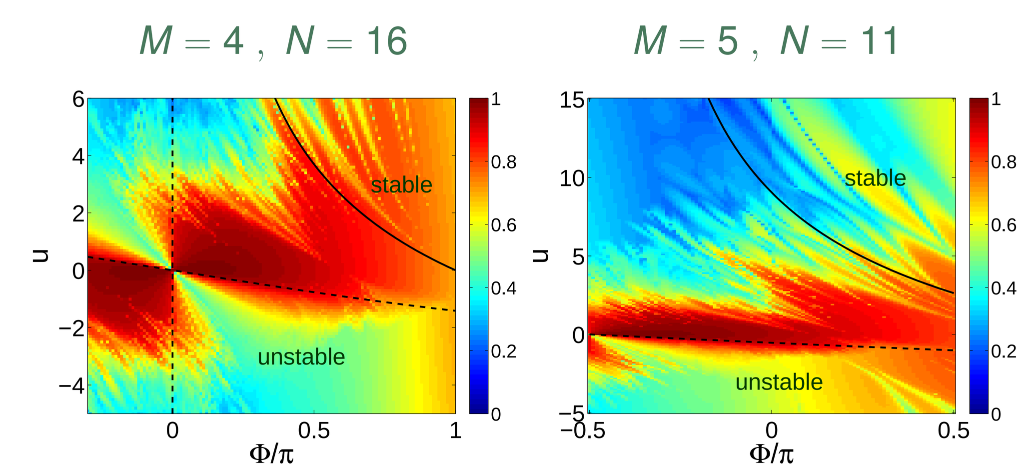
$$I_m = \frac{N}{M}K\sin\left(\frac{1}{M}(2\pi m - \Phi)\right) \quad , \quad (\text{here } m=1)$$

Spectral stability: (solid) $u > \dfrac{3 - 12\sin^2\left(\frac{\Phi}{3} - \frac{\pi}{6}\right)}{4\sin\left(\frac{\Phi}{3} - \frac{\pi}{6}\right)}$

Dynamical instability: (dashed) $u > \dfrac{9}{4}\sin\left(\dfrac{\pi}{6} - \dfrac{\Phi}{3}\right)$ & $\Phi < \dfrac{\pi}{2}$

Swap transition: (dotted) $u = 18\sin\left(\dfrac{\pi}{6} - \dfrac{\Phi}{3}\right)$

Regime diagram for "Large" systems ($M \geq 4$)

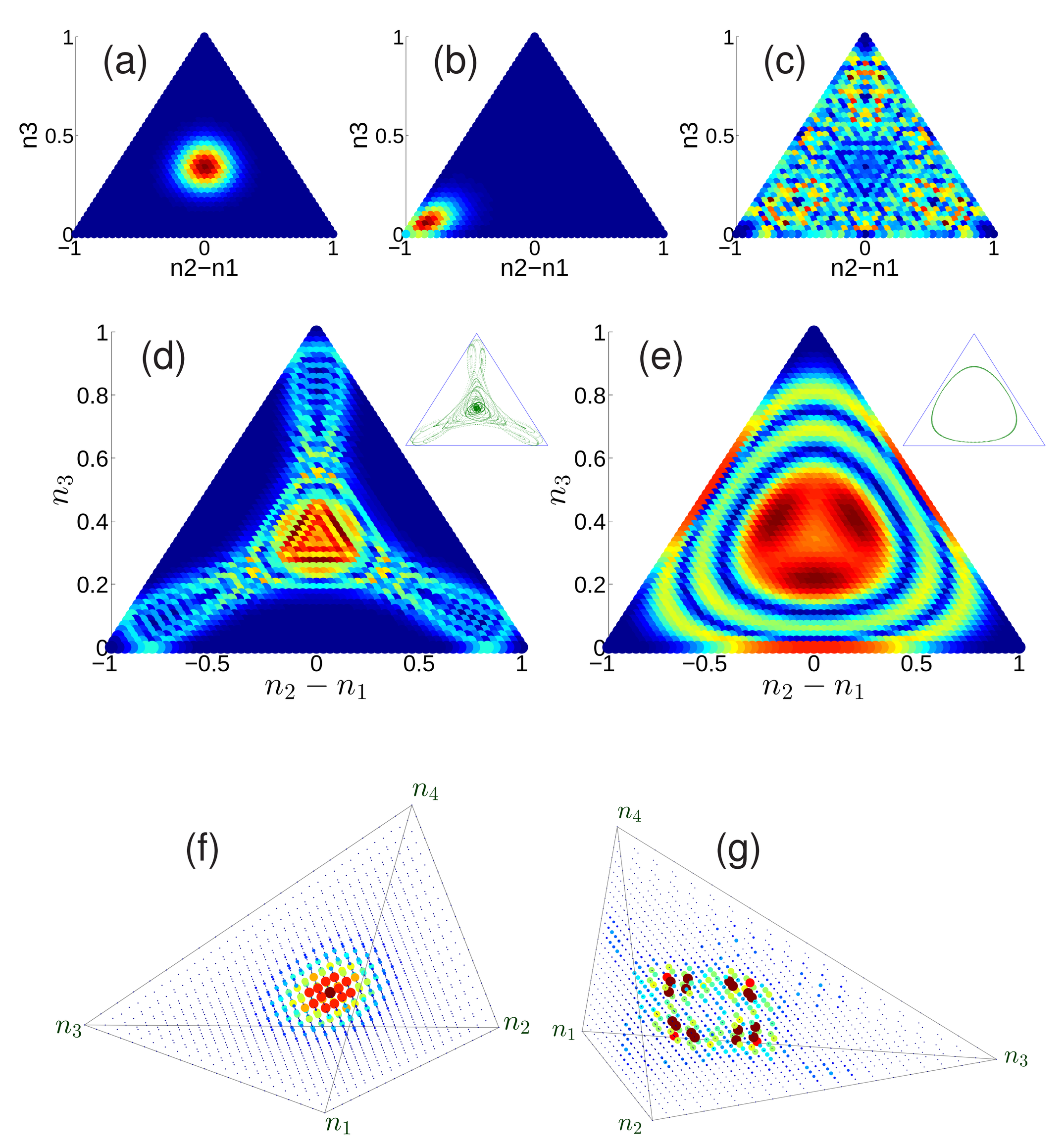


- Energy surface is $2d - 1$ dimensional
- KAM tori are d dimensional
- Arnold diffusion: the KAM tori in phase space are not effective in blocking the transport on the energy shell if $d > 2$.
- As u becomes larger this non-linear leakage effect is enhanced, stability of the motion is deteriorated, and the current is diminished.
- Due to the finite uncertainty width of the vortex state superfluidity can be diminished even in the spectrally stable region.

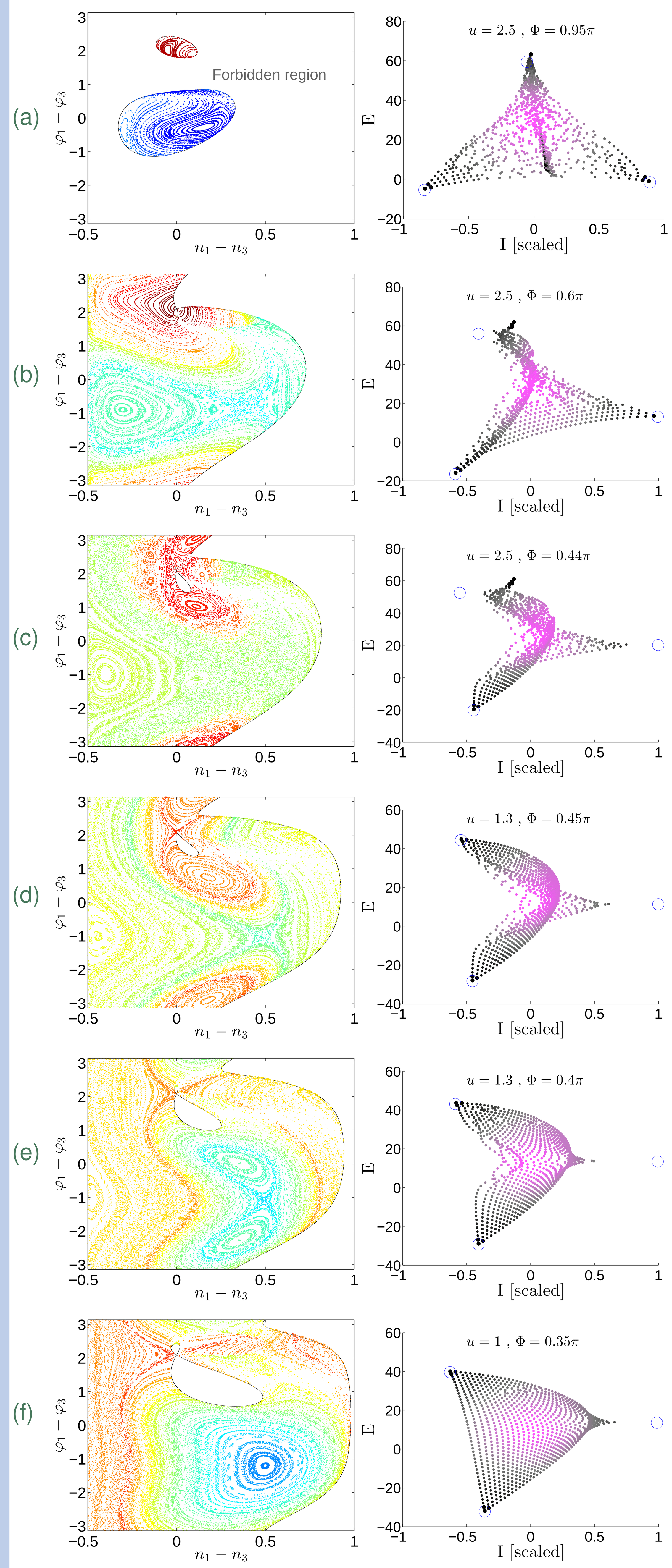
Representative Wavefunctions $M = 3, 4$

(a) Regular coherent vortex state.
(b) Self-trapped state ("bright soliton").
(c) Typical state in the chaotic sea.
(d) Chaotic vortex state.
(e) Breathing vortex state.
(f) Regular coherent vortex state.
(g) Irregular vortex state.

Images of $|\langle \mathbf{n}|E_\alpha\rangle|^2$ (Fock basis representation).
Insets: underlying classical dynamics.
Panels (a-e) are for $M = 3$, panels (f-g) are for $M = 4$.



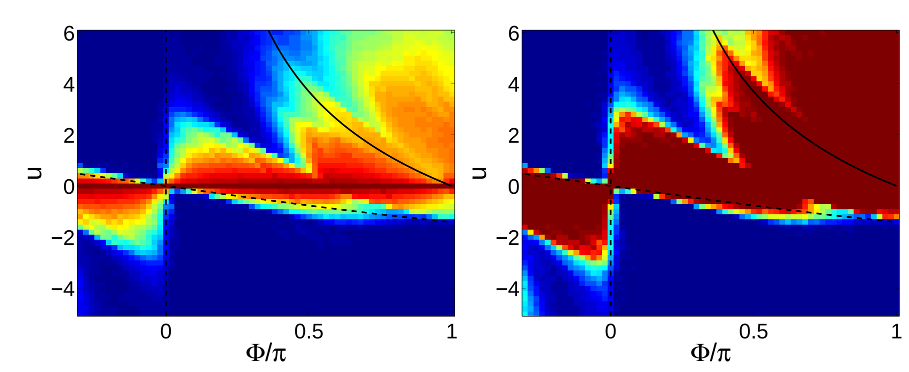
Phase-space structure $M = 3$



Poincare sections: red (blue) = large positive (negative) current

Semiclassical reproduction of the regime diagram $M = 4$

We launch a Gaussian cloud of trajectories that have an uncertainty width that corresponds to N.
The fraction of trajectories that escape is used as a measure for the stability.



Results are displayed for clouds that have uncertainty width $\Delta\varphi \sim \pi/2$ (left) and $\Delta\varphi \sim \pi/4$ (right).

Conclusions

- The recent experimental realization of confining potentials with toroidal shapes and tunable weak links has opened a new arena of studying superfluidity in low dimensional rings. In particular a discrete ring has been realized.
- We challenge the application of traditional BdG analysis to low-dimensional superfluid circuits.
- We have highlighted a novel type of superfluidity that is supported by irregular or chaotic or breathing vortex states.
- In a larger perspective we emphasize that the role of chaos should be recognized in the analysis of superfluidity. Furthermore we believe that a global understanding of the mixed phase-space structure is essential in order to analyse dynamical processes such as phase-slips.

References

[1] G. Arwas, A. Vardi, D. Cohen (PRA 2014)
[2] G. Arwas, A. Vardi, D. Cohen (arXiv 2014)

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