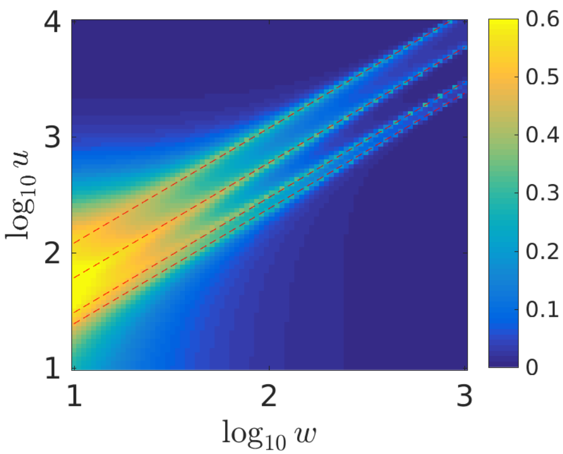
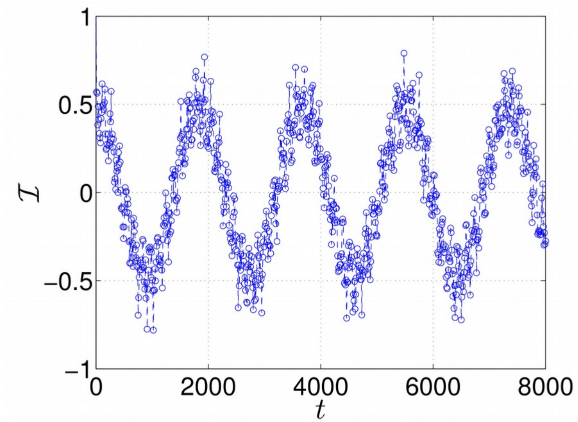
Persistent currents and chaos for ultracold bosons on a lattice ring

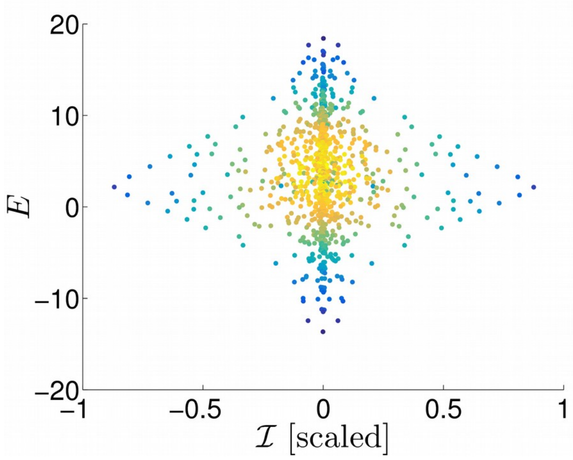
Geva Arwas, Doron Cohen

Ben-Gurion University

IPS 2017



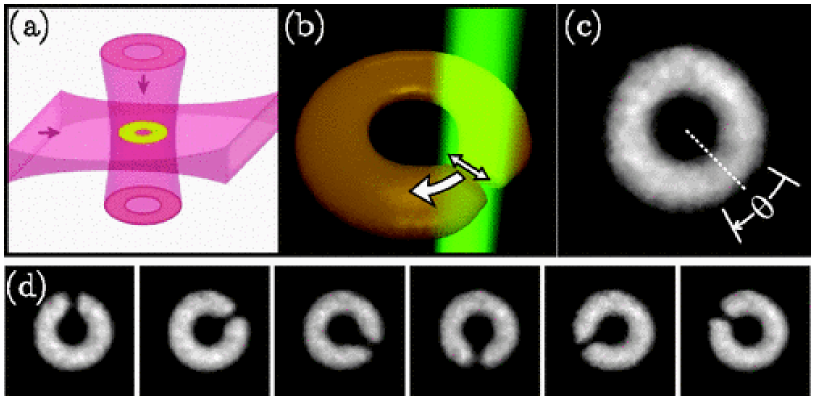




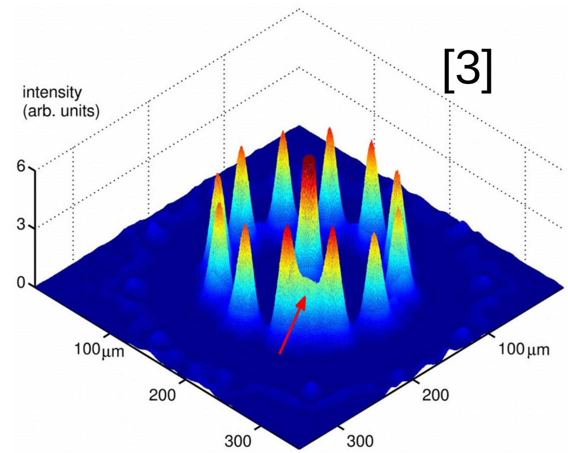
- G. Arwas, D. Cohen, F. Hekking, A. Minguzzi, Phys. Rev. A (2017) Editors' Suggestion
- G. Arwas, D. Cohen, New J. Phys. 18, 015007, (2016)

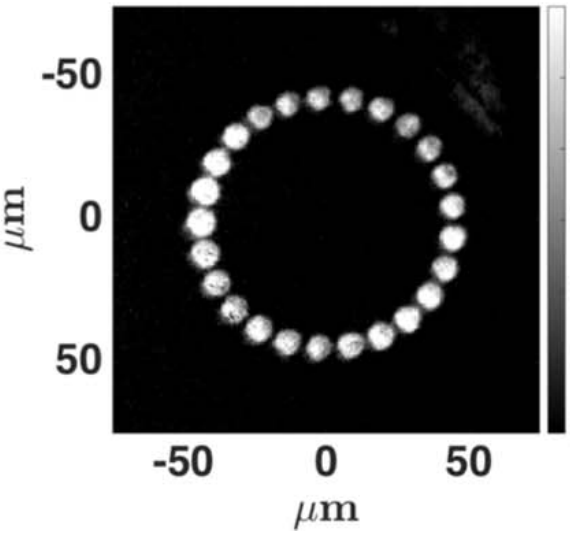
Ultracold bosons on ring traps are the object of an active experimental investigation

Toroidal ring [1,2]



Discrete ring (optical lattice) [4]





Such circuits are proposed to be used as Qbits, or as an “atomtronic” analog to the electronic SQUID for sensing of acceleration or gravitation

[1] Wright, Blakestad, Lobb, Phillips, Campbell (PRL 2013)
[2] Ekel, Lee, Jendrzeejewski, Murray, Clark, Lobb, Phillips, Edwards, Campbell (Nature 2014)
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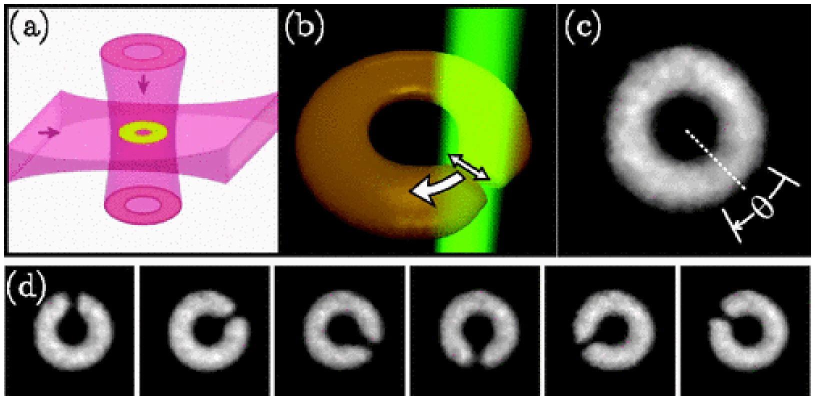

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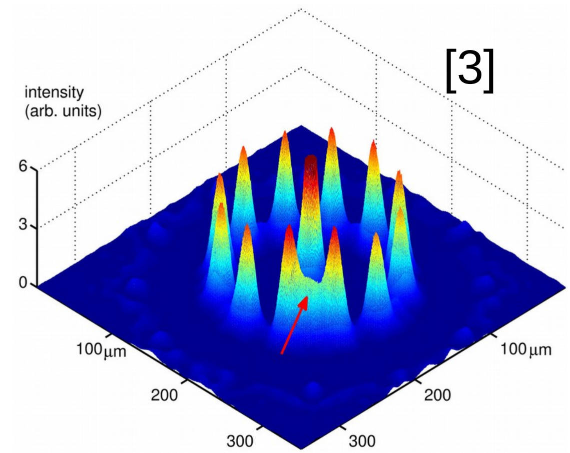

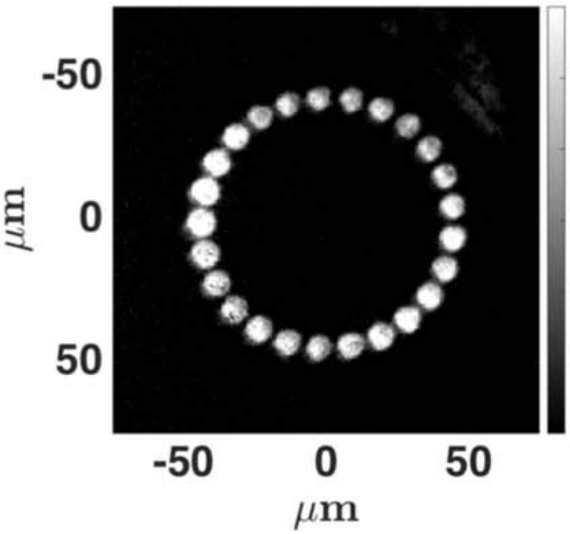

Such circuits are proposed to be used as Qbits, or as an “atomtronic” analog to the electronic SQUID for sensing of acceleration or gravitation

We are studying the feasibility for a coherent operation of such device

- Persistent currents [a]
- Coherent Rabi oscillations between superfluid flow-states [b]

[a] G. Arwas, D. Cohen, F. Hekking, A. Minguzzi, Phys. Rev. A (2017) Editors’ Suggestion
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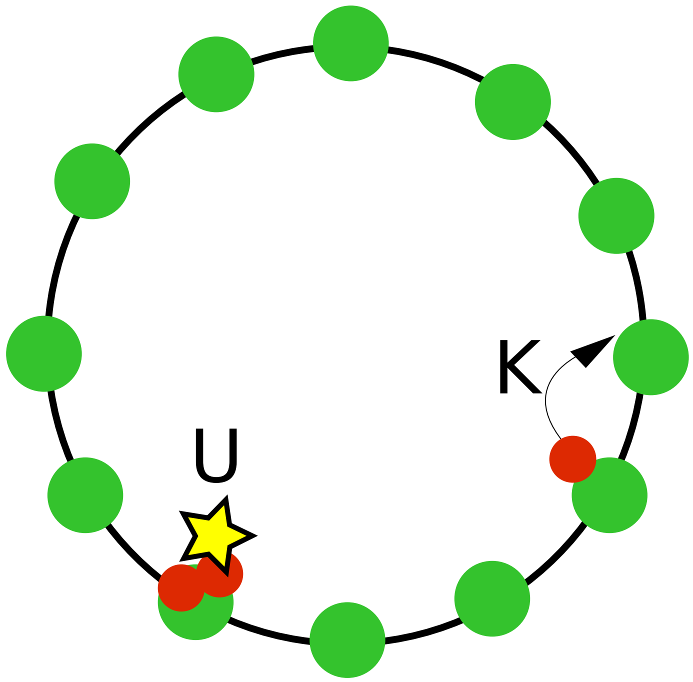
Bose Hubbard ring

A system of N Bosons in an M-site ring

$$\mathcal{H} \;=\; \sum_{j=1}^{M}\left[\frac{U}{2}a_j^{\dagger}a_j^{\dagger}a_j a_j - \frac{K}{2}\left(a_{j+1}^{\dagger}a_j + a_j^{\dagger}a_{j+1}\right)\right]$$

Dimensionless
interaction strength

$$u \;=\; \frac{MNU}{K}$$



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A system of N Bosons in a rotating M-site ring

$$\mathcal{H} = \sum_{j=1}^{M} \left[\frac{U}{2} a_j^\dagger a_j^\dagger a_j a_j - \frac{K}{2} \left(\mathrm{e}^{i(\Phi/M)} a_{j+1}^\dagger a_j + \mathrm{e}^{-i(\Phi/M)} a_j^\dagger a_{j+1} \right) \right]$$

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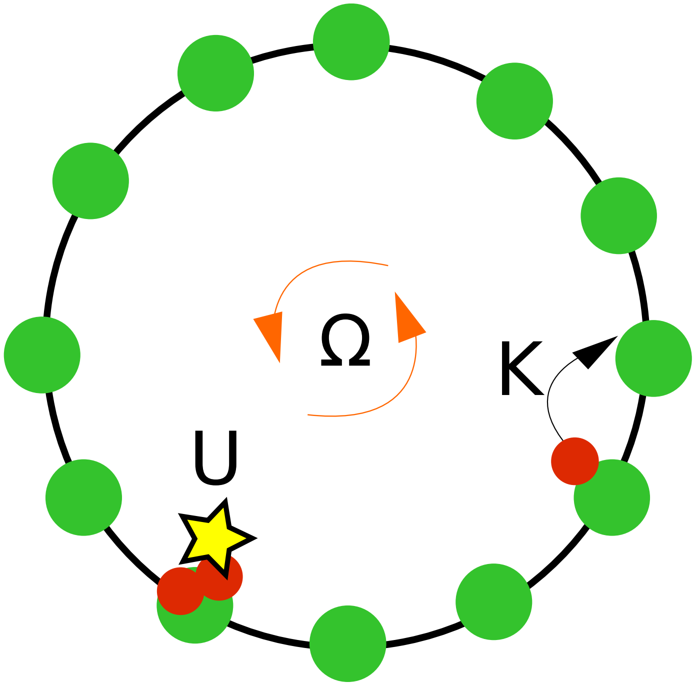

In the rotating reference frame

Flux ↔ Rotation frequency

$$\Phi = 2\pi R^2 m \Omega$$

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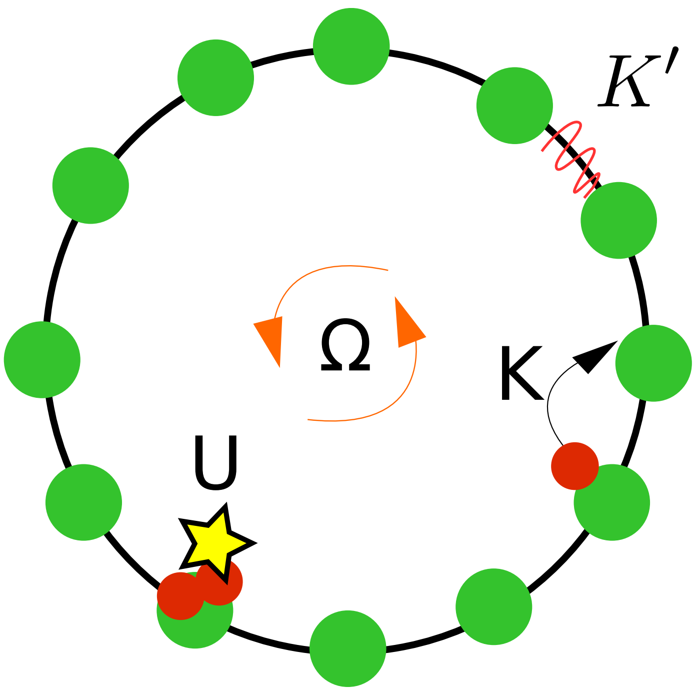

Weak-link

$$K' < K$$

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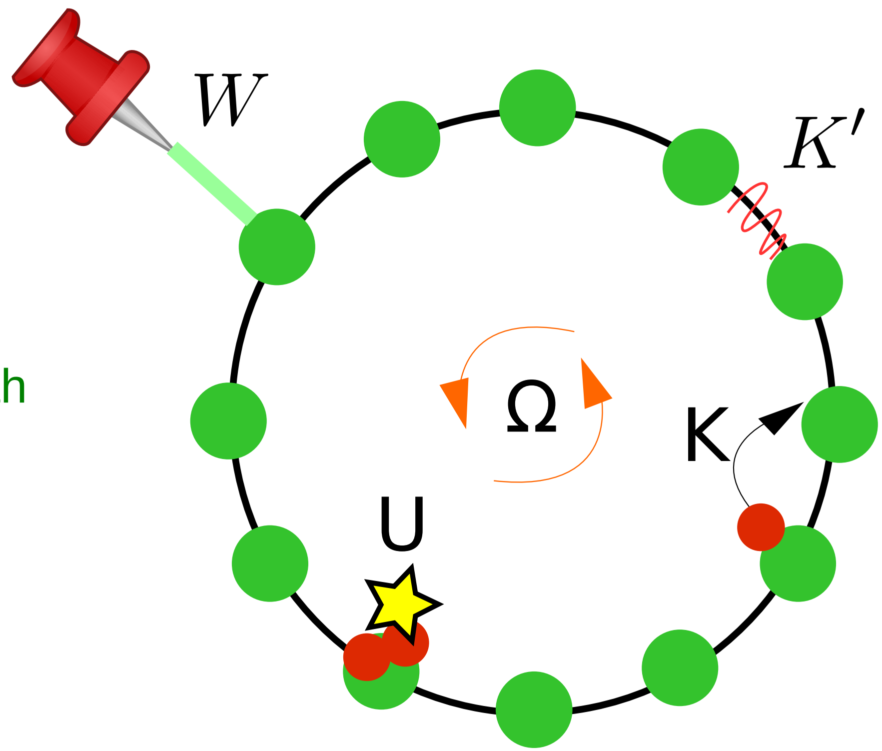
Energy barrier



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Dimensionless interaction strength

$$u = \frac{MNU}{K}$$

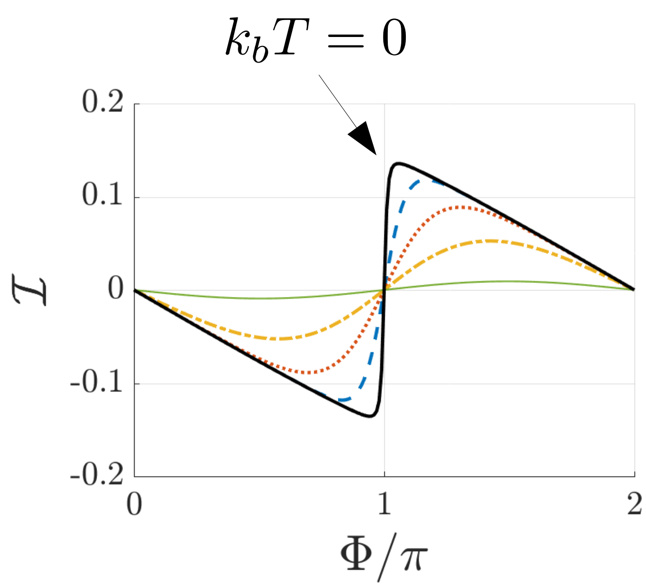
Dimensionless barrier strength

$$w = \frac{MW}{K}$$

Persistent current amplitude

Consider a ring with a rotating potential barrier
The current is given by:

$$\mathcal{I} \quad = \quad -\frac{\partial \mathcal{H}}{\partial \Phi}$$



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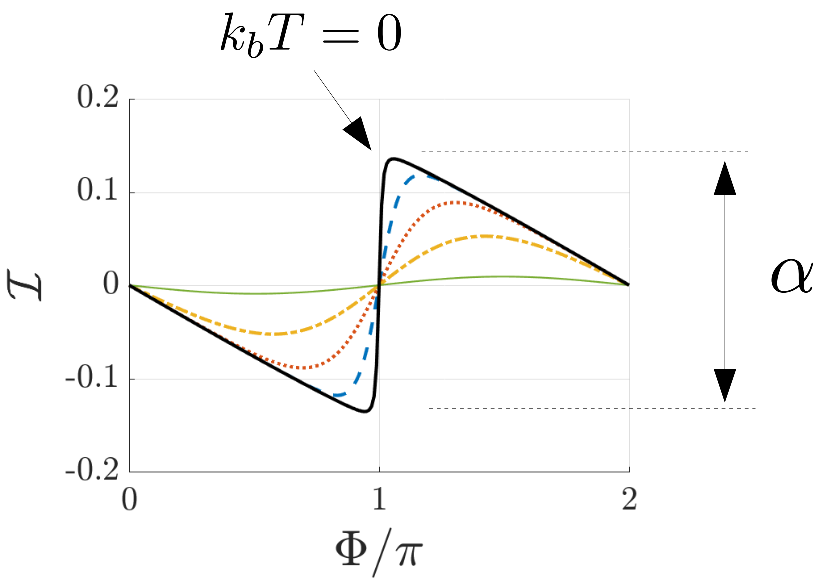

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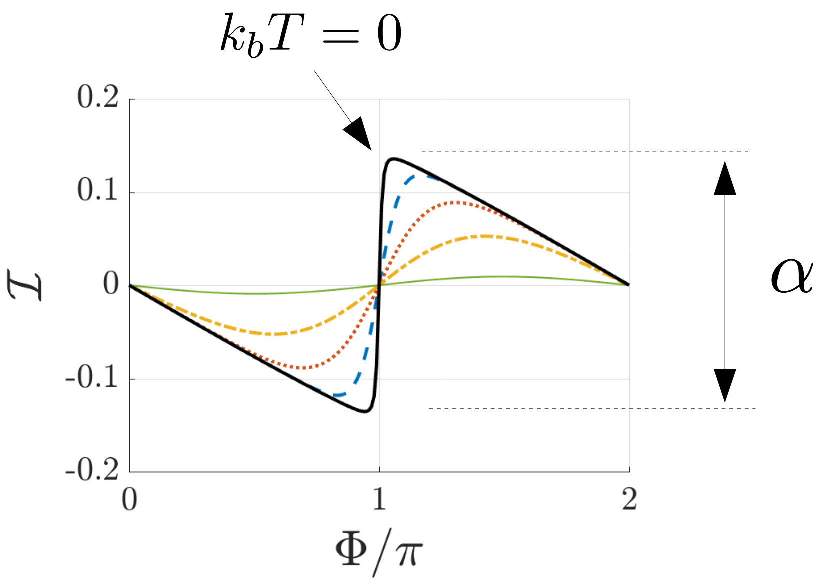
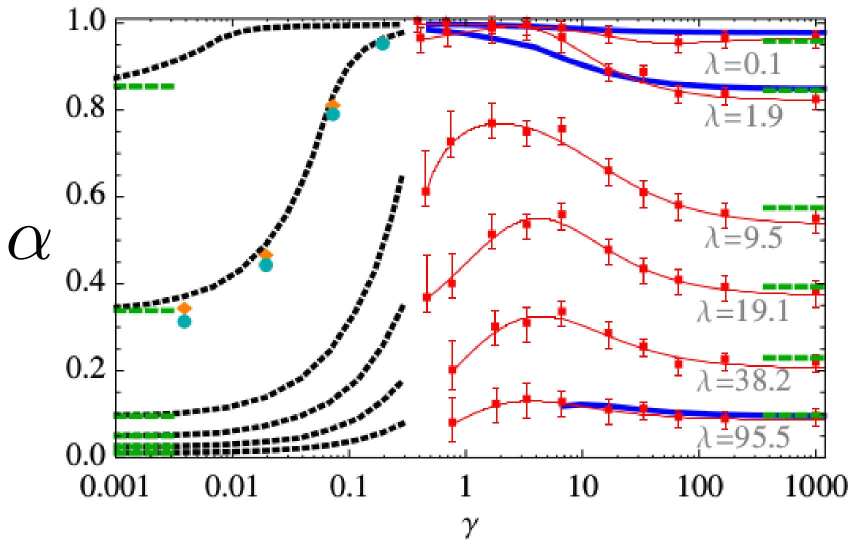


Without optical lattice: Non-monotonic behavior of the persistent current amplitude with respect to the interaction strength



M. Cominotti, D. Rossini, M. Rizzi,
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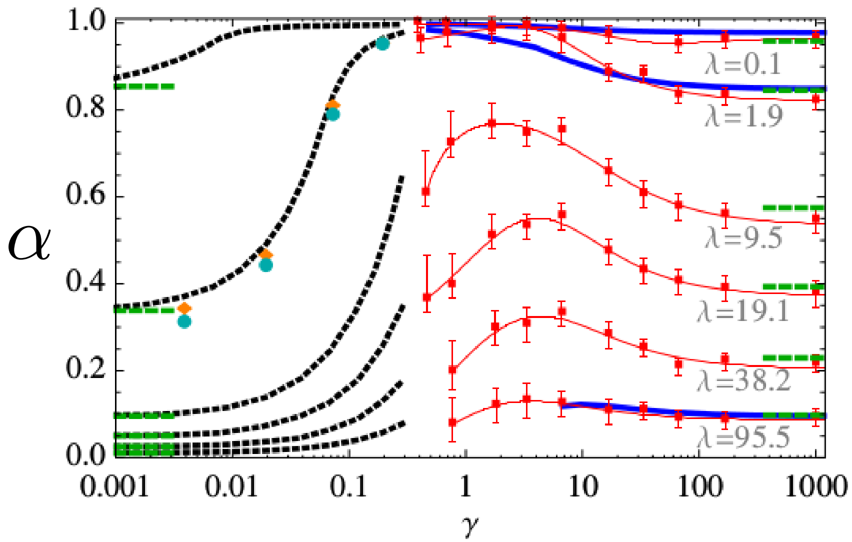

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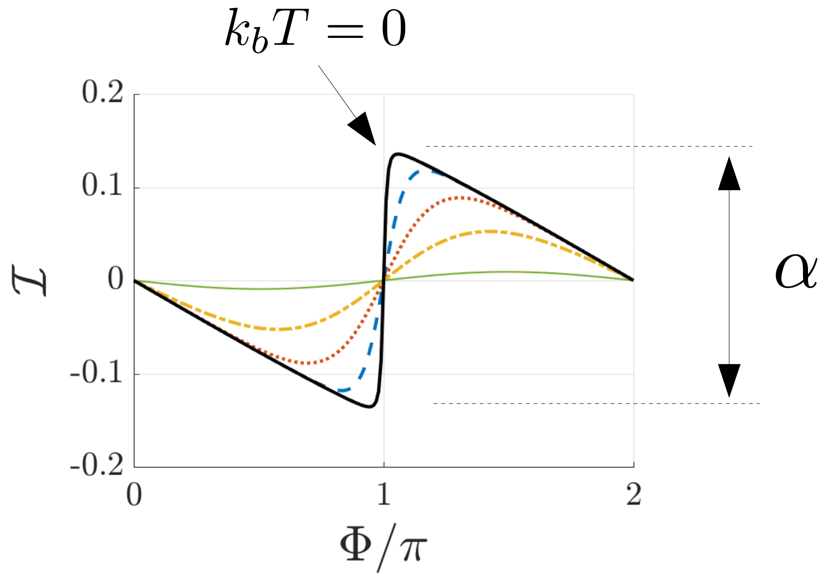
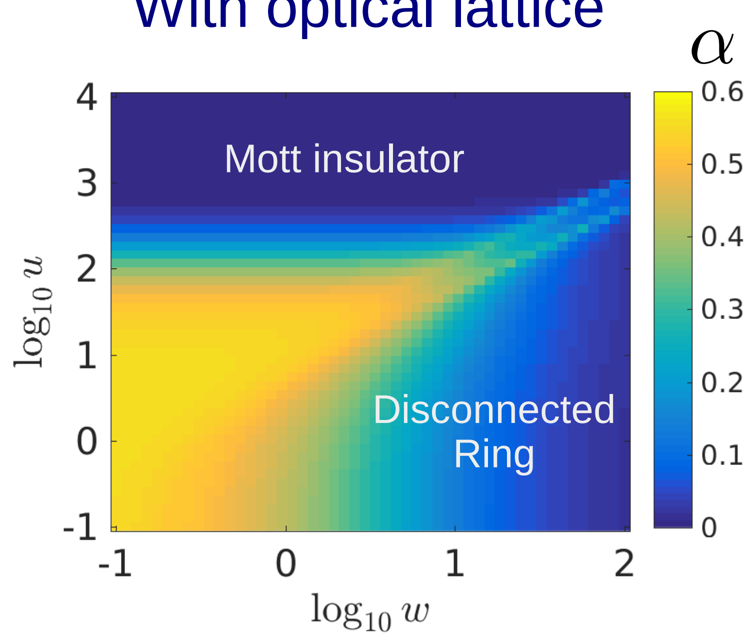
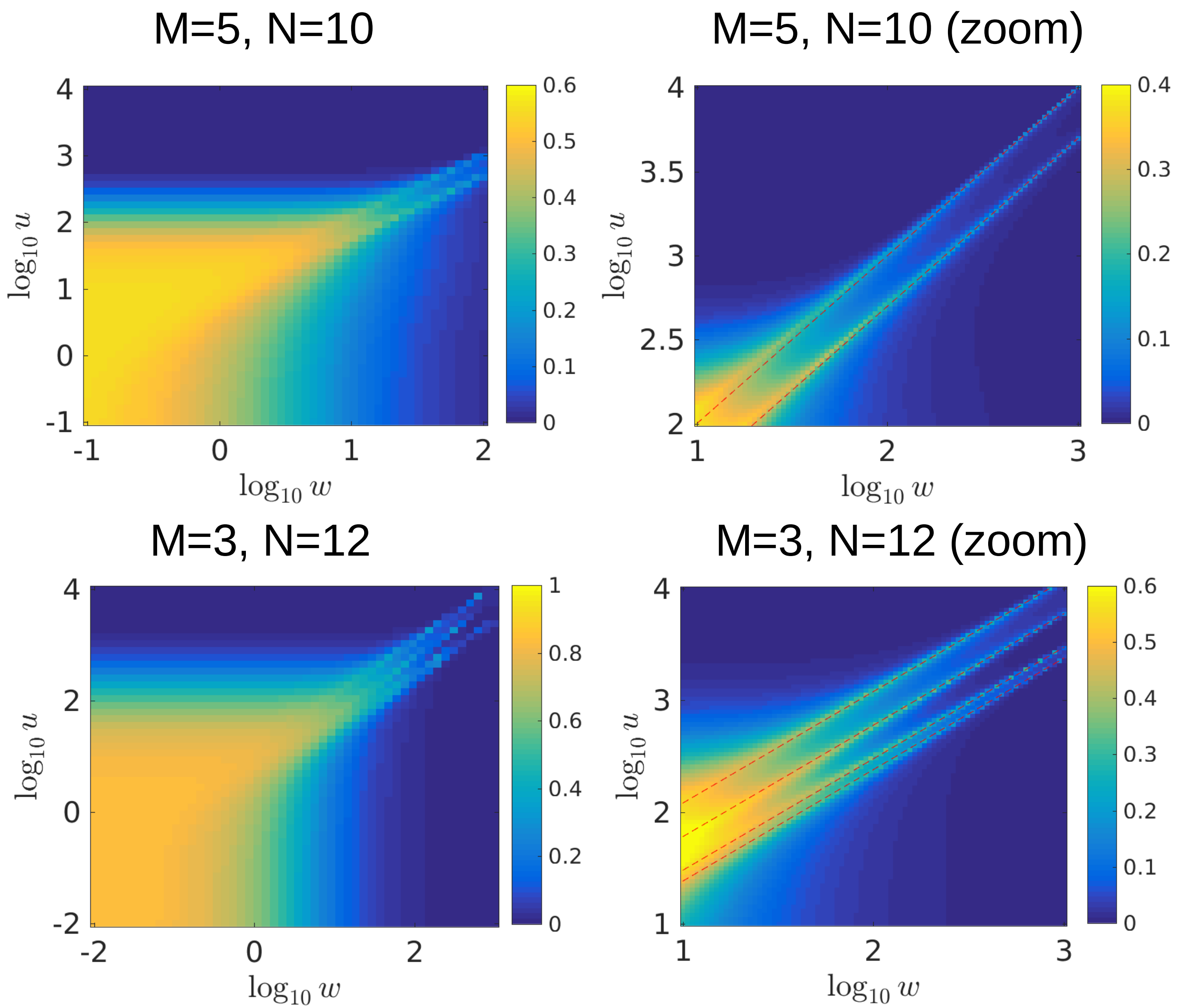


With optical lattice



Resonant Persistent Currents



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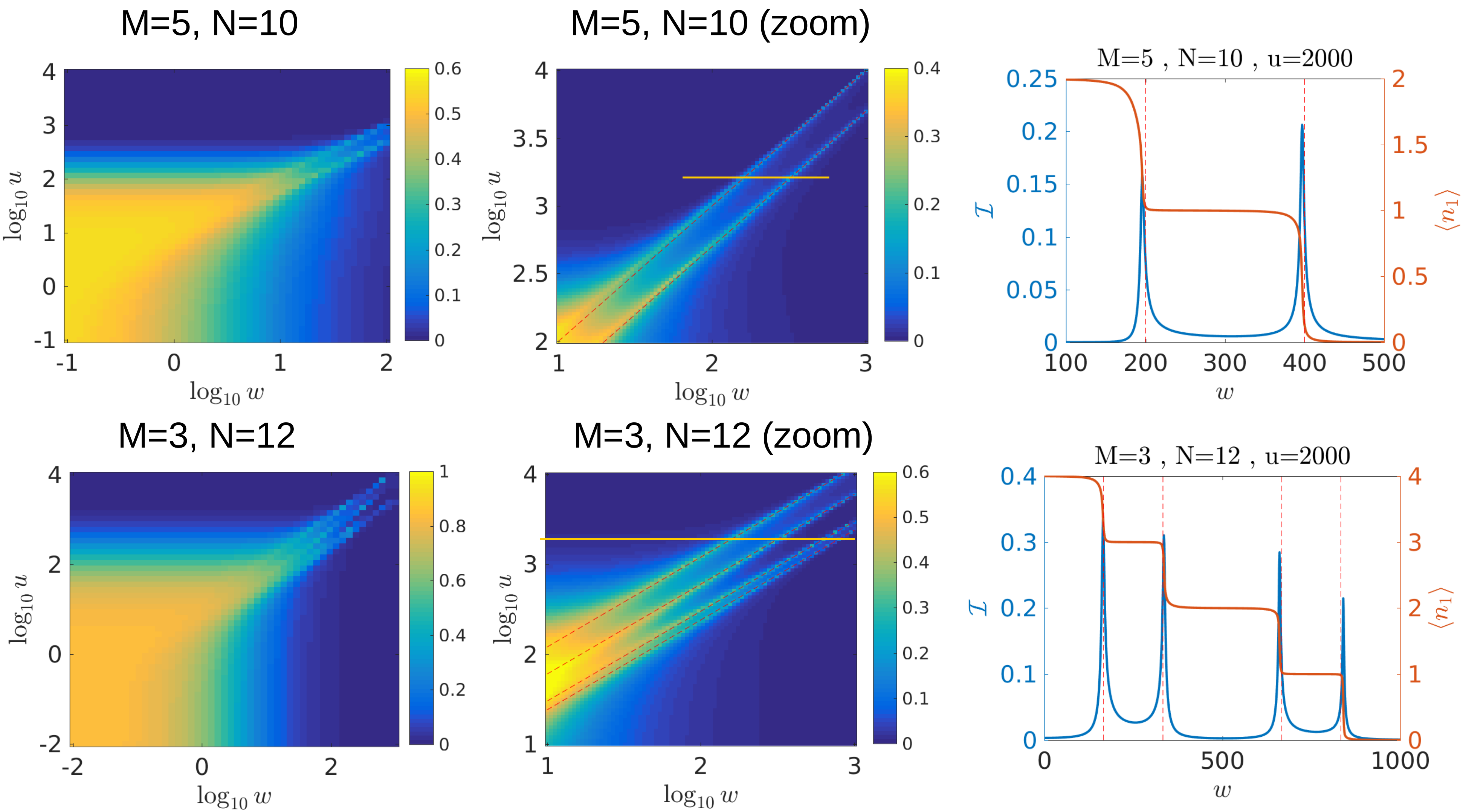

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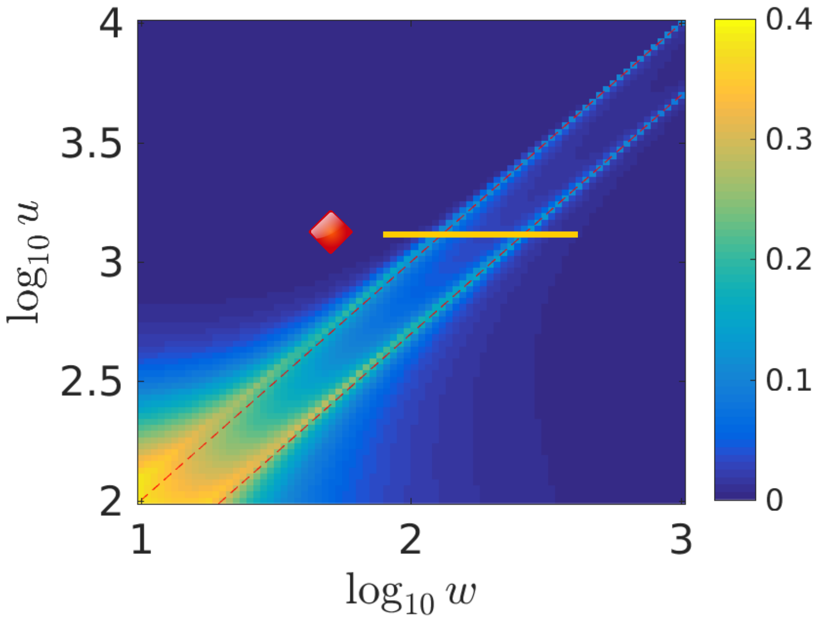
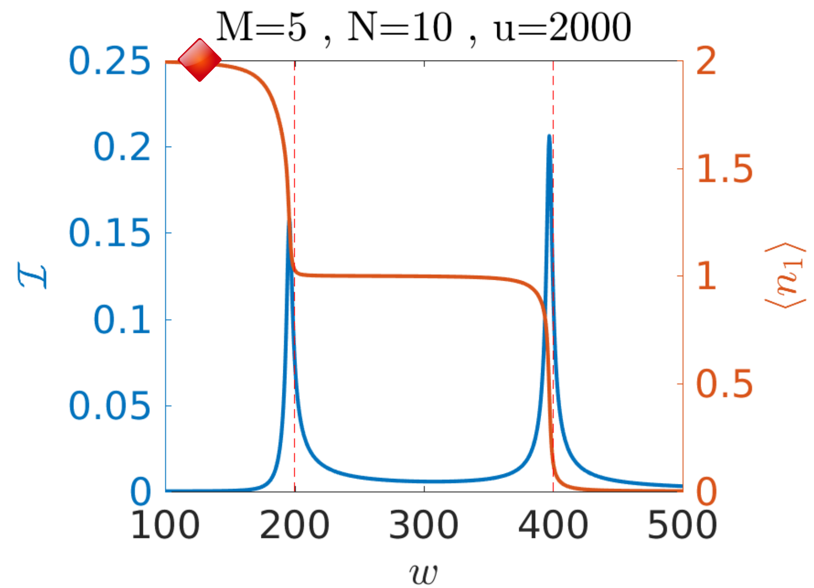
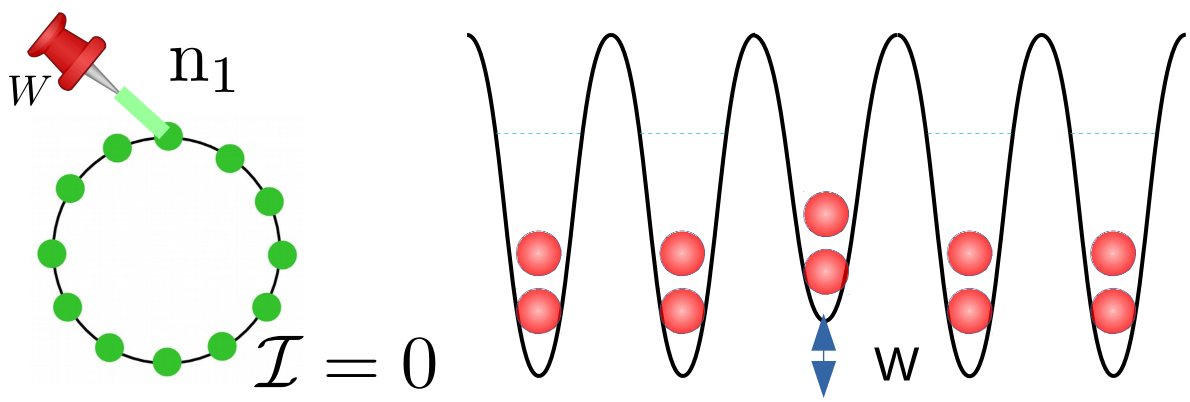


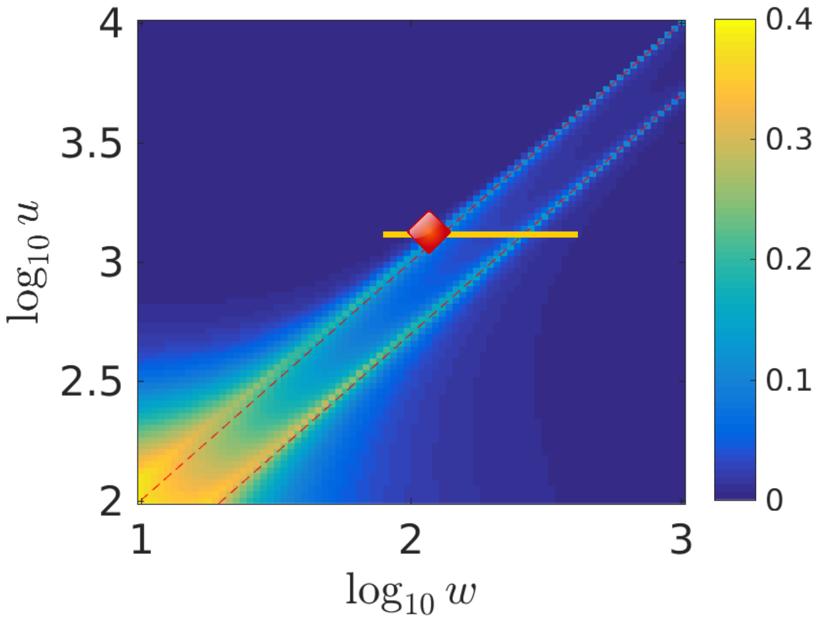


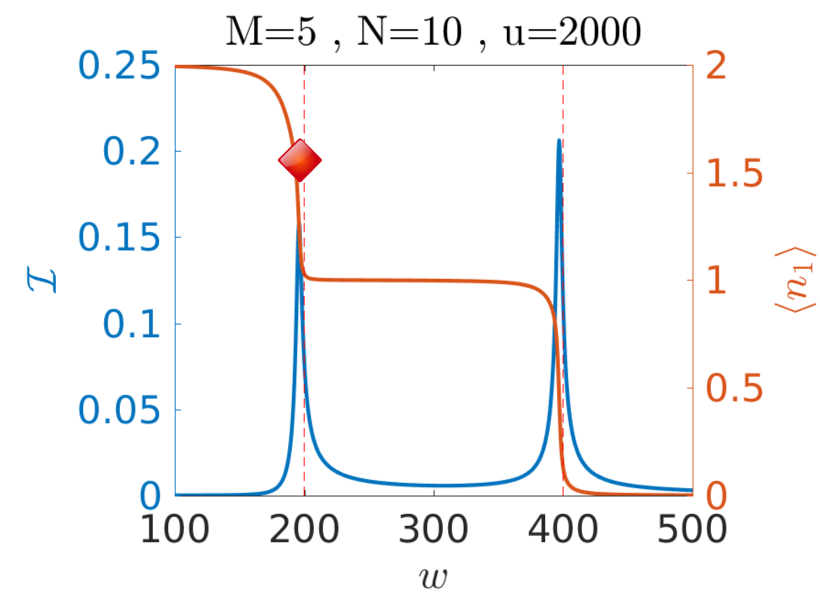


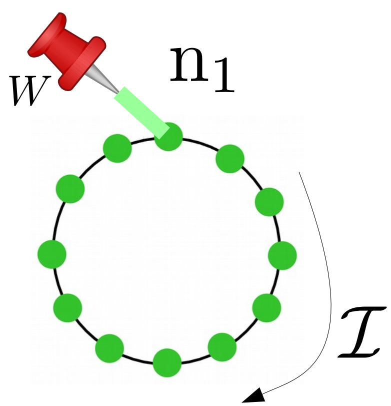
$$\langle n_1 \rangle = 2$$

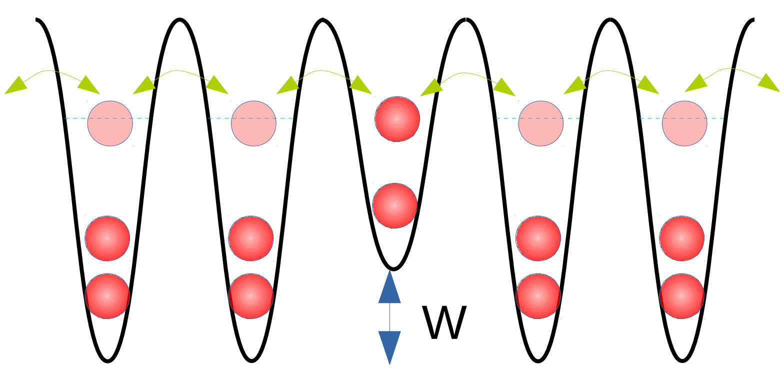
Resonant Persistent Currents

M=5 sites, N=10 particles









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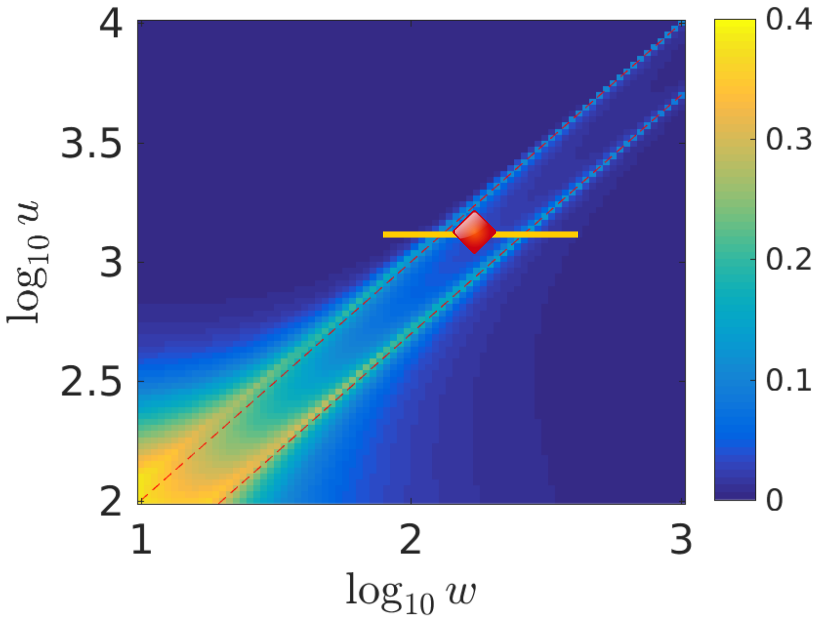

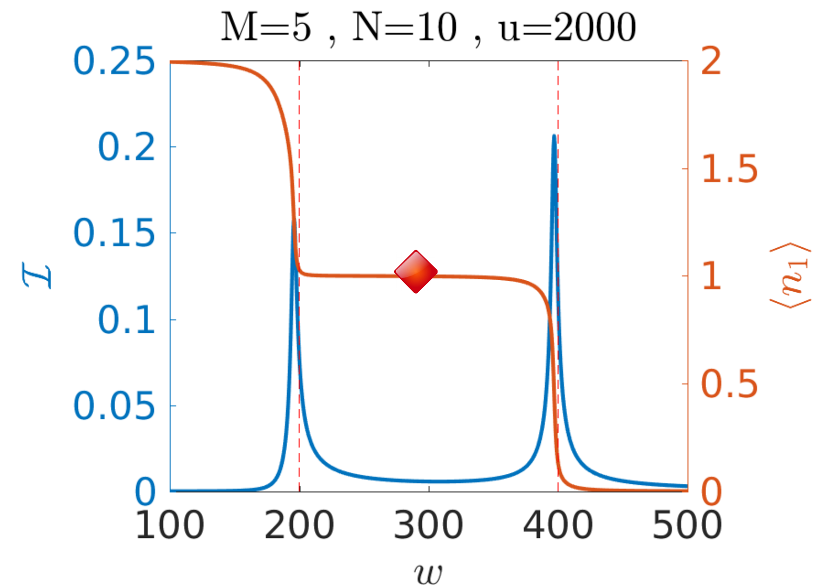

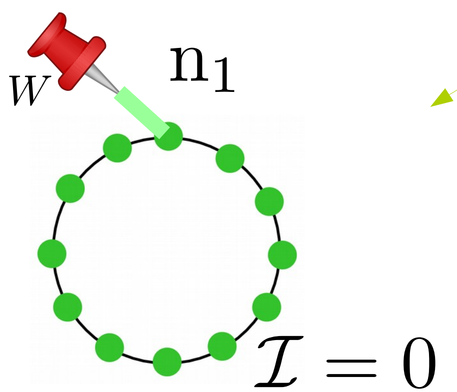

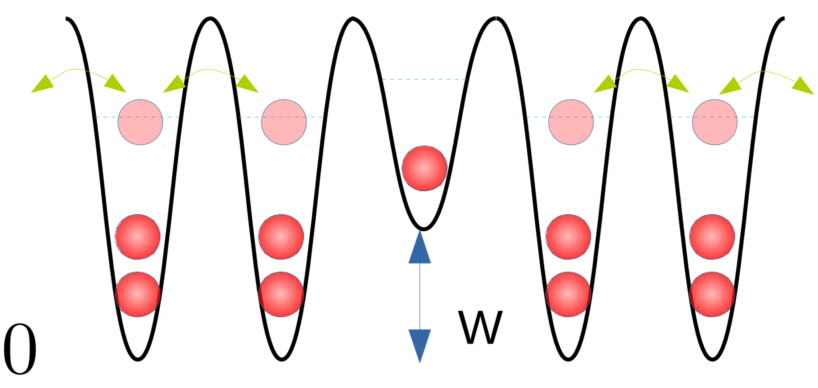

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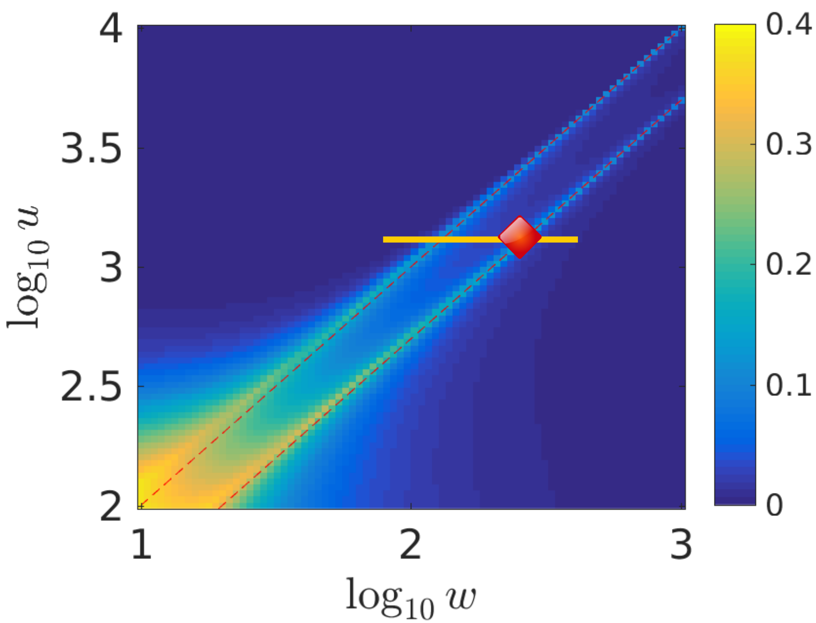

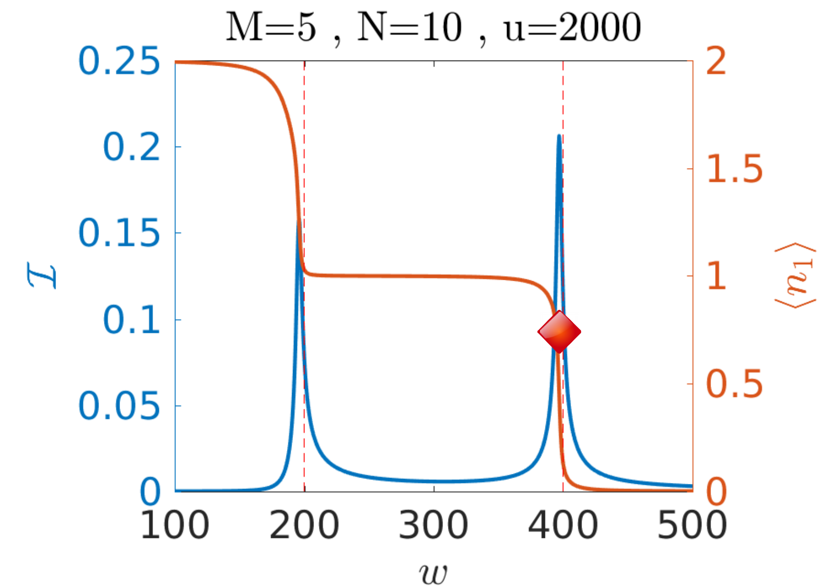

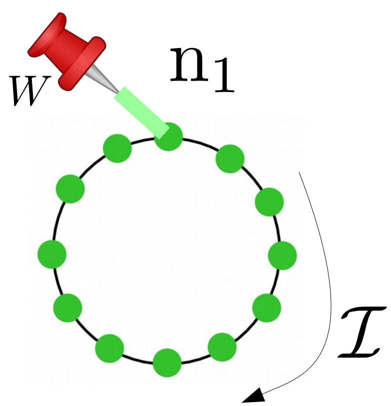

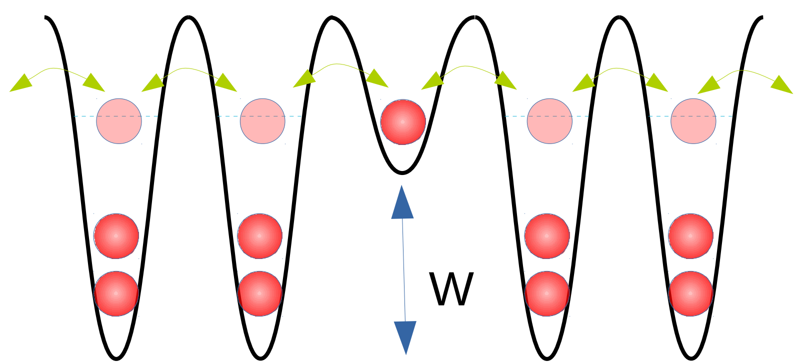

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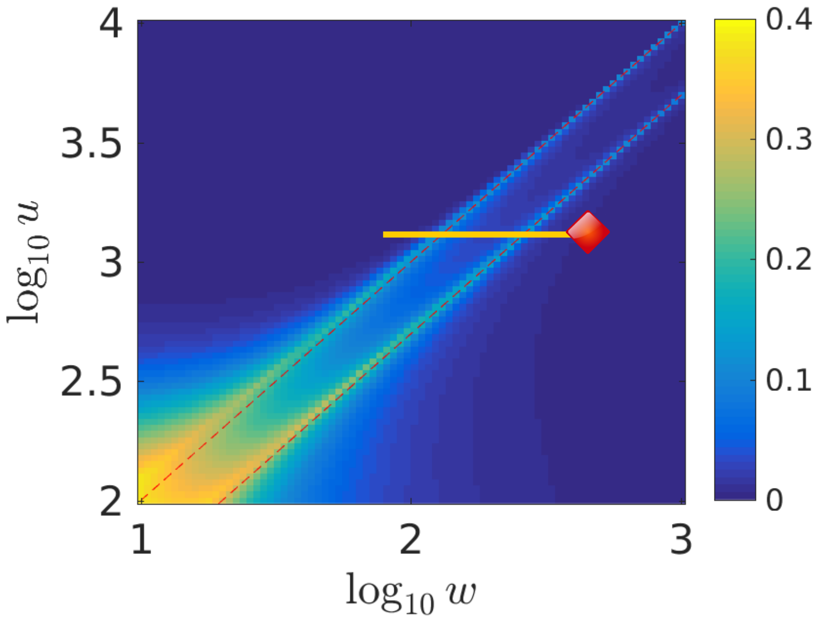
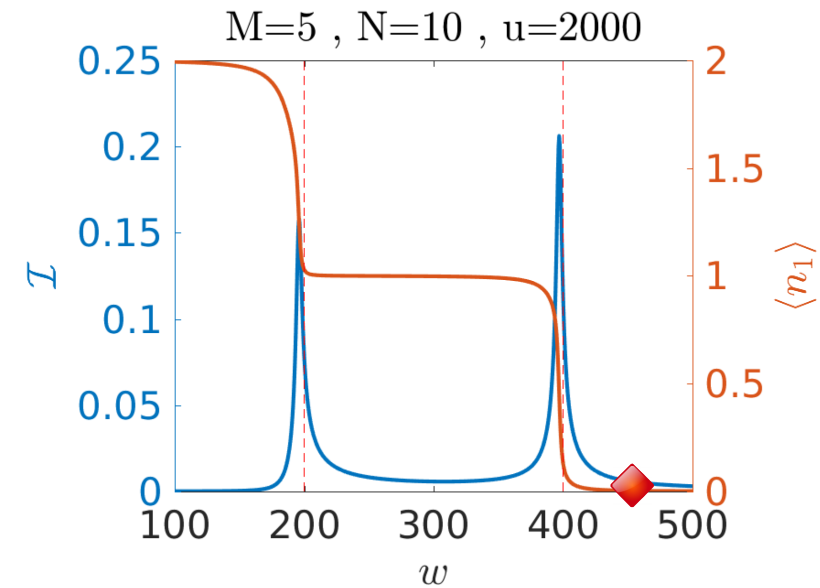
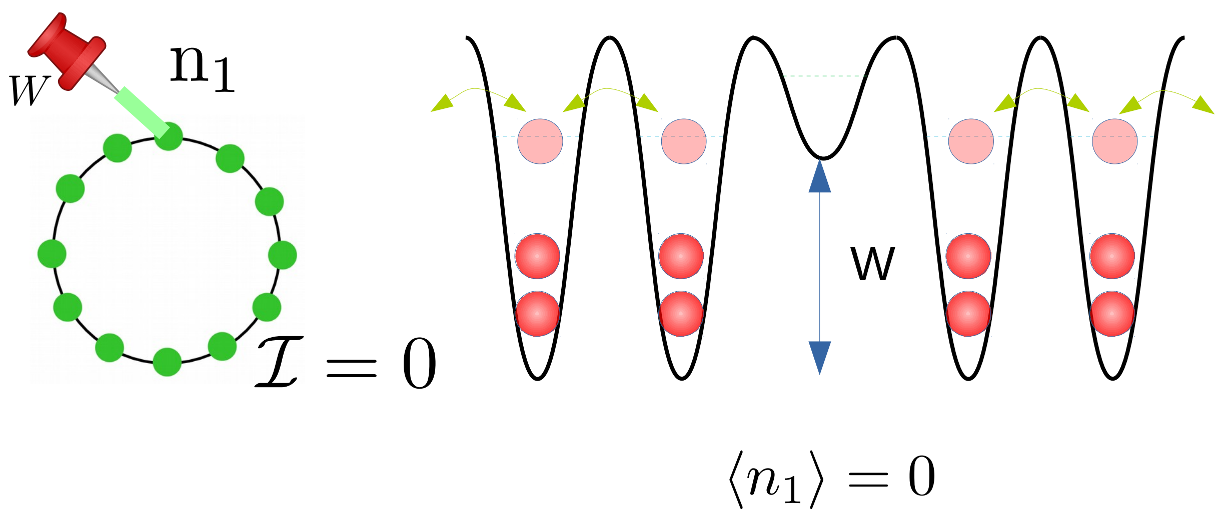






Resonant Persistent Currents

Technical details

$$\mu_{\text{barrier}}(k) = Uk + W$$

$$\mu_{\text{chain}}(q) = Uq - 2J_q$$

q= # of particles in chain sites

k= # of particles in barrier site

$$J_q = (q+1)J$$

Resonance Lines:

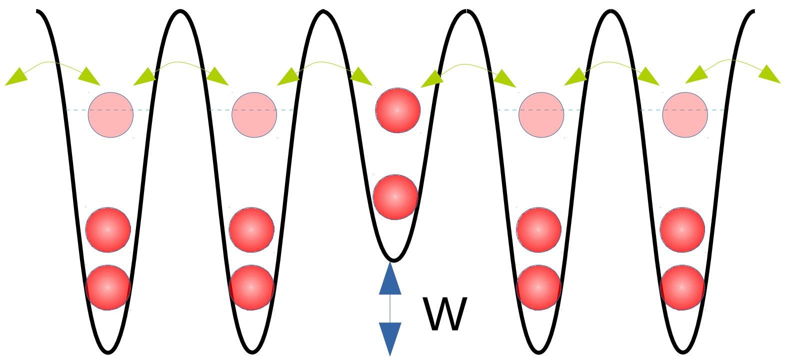
$$U = \frac{W + (2J_q - \Delta)}{q_{k+1} - k}$$

$$k = 0, 1, ..., q_c$$

$$q_c = \left\lfloor \frac{N}{M} \right\rfloor$$ (occupation floor)

$$q_k = q_c + \left\lfloor \frac{p + (q_c - k))}{M - 1} \right\rfloor$$

Resonance: $\mu_{\text{barrier}}(k) = \mu_{\text{chain}}(k)$



$$\mathrm{N} = \mathrm{M}q_c + p = k + (M-1)q_k + p_k$$

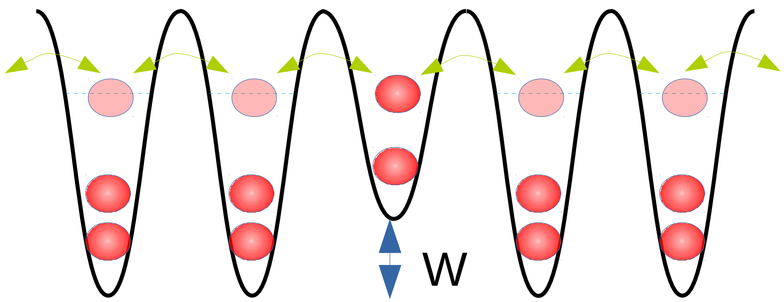
Self-energy correction

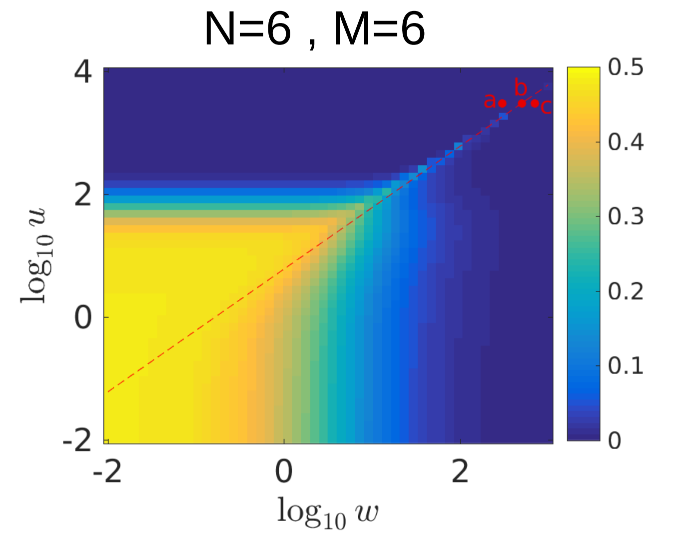
$$\Delta = \frac{2J_w^2(M-1+\cos\Phi)}{J_q M}$$

Effective one particle Hamiltonian

Bose Hubbard model:

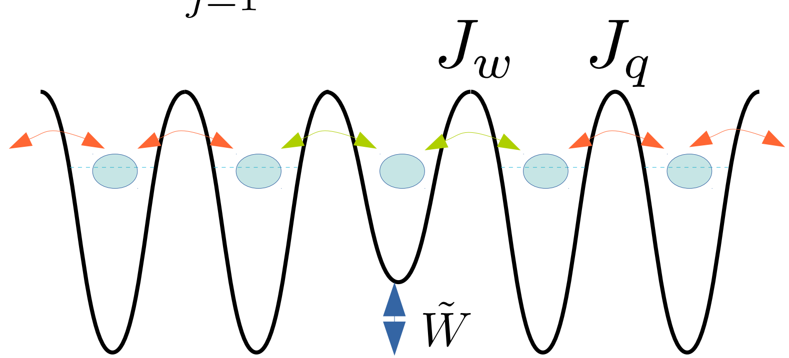
$$\mathcal{H}_{\rm BH} \;=\; \sum_{j=1}^{M}\left[\frac{U}{2}a_j^\dagger a_j^\dagger a_j a_j - \frac{K_j}{2}\left(\mathrm{e}^{i(\Phi/M)}a_{j+1}^\dagger a_j + \mathrm{e}^{-i(\Phi/M)}a_j^\dagger a_{j+1}\right)\right] + W a_1^\dagger a_1$$

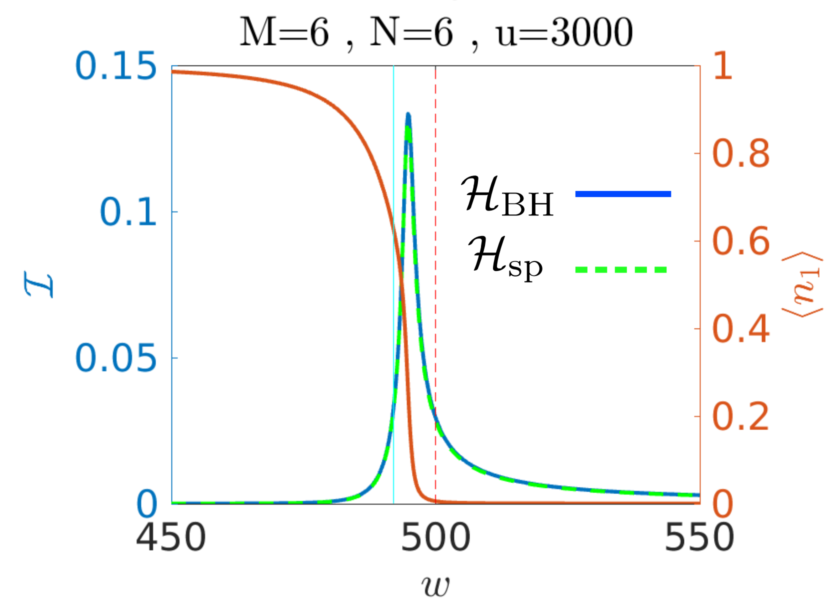




Effective one particle model:

$$\mathcal{H}_{\rm sp} \;=\tilde{W}|1\rangle\langle 1| - \sum_{j=1}^{M}\tilde{J}_j \mathrm{e}^{i(\Phi/M)}|j+1\rangle\langle j| + \mathrm{h.c.}$$





With:

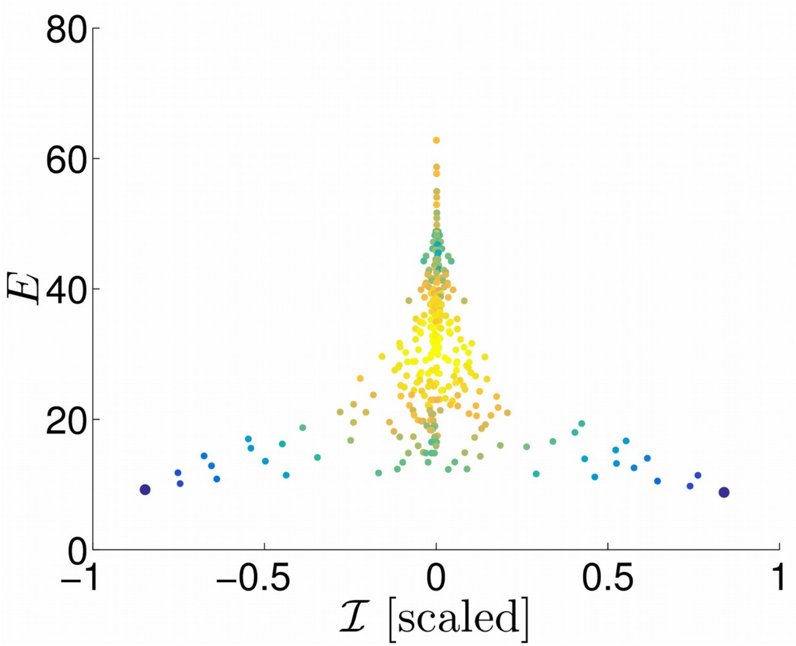
$$J_q = (q+1)J$$

$$J_w = \sqrt{(q+1)(k+1)}\,J$$

$$\tilde{W} = W - U(q-k)$$

Coherent Rabi oscillations between flow-states

Many-body Spectrum (M=3 , N= 24)



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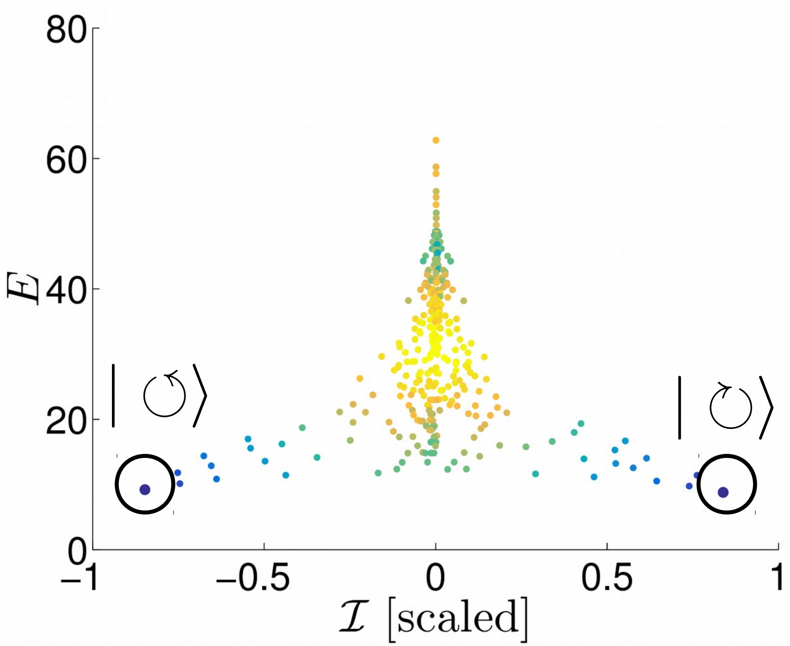

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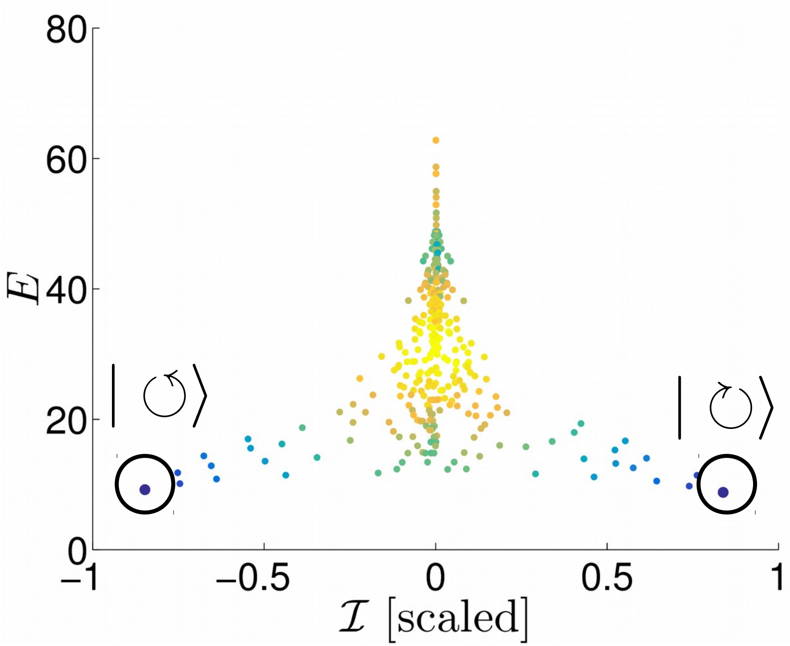

For $\Phi \approx \pi$ we have an effective two level system

$$\mathcal{H}_{\mathrm{TLS}} = \begin{pmatrix} 0 & \Delta_s/2 \\ \Delta_s/2 & 0 \end{pmatrix}$$

Pure Rabi oscillations

$$|\Psi\rangle = \cos\left(\frac{\Delta_s t}{2}\right)|\circlearrowright\rangle - i\sin\left(\frac{\Delta_s t}{2}\right)|\circlearrowleft\rangle$$

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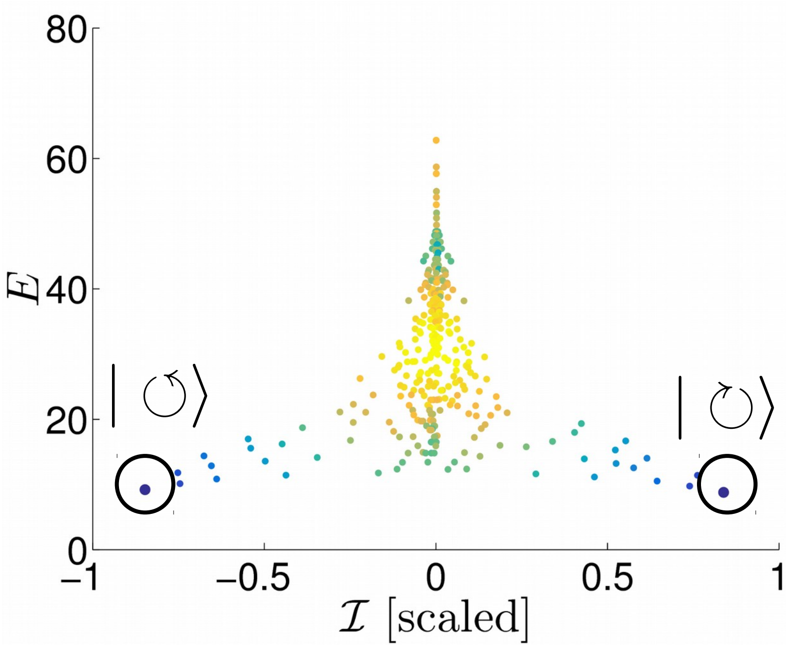

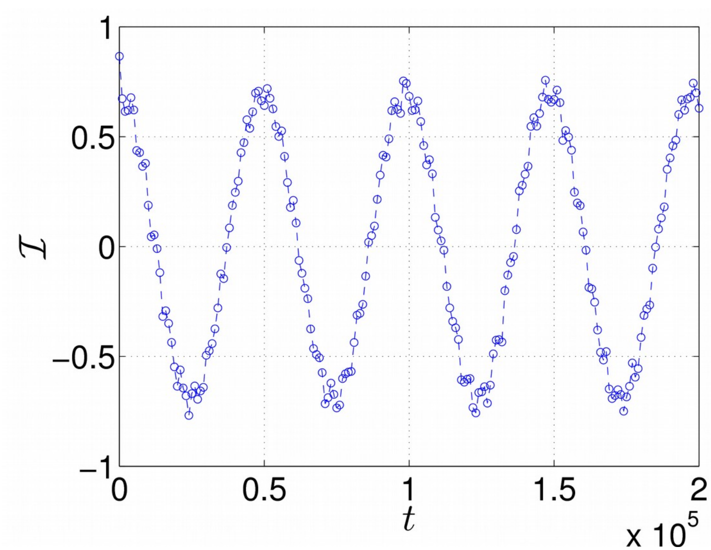

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Implying alternating current with frequency Δ_s

Coherent Rabi oscillations between flow-states

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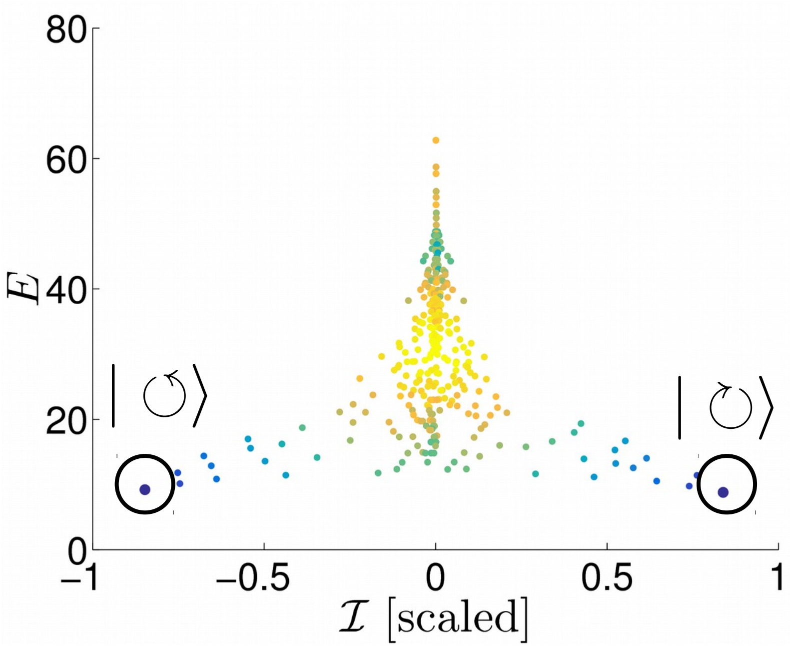

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Pure Rabi oscillations

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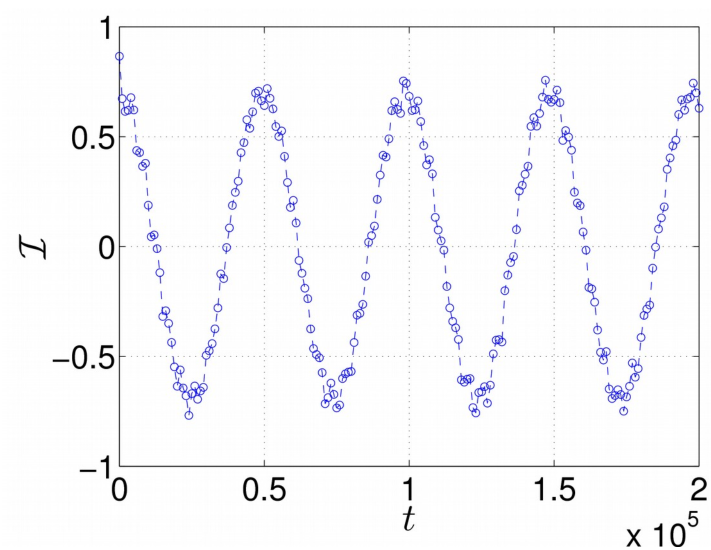
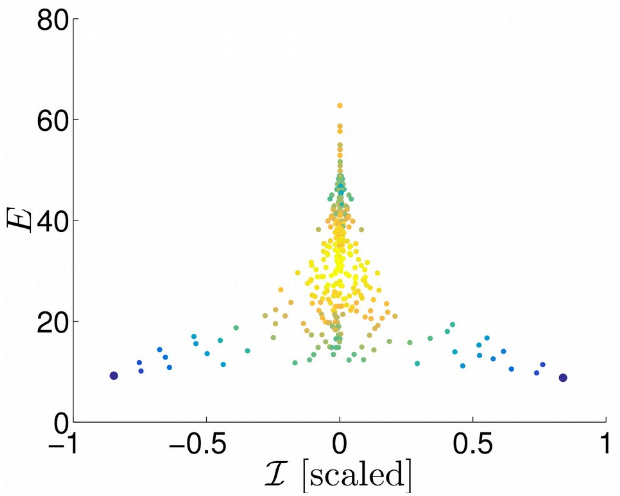


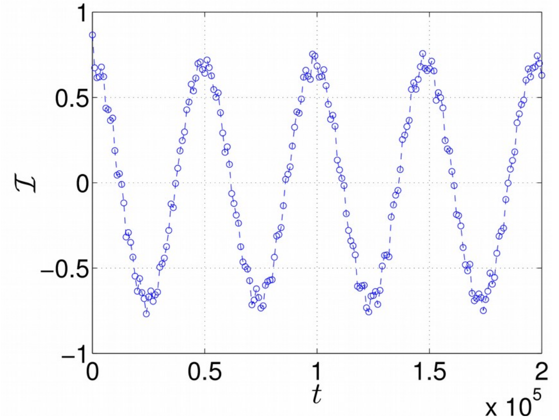
Implying alternating current with frequency Δ_s

Δ_s Decreases exponentially with N !

Coherent Rabi oscillations between flow-states

Under the barrier tunneling





N=24 bosons in M=3
ring with u=5 and $\Phi = \pi$

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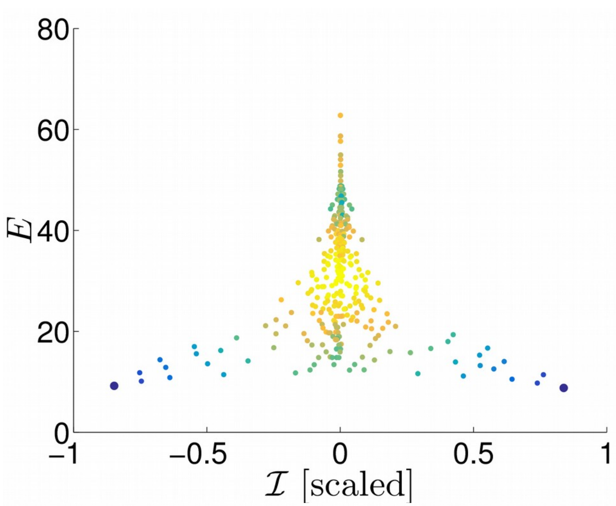

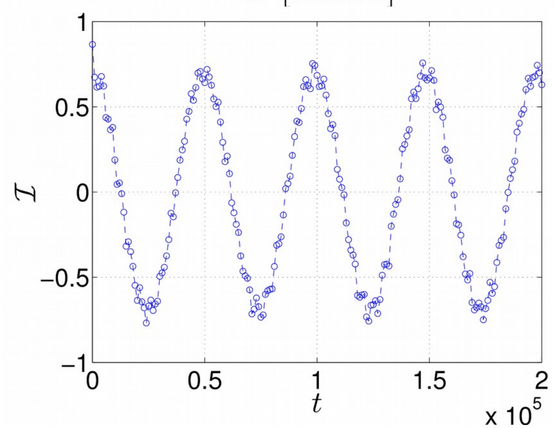

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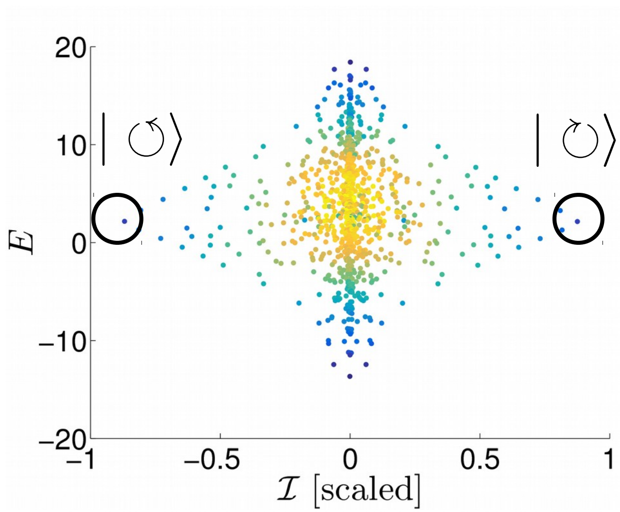
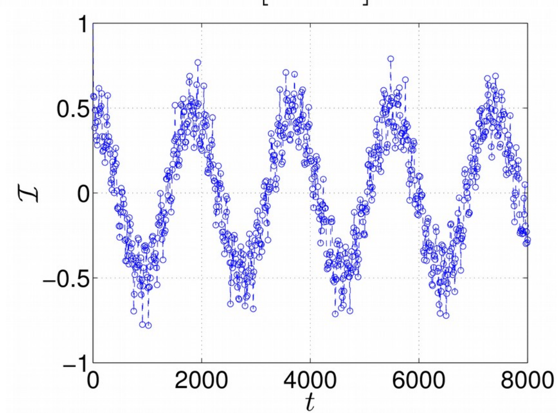
Chaos-assisted tunneling





N=16 bosons in M=4
ring with u=1 and $\Phi = 0$
(non-rotating).

Coherent Rabi oscillations between flow-states

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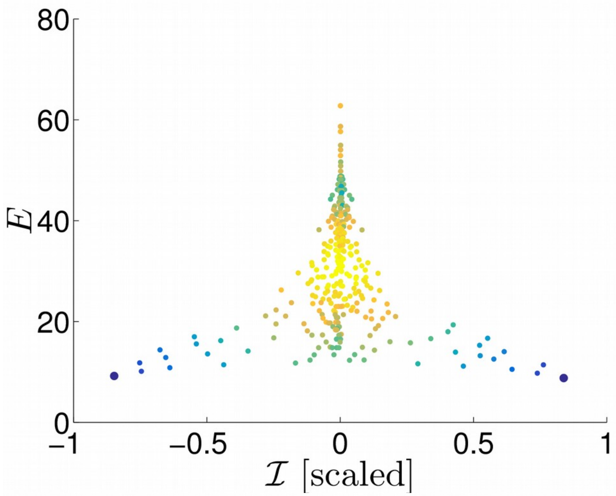

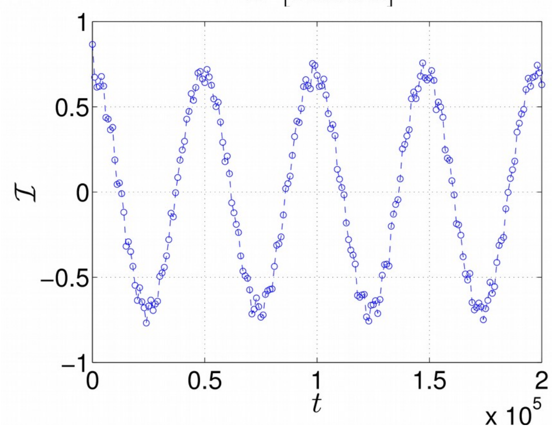

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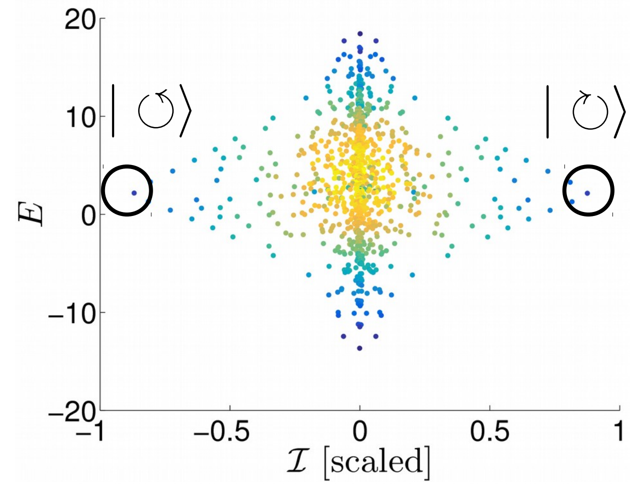

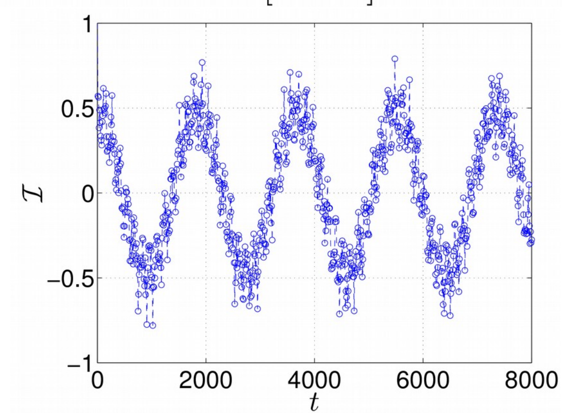

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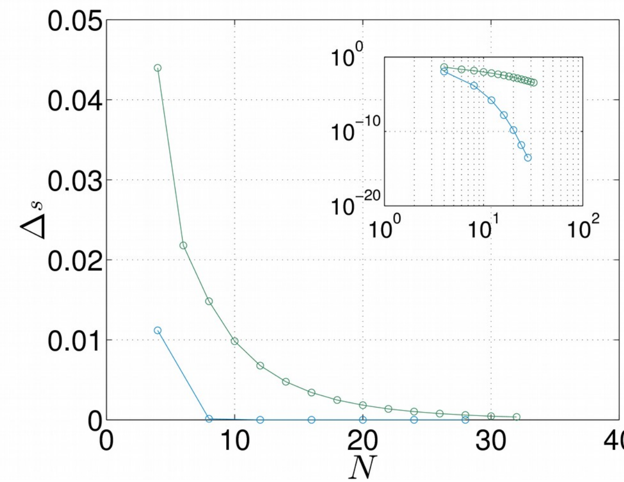
N dependence



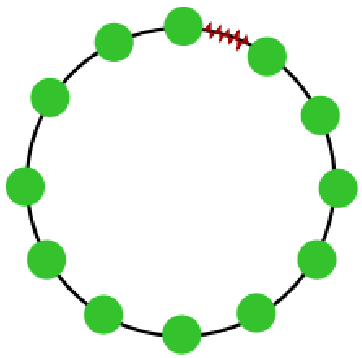
Contrasting $\Phi = \pi$ with $\Phi = 0$ For the M=4 ring.

The Introduction of a Weak Link

In a semiclassical framework $a \mapsto \sqrt{n}e^{i\varphi}$

This is like M coupled oscillators

$$H = \sum_{j=1}^{M} \left[\frac{U}{2} n_j^2 - K_j \sqrt{n_{j+1} n_j} \cos\left(\varphi_{j+1} - \varphi_j - \frac{\Phi}{M} \right) \right]$$



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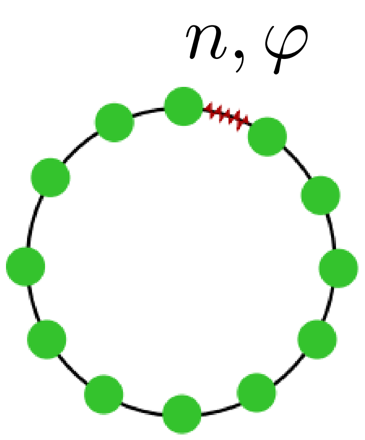

With the weak link we can describe the system as one DOF + Bath
Consequently we obtain the Josephson circuit Hamiltonian:

$$\mathcal{H}_{\mathrm{JCH}} = E_C \, n^2 + \frac{1}{2} E_L \varphi^2 - E_J \, \cos(\varphi - \Phi) + \mathcal{H}_{\mathrm{bath}}$$

With: $E_C = U$, $E_L = [(N/M)/(M-1)]K$, $E_J = (N/M)K'$

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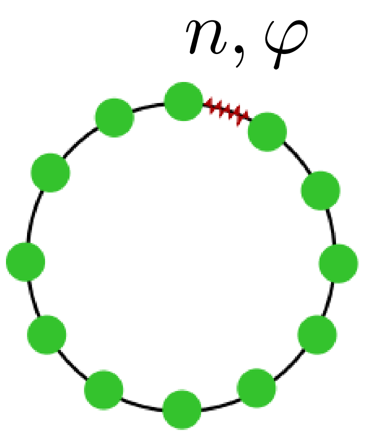

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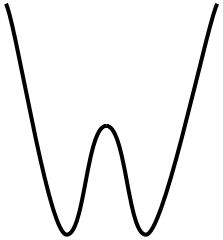
$E_J > E_L$



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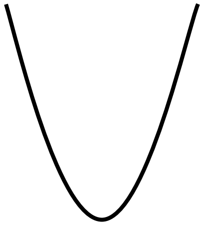

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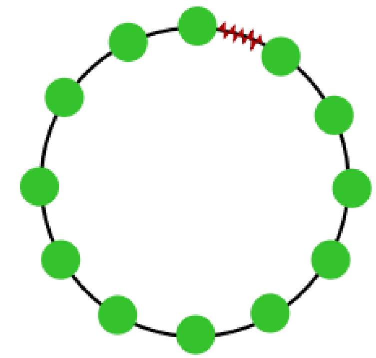
n, φ



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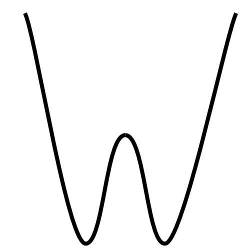

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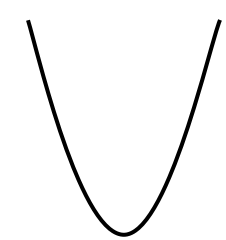

$$\eta \;=\; \frac{\pi}{\sqrt{\gamma}} \qquad , \quad \gamma = \frac{u}{N^2}$$

The condition for witnessing coherent oscillations is $\eta < \pi$, which requires $\gamma > 1$. This coincides with the border of the Mott regime!

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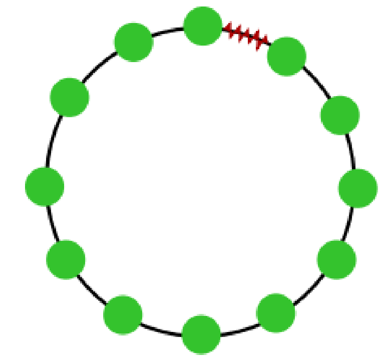

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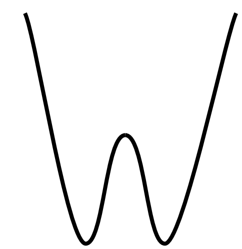

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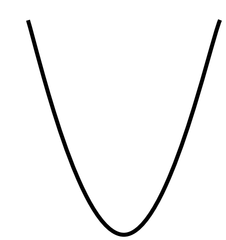

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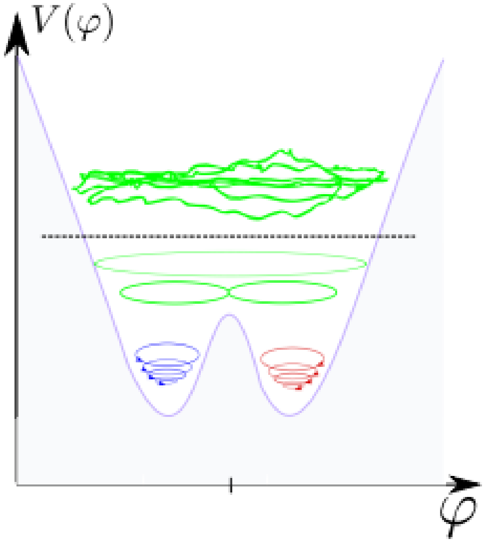
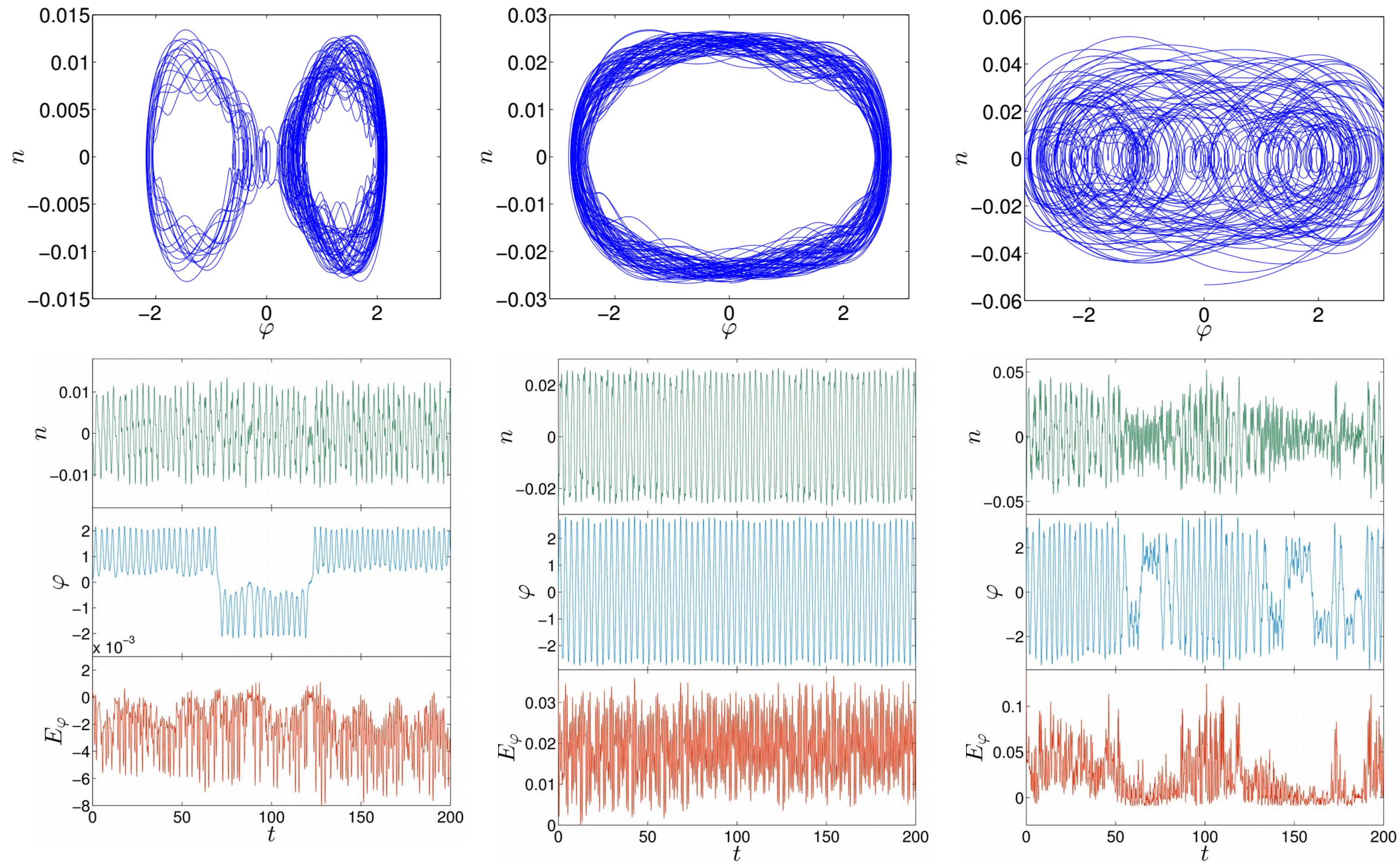
The “system plus bath” perspective is expected to be valid for *large* M ...

Threshold for chaotic motion





This picture is valid provided

$\mathrm{M} \geq 6$

Double well for

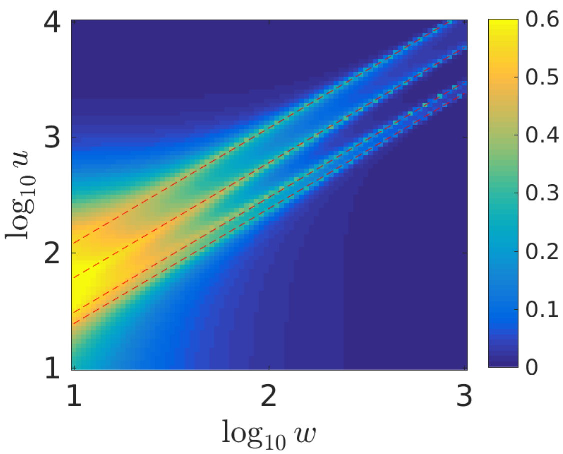
$$\frac{E_J}{E_L} = (M-1)\frac{K'}{K} > 1$$

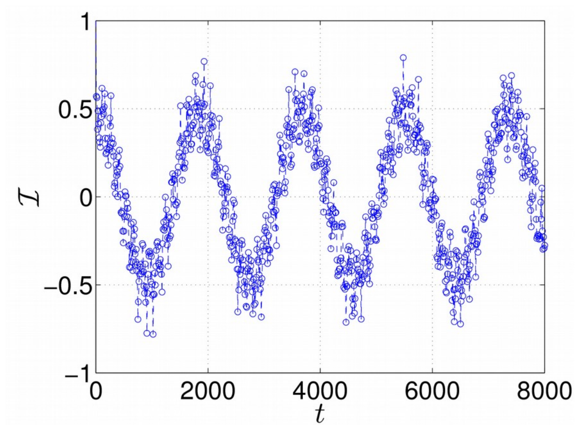
Here: M=6 , u=200 , $\Phi = \pi, K'/K = 0.3$

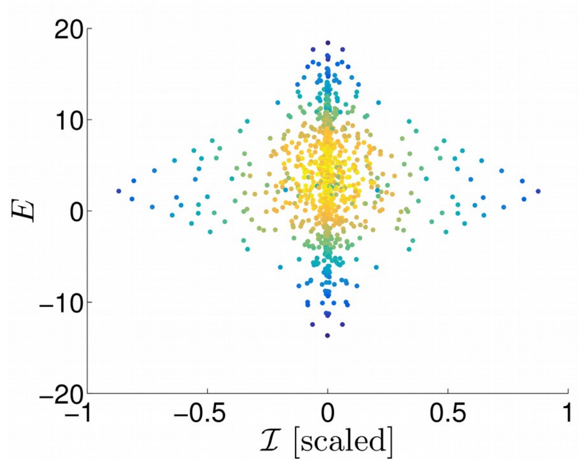
Chaos threshold > Barrier

Concluding Remarks

- We have shown that the a resonant behavior occurs for the persistent currents, between the Mott insulator and the “disconnected ring” regimes.
- We have highlighted the feasibility of chaos-assisted Rabi oscillation between metastable flow-states of a non-rotating device.
- The JCH description of a ring with a weak link is valid for a ring with M>5 sites. However, it is not likely to observe coherent operation.
- In a broader perspective we would like to demonstrate that tools of semiclassics and quantum chaos are extremely advantageous in an arena that is largely dominated by field-theoretical many-body methods.







- G. Arwas, D. Cohen, F. Hekking, A. Minguzzi, Phys. Rev. A (2017) Editors’ Suggestion
- G. Arwas, D. Cohen, New J. Phys. 18, 015007, (2016)