Superfluidity of Bose-Hubbard circuits: beyond the traditional paradigm

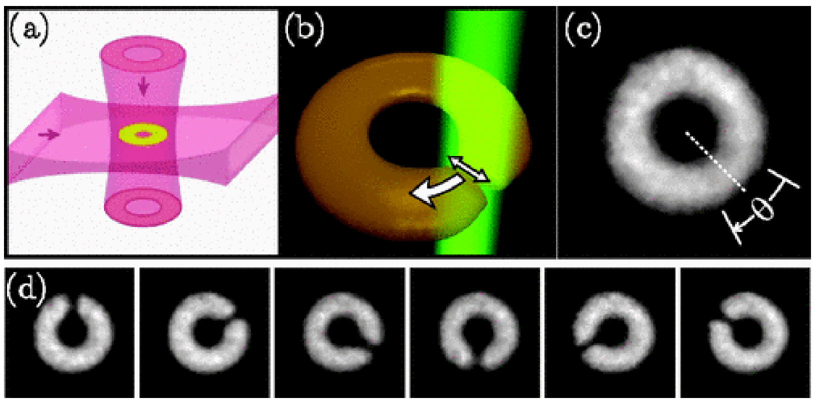
Geva Arwas, Doron Cohen

Ben-Gurion University

IPS 2016

Recent experiments with interacting bosonic particles have opened a new arena: Superfluidity in low dimensional circuits

Toroidal ring [1,2]



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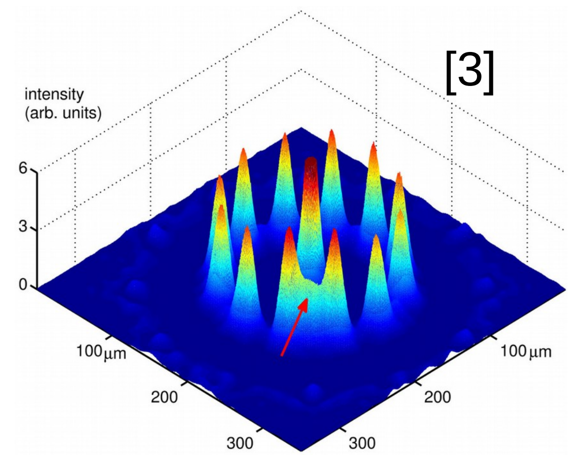

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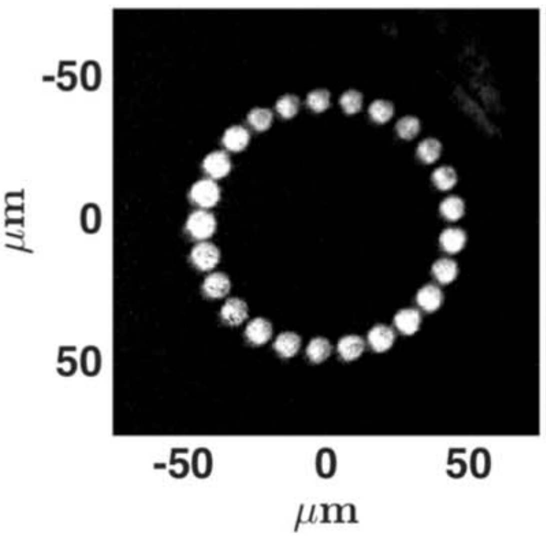

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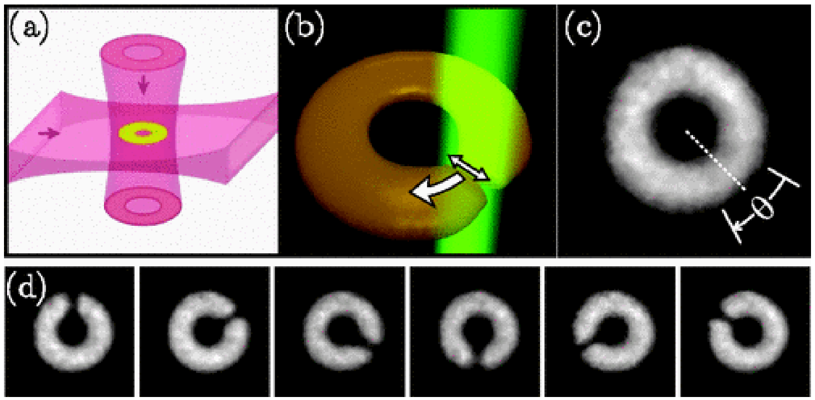

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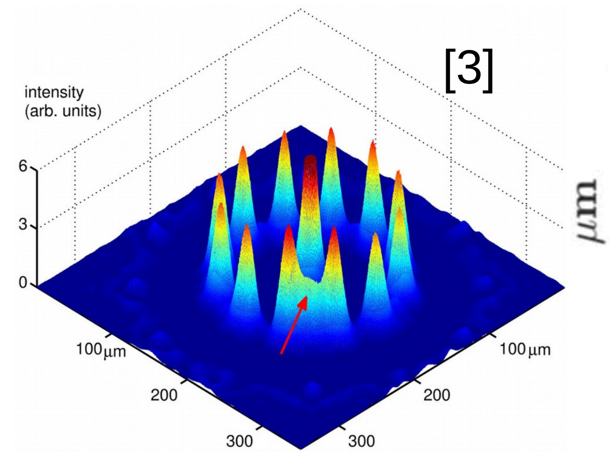

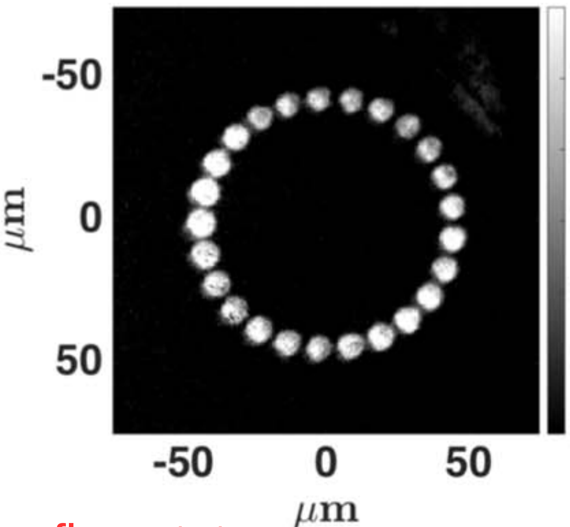

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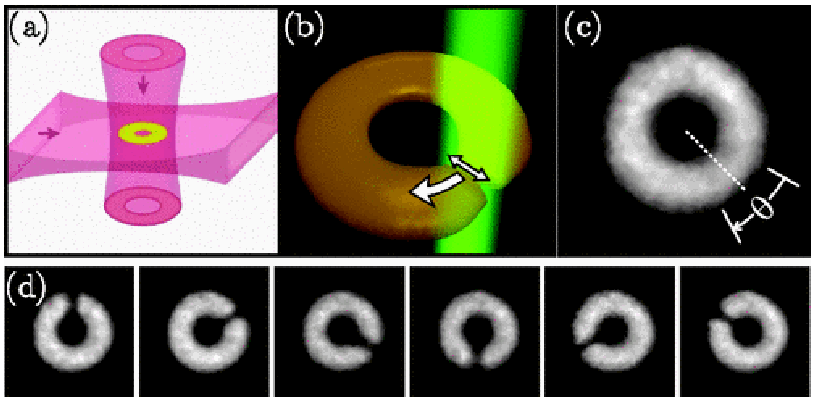

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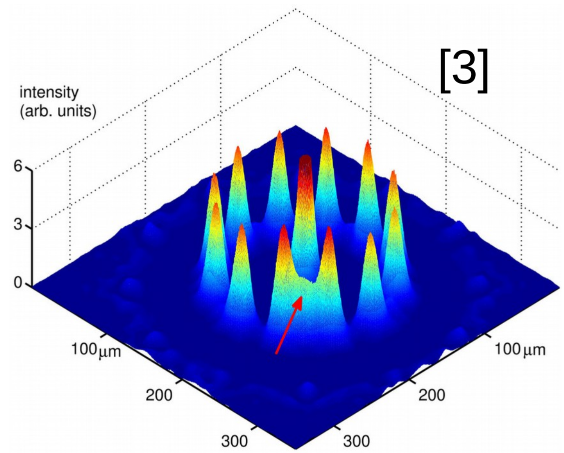

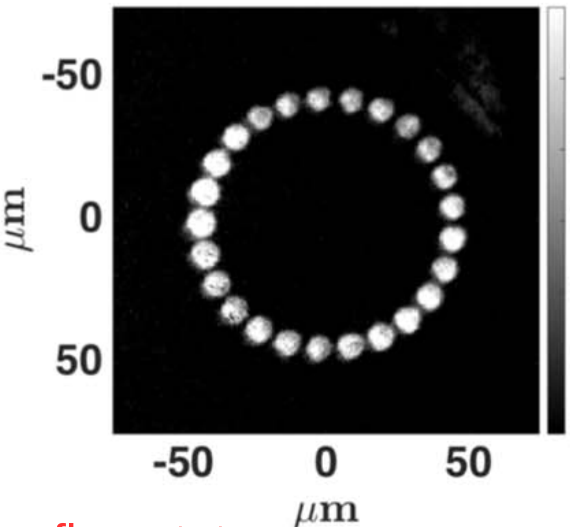

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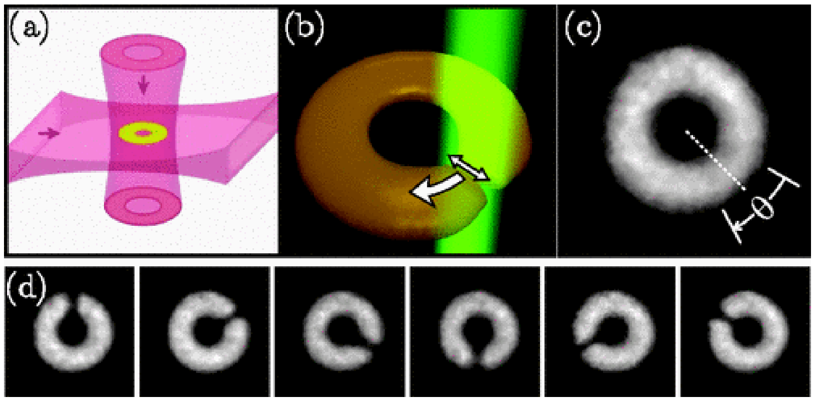

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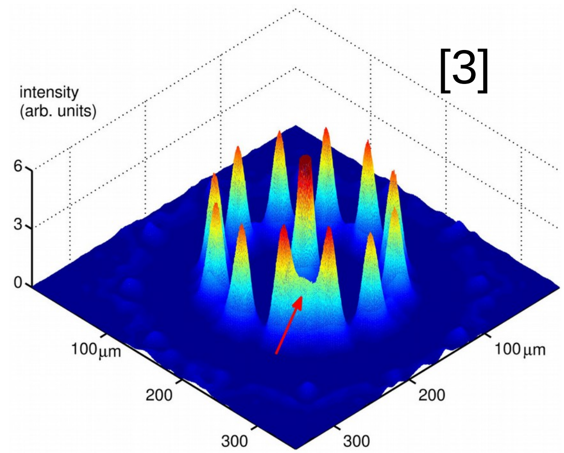

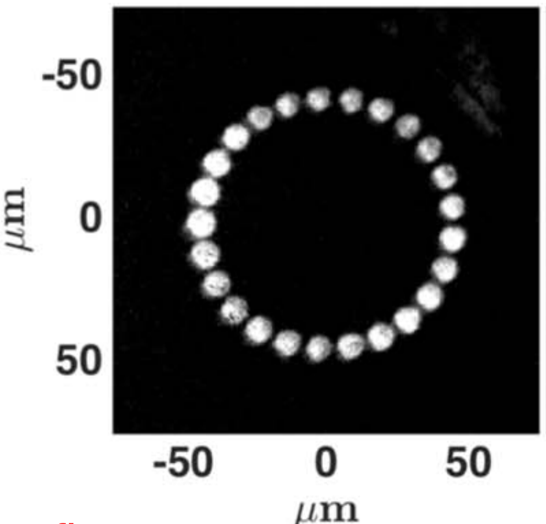

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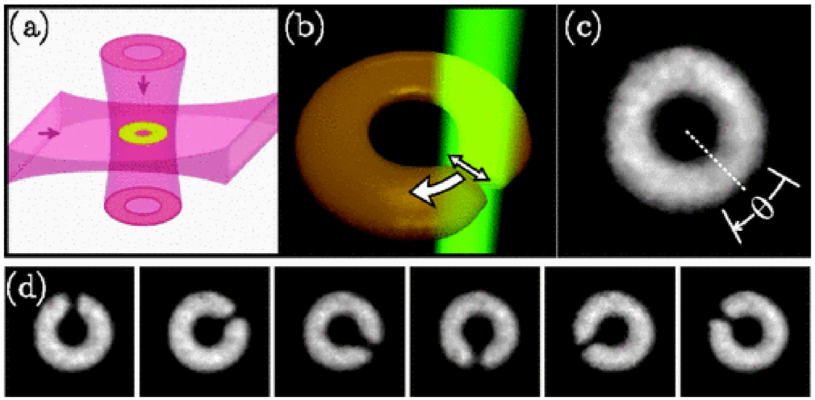

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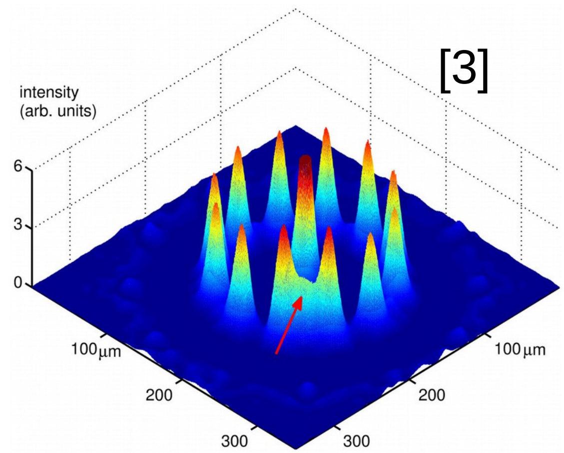

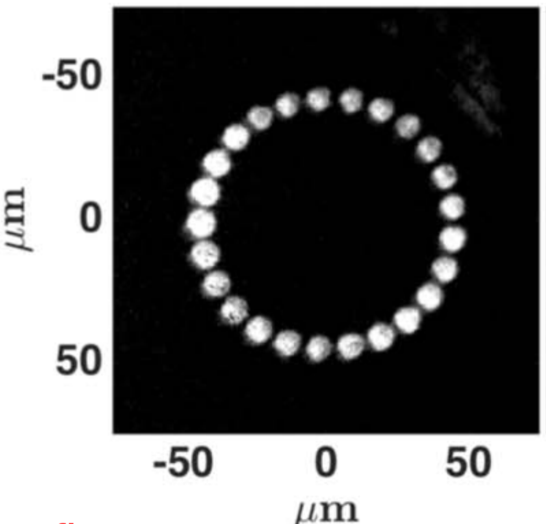

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- A stability regime diagram of the flow states in the toroidal ring has been explained by following the reasoning of the Landau superfluidity criterion
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- We demonstrate [b] how the stability is affected by non-linear resonances, in regimes where the dynamics is traditionally considered to be stable

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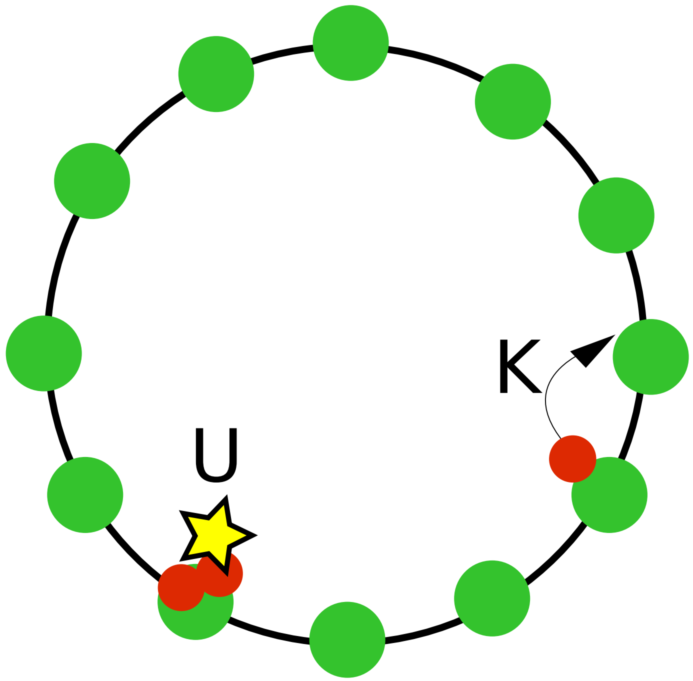
Bose Hubbard ring

A system of N Bosons in an M-site ring

$$\mathcal{H} \;=\; \sum_{j=1}^{M}\left[\frac{U}{2}a_j^{\dagger}a_j^{\dagger}a_ja_j - \frac{K}{2}\left(a_{j+1}^{\dagger}a_j + a_j^{\dagger}a_{j+1}\right)\right]$$



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$$u \;=\; \frac{NU}{K}$$

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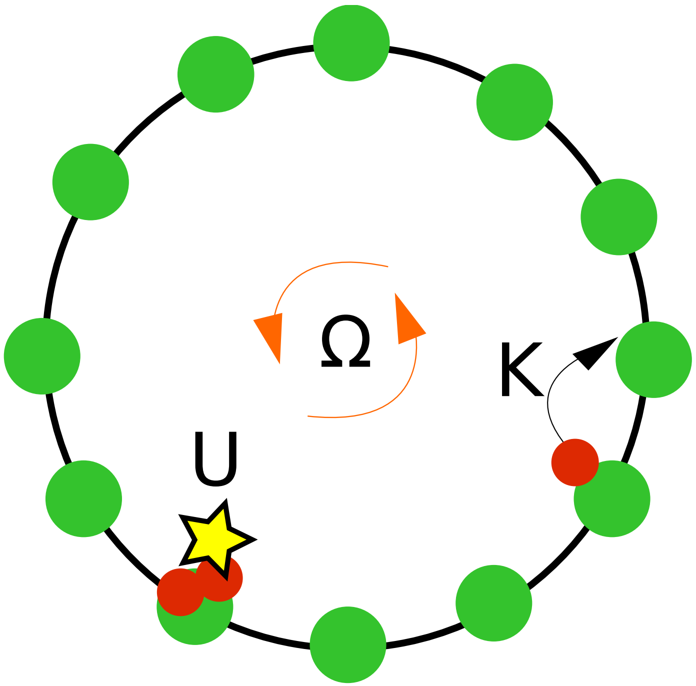

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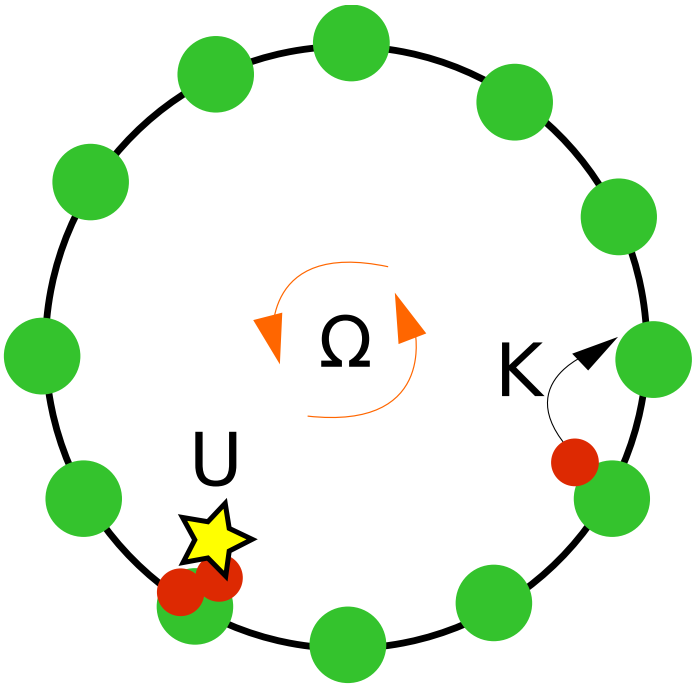

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A coherent flow-state is formed by condensing all bosons into a single momentum orbital:

$$|m\rangle = \left(b_m^\dagger\right)^N |0\rangle \ , \ m = 1...M$$

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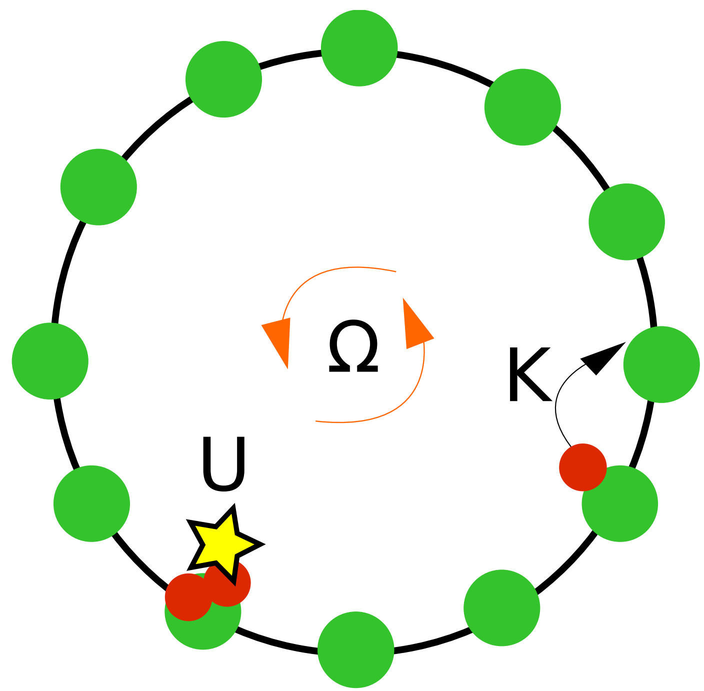

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Given (u, Φ, M, N)

Does a prepared flow state is metastable (superfluid)?

Or does it decay?

Linear stability analysis

The BH Hamiltonian in momentum space is given by:

$$\mathcal{H} = \sum_k \epsilon_k b_k^\dagger b_k + \frac{U}{2M} \sum_{\langle k_1 .. k_4 \rangle} b_{k_4}^\dagger b_{k_3}^\dagger b_{k_2} b_{k_1}$$

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Assuming condensation at $\mathrm{k{=}k}_m$
Expanding around $\mathrm{b}_m \sim \sqrt{N}$ and using Bogoliubov transformation:

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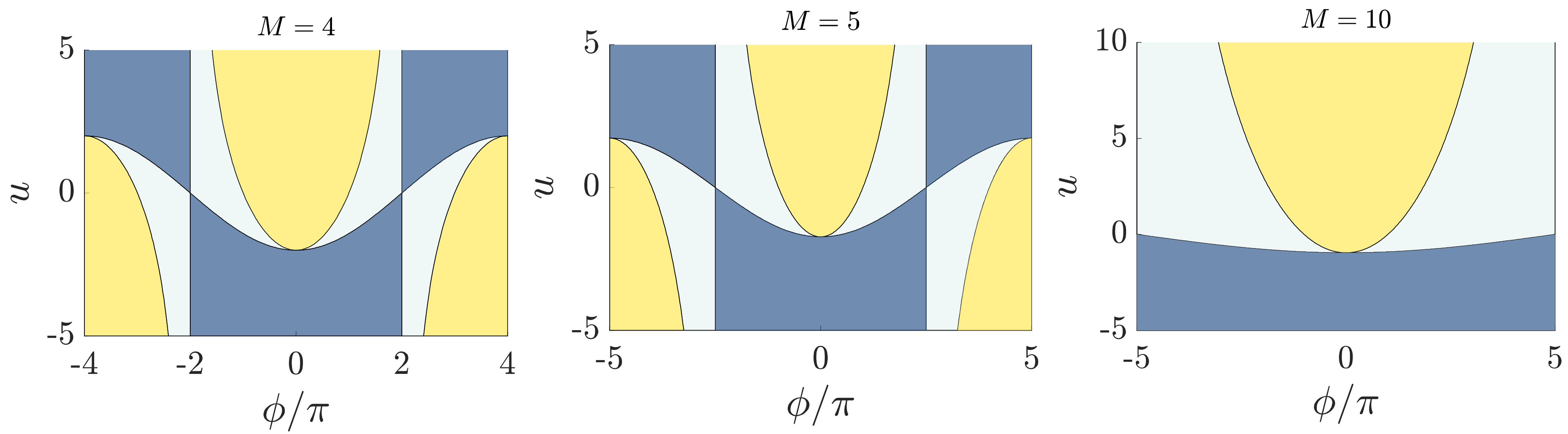
The Bogoliubov frequencies ω_q determine the linear stability of $|m\rangle$

- Energetic (Landau) stability = all ω_q real & have same sign
- Linear dynamical stability = all ω_q real
- Dynamical instability = some ω_q have imaginary part



where $\phi = \Phi - 2\pi m$

Non-linear resonances

Approximated Hamiltonian at the vicinity of the condensate:

$$\mathcal{H} \;=\; \sum_q \omega_q c_q^\dagger c_q \;+\; \frac{\sqrt{N}U}{M} \sum_{\langle q_1,q_2\rangle} \left[A_{q_1,q_2}\left(c_{-q_1-q_2} c_{q_2} c_{q_1} + \mathrm{h.c.}\right) + B_{q_1,q_2}\left(c_{q_1+q_2}^\dagger c_{q_2} c_{q_1} + \mathrm{h.c.}\right)\right]$$

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"B" terms,
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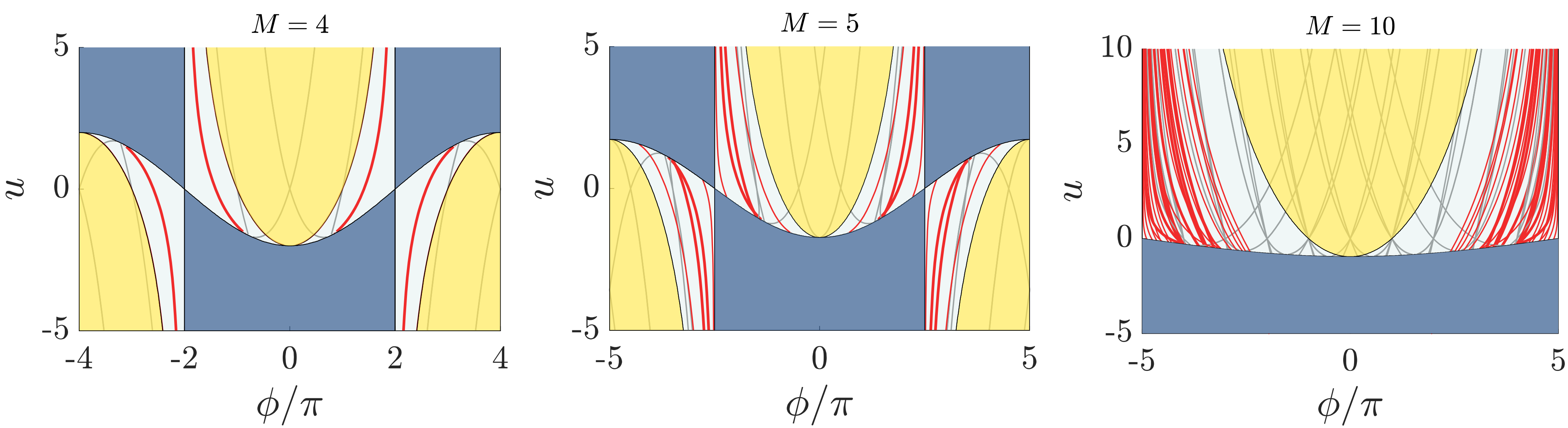

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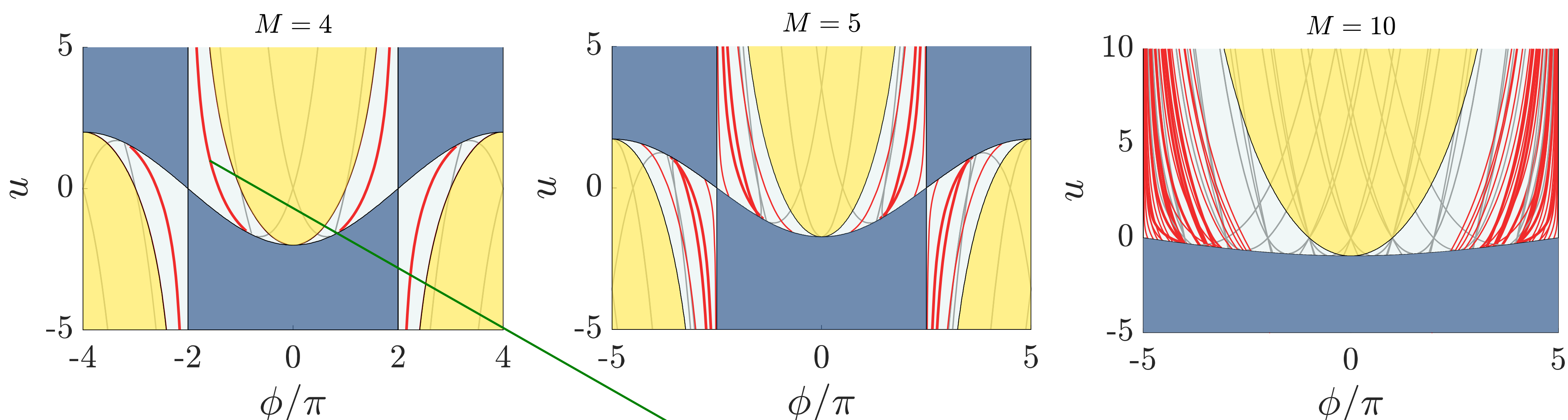

For example: “1:2” resonance
for the m = 1 flow-state: $\mathrm{u} = 4\cot\left(\frac{\Phi}{4}\right)\left[3\cos\left(\frac{\Phi}{4}\right) - \sqrt{6 + 2\cos\left(\frac{\Phi}{2}\right)}\right]$

(setting $\mathrm{q}_1 = q_2 = q = \frac{2\pi}{4}$ we get from $2\omega_q + \omega_{-2q} = 0$ the resonance line)

The quench scenario

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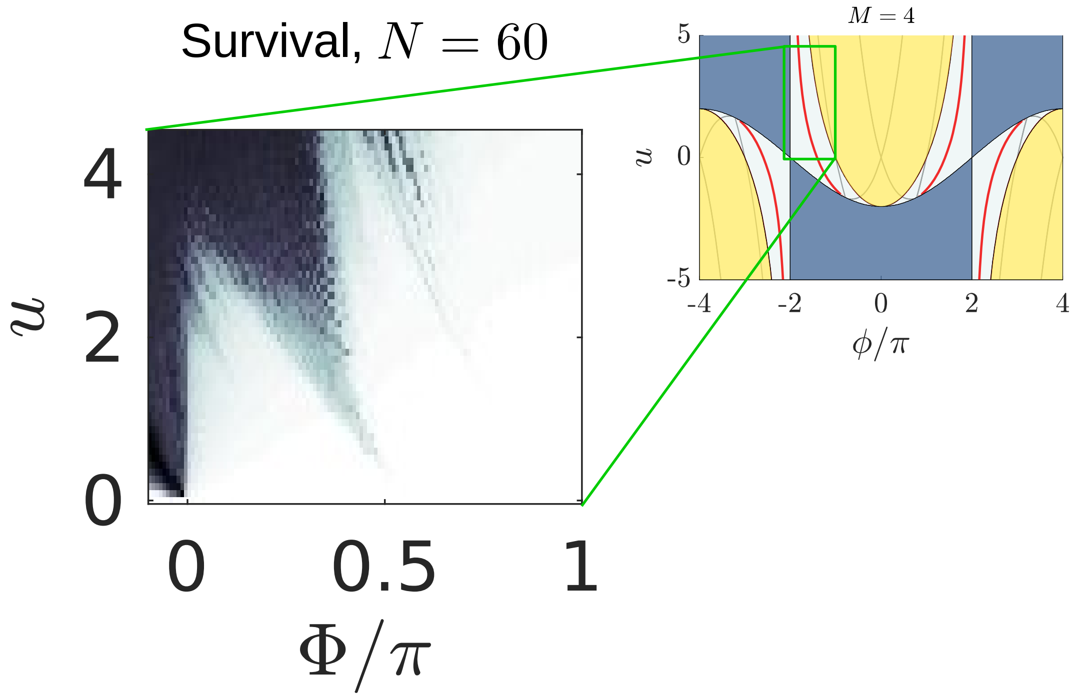

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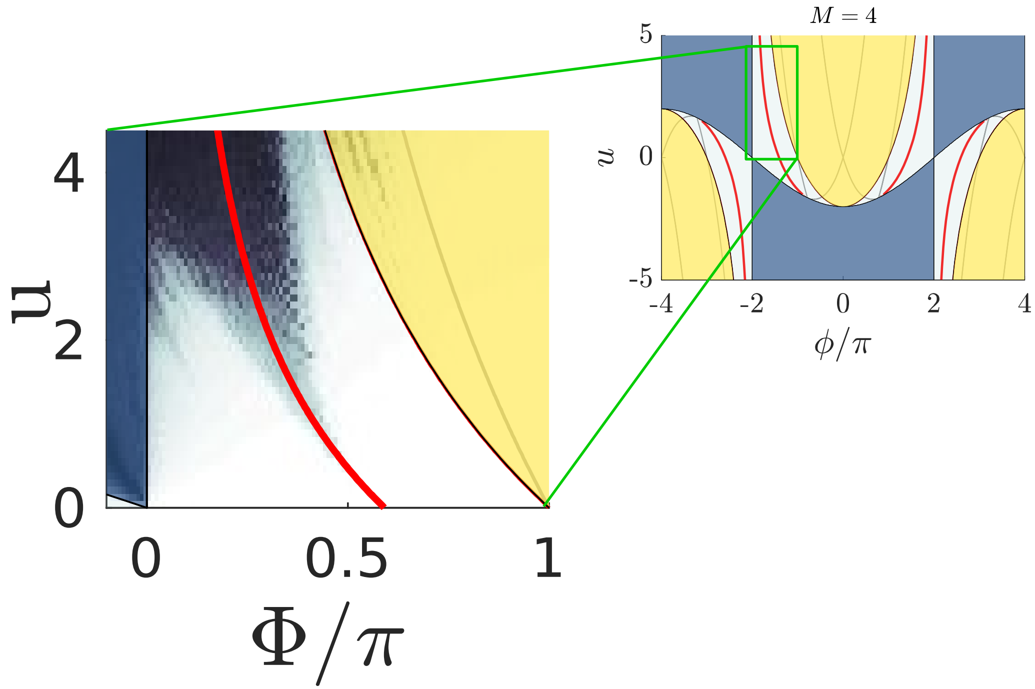

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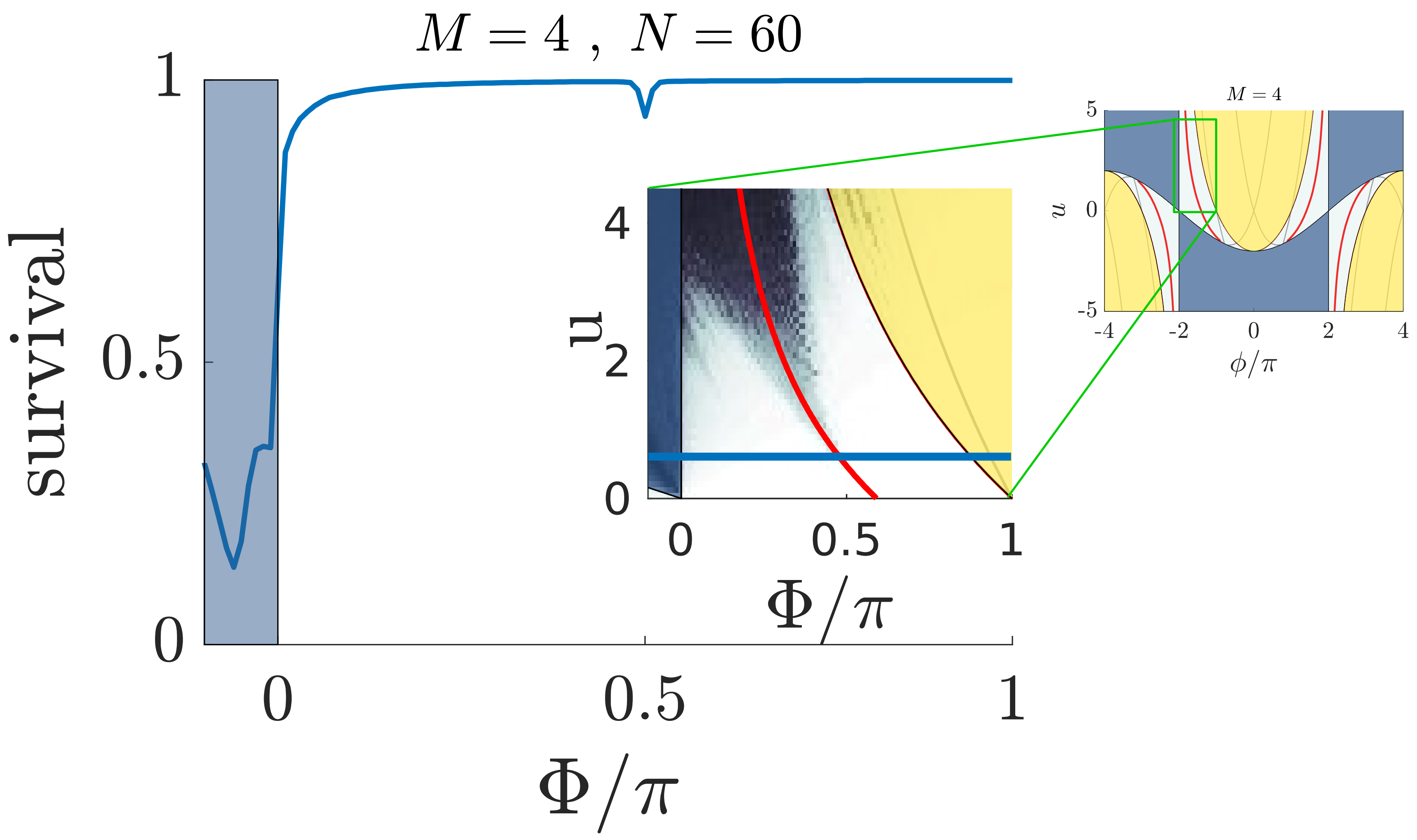

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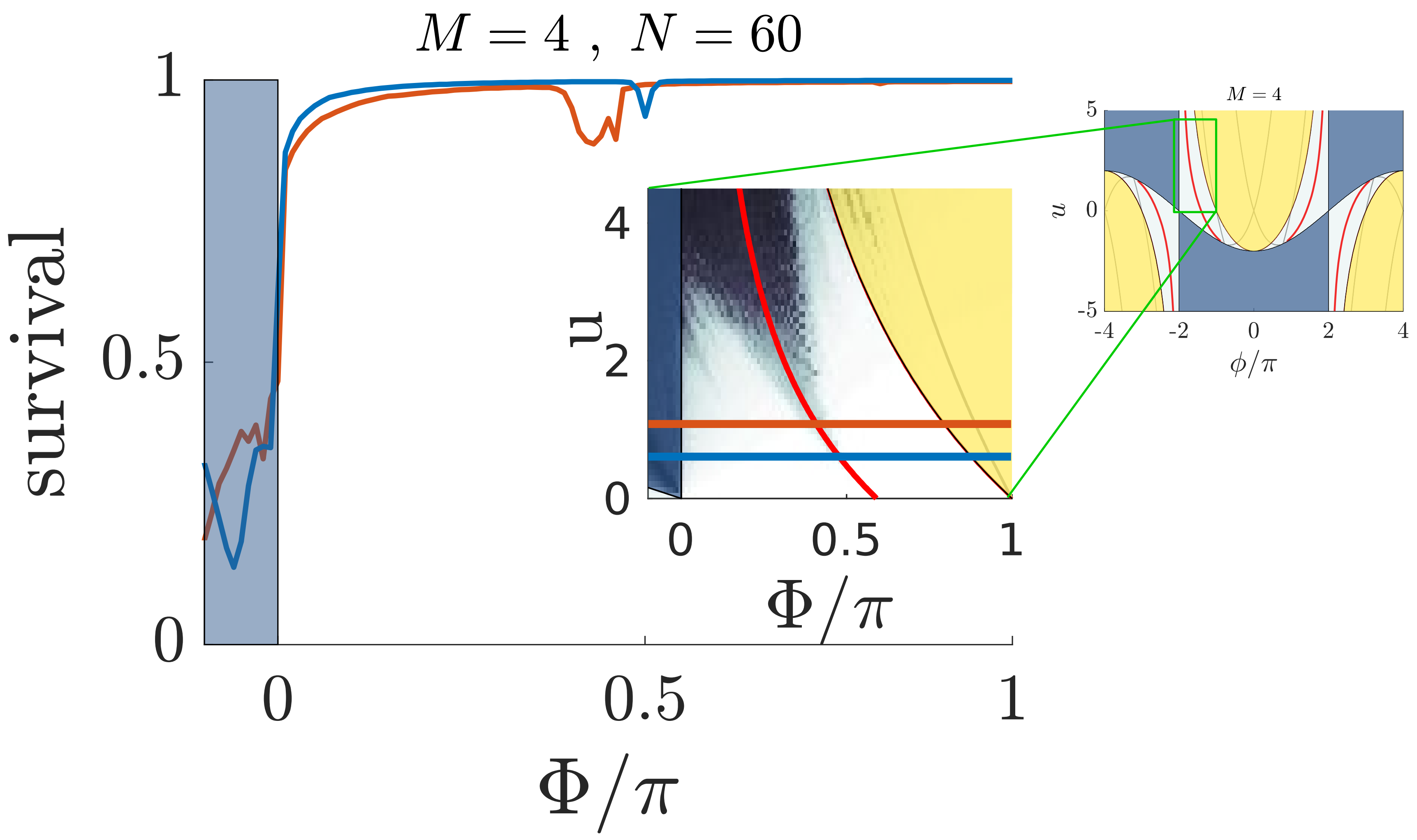

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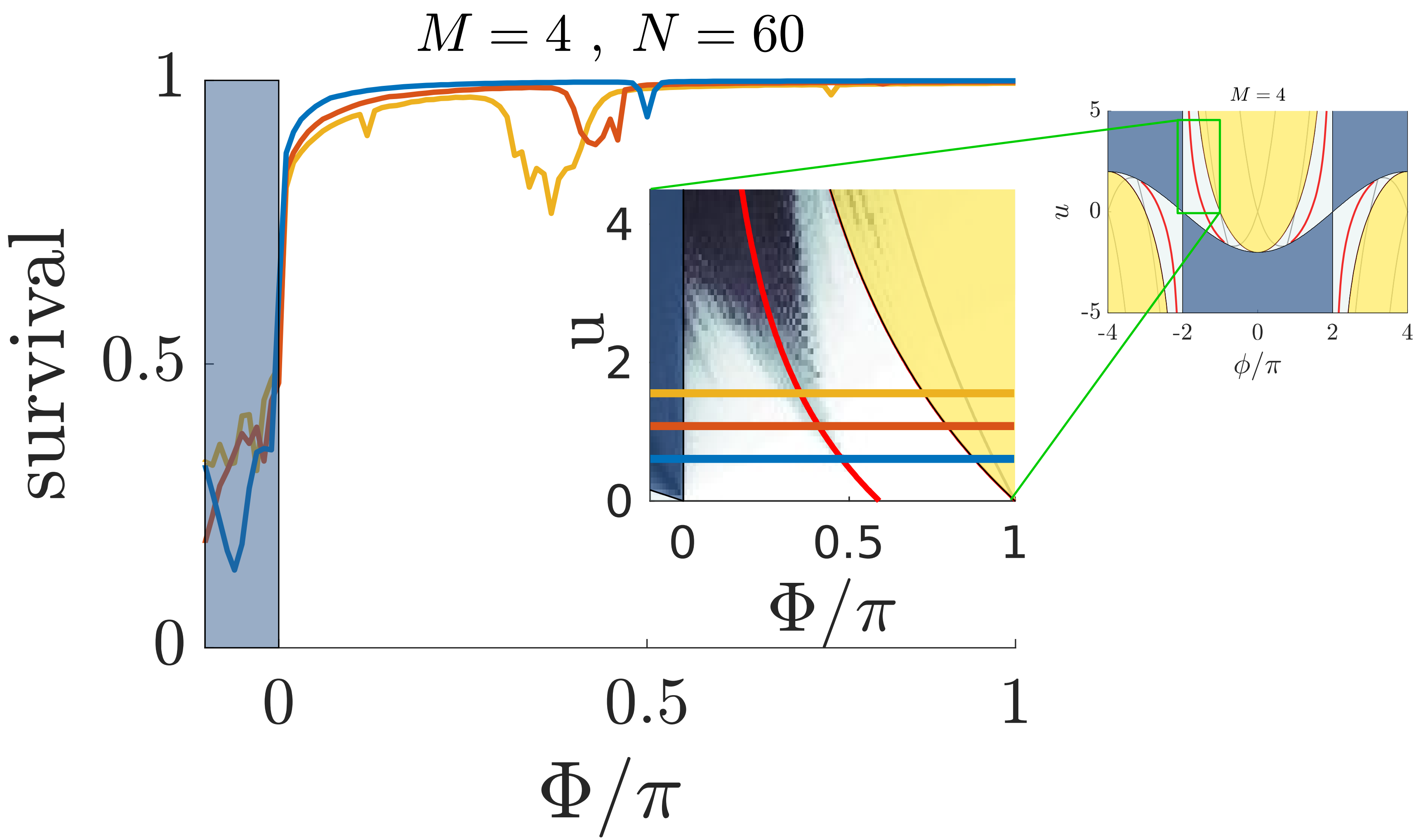

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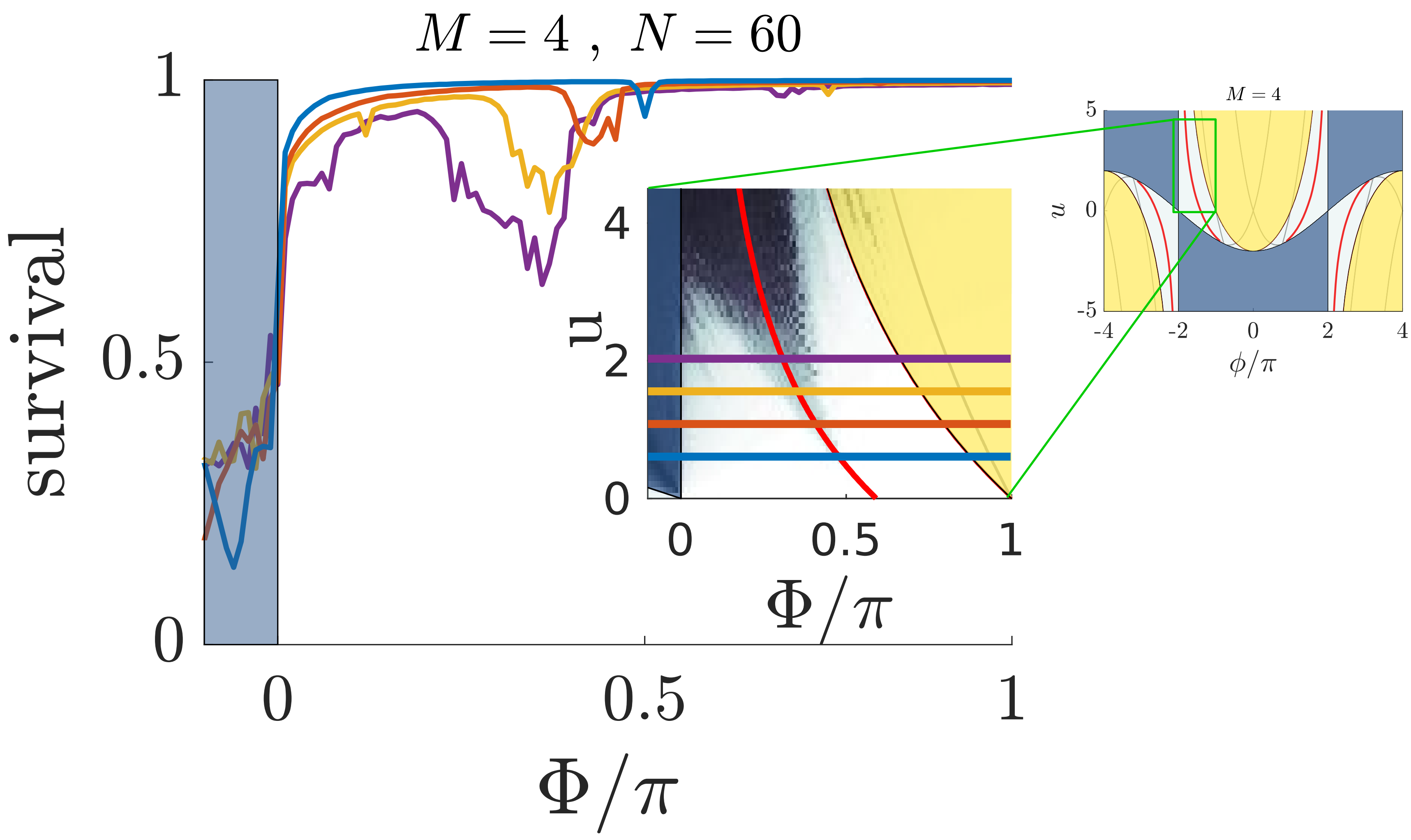

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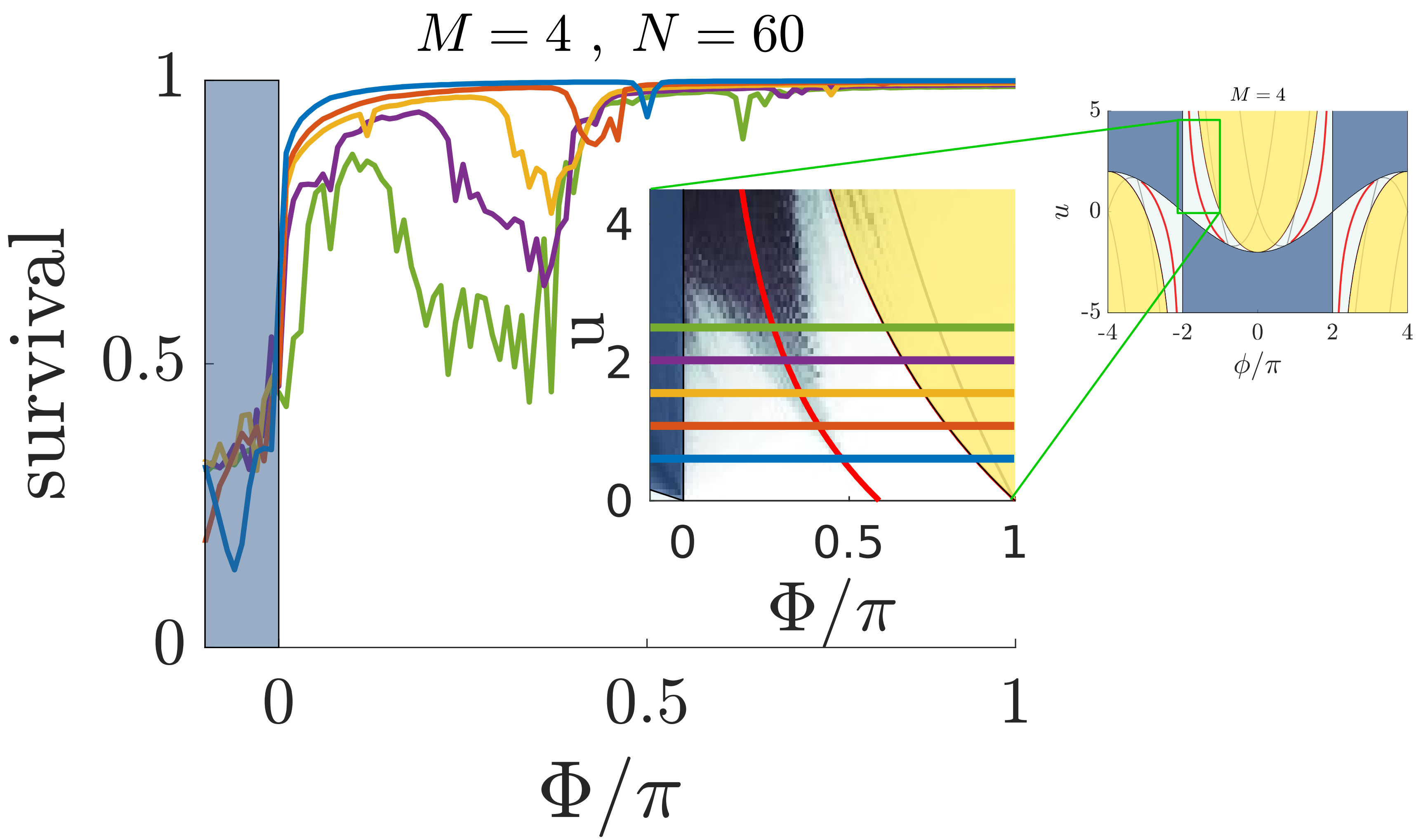

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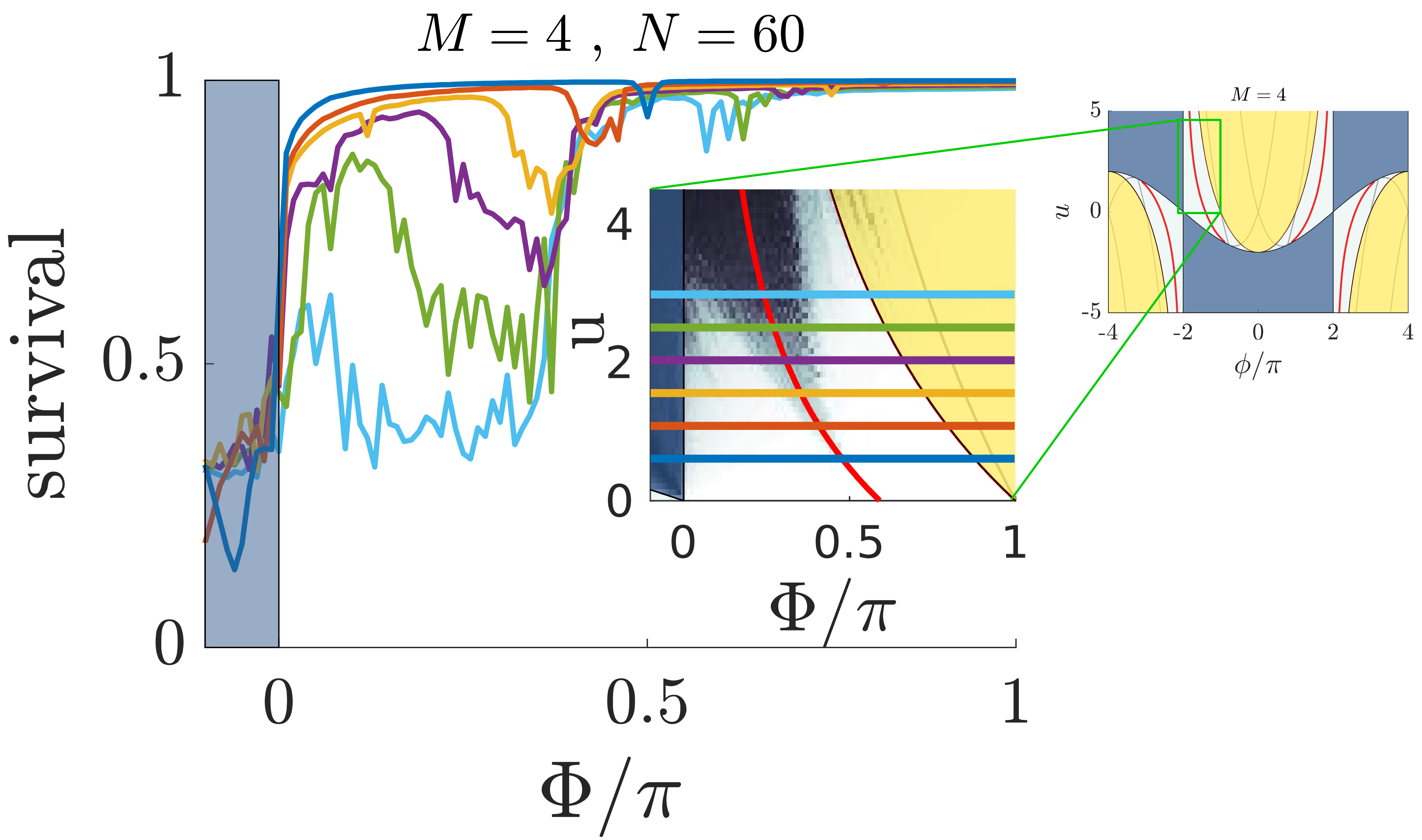

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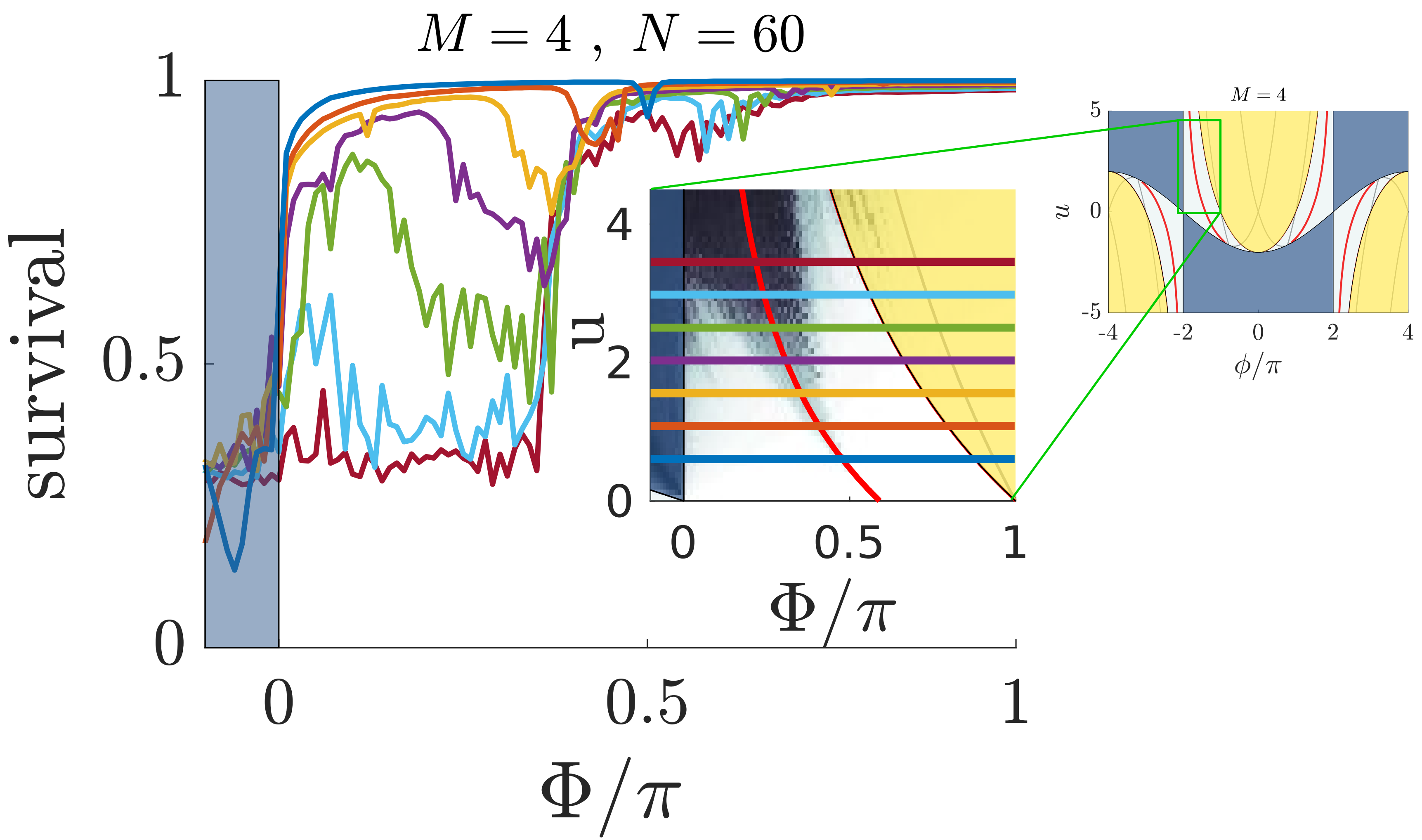

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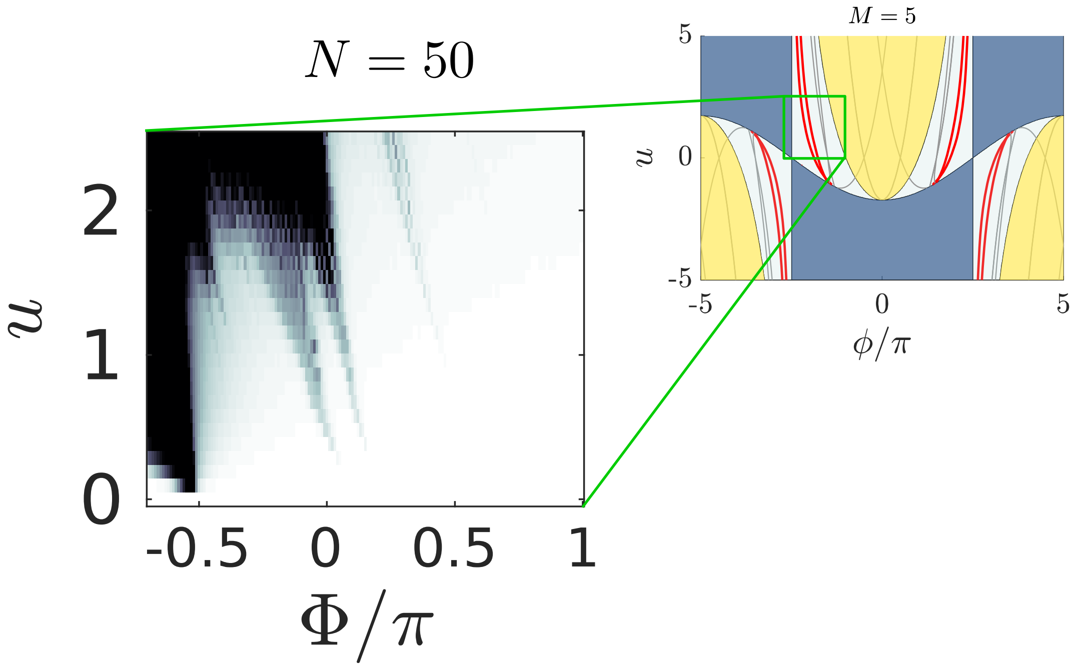

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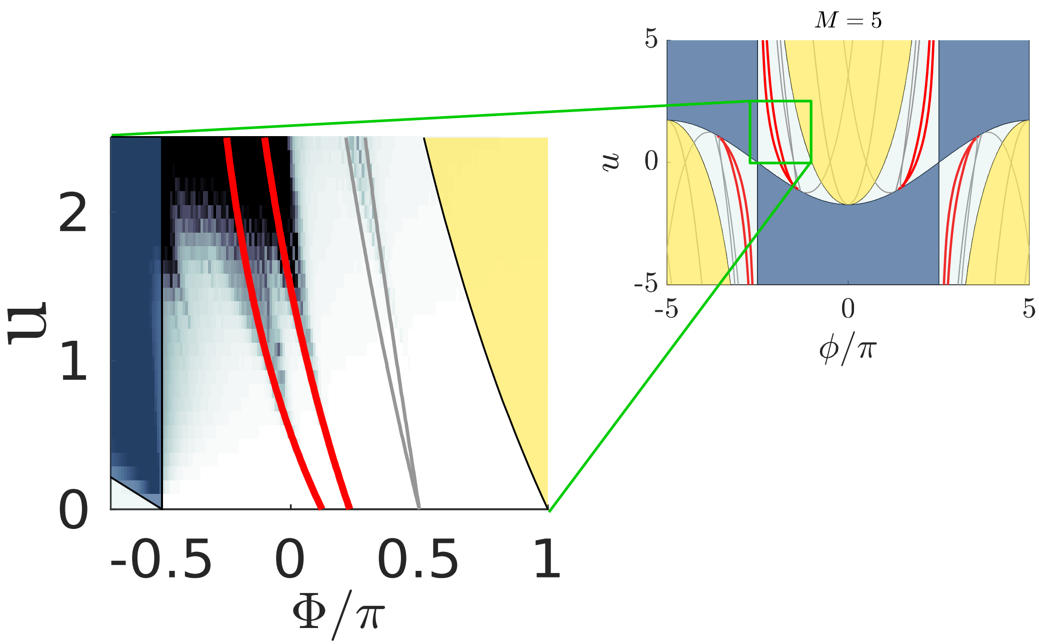

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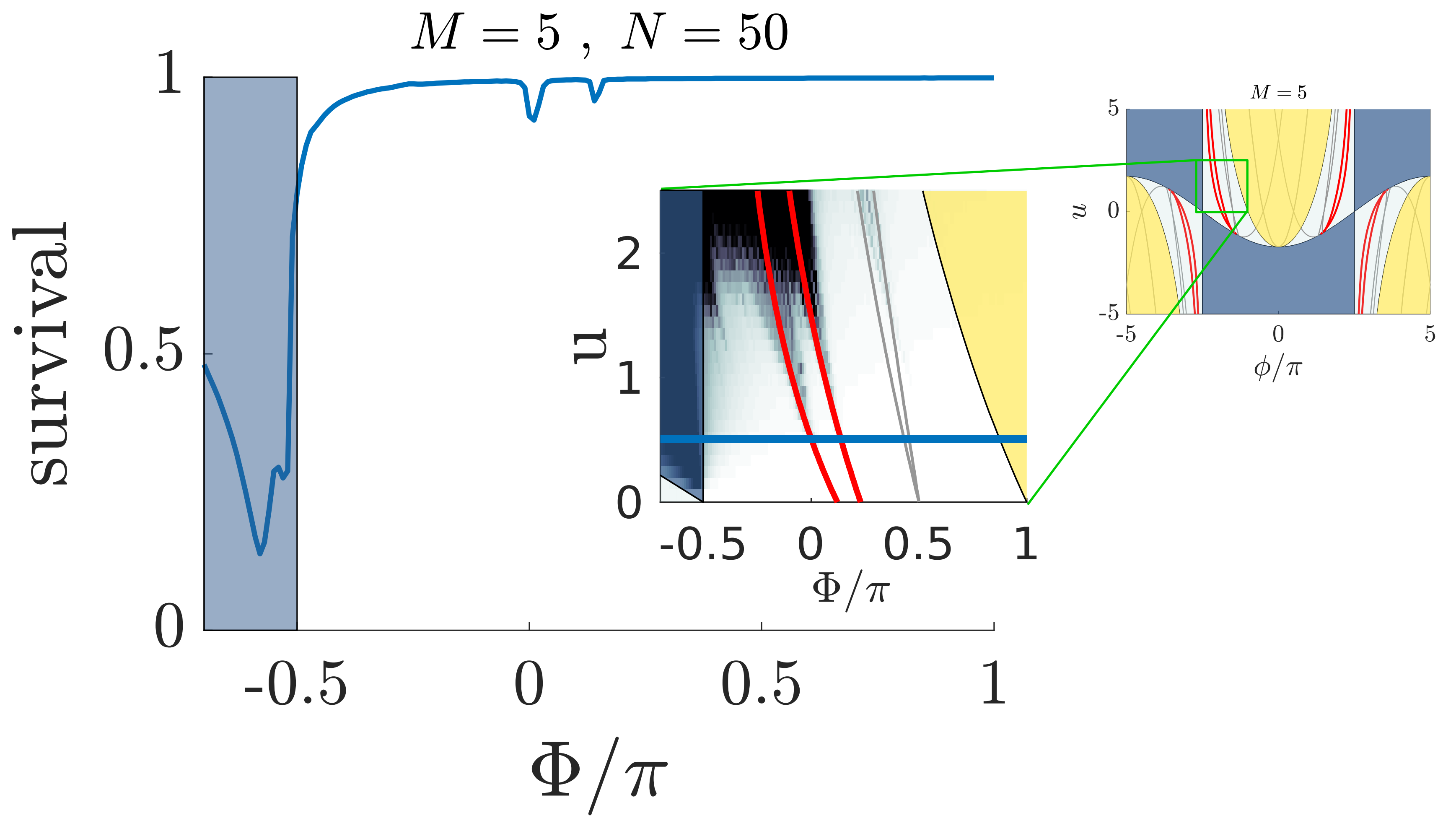
We test numerically whether a prepared flow-state $|m = 1\rangle$ is metastable or not:



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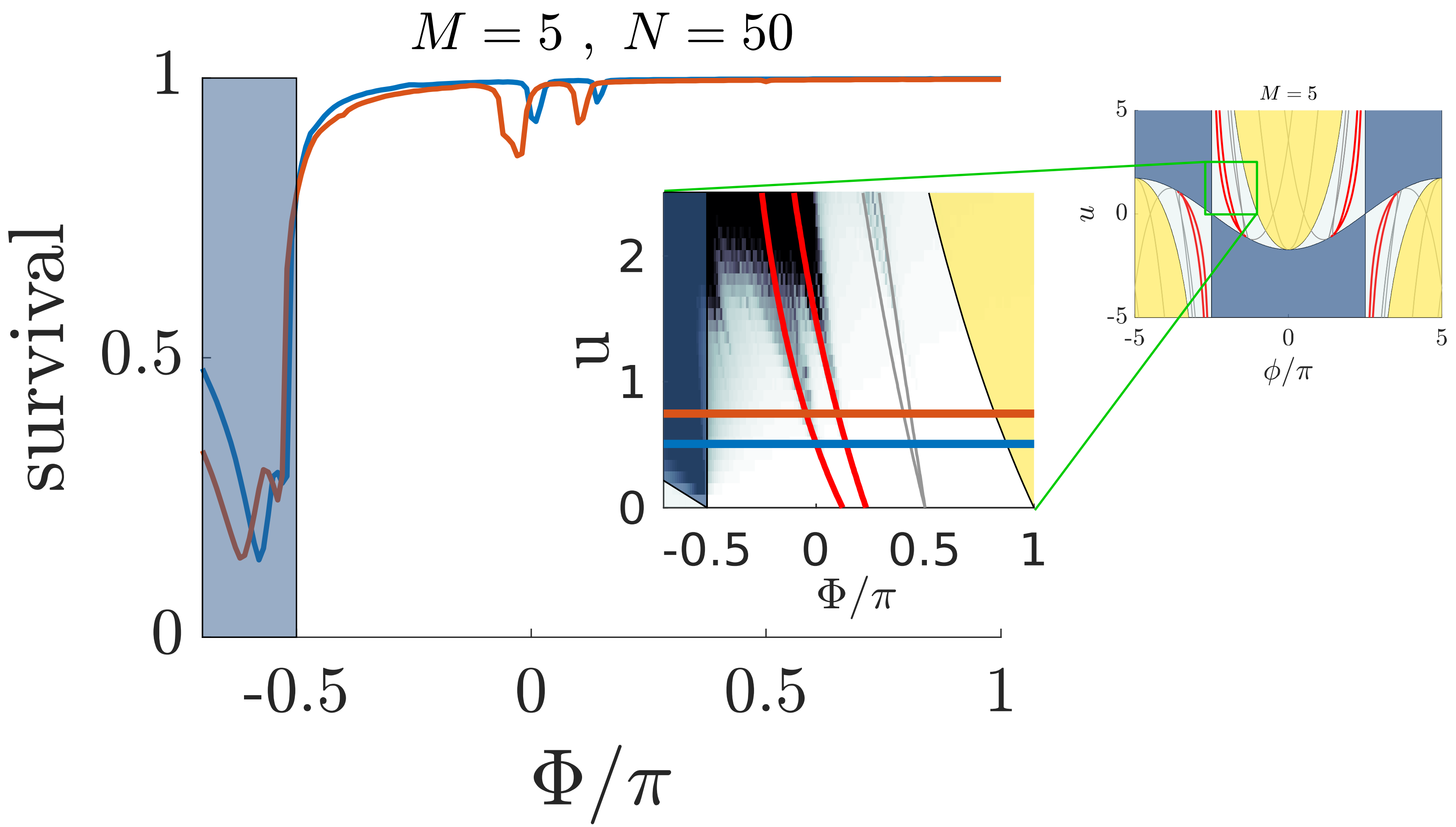

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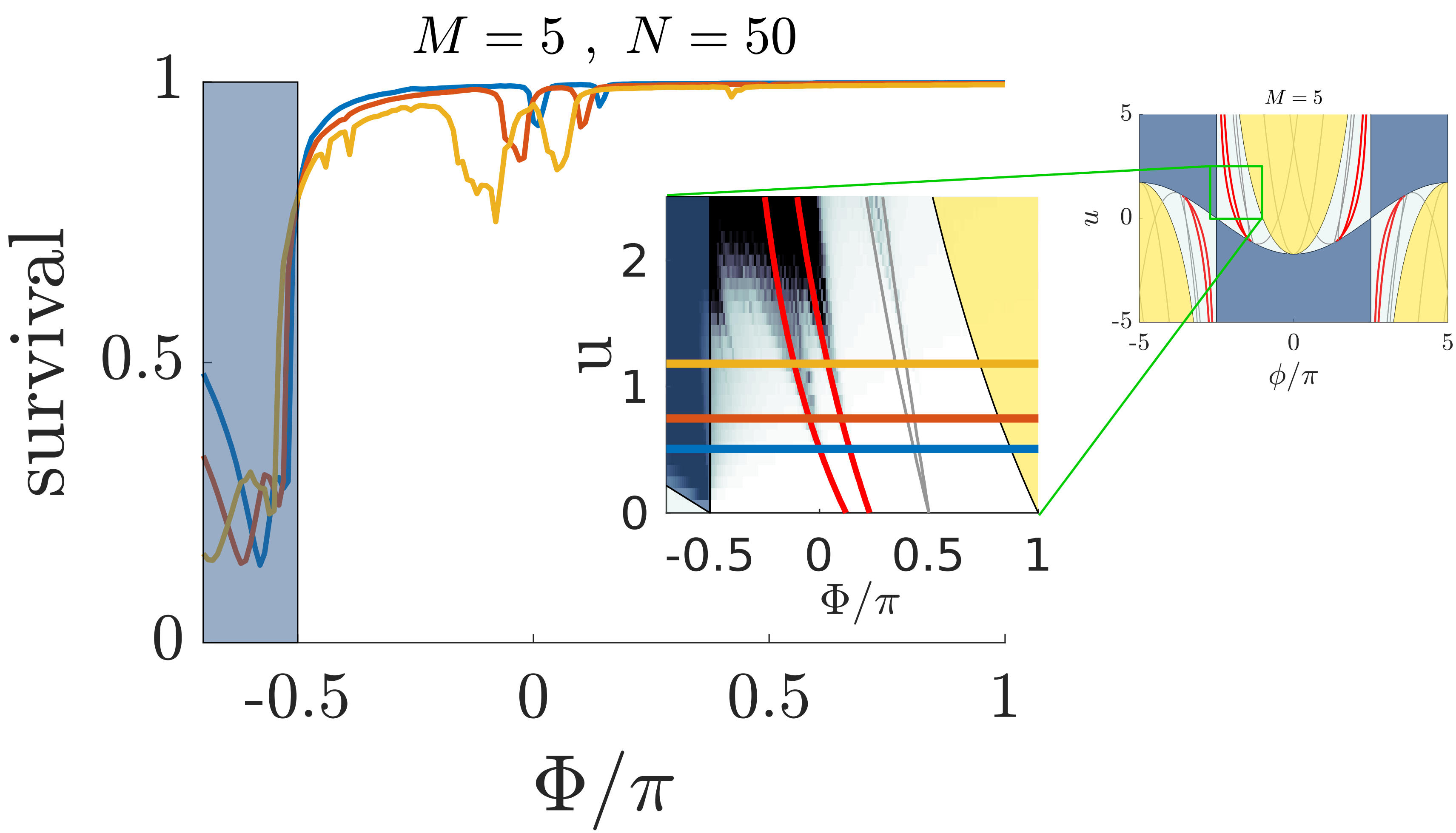

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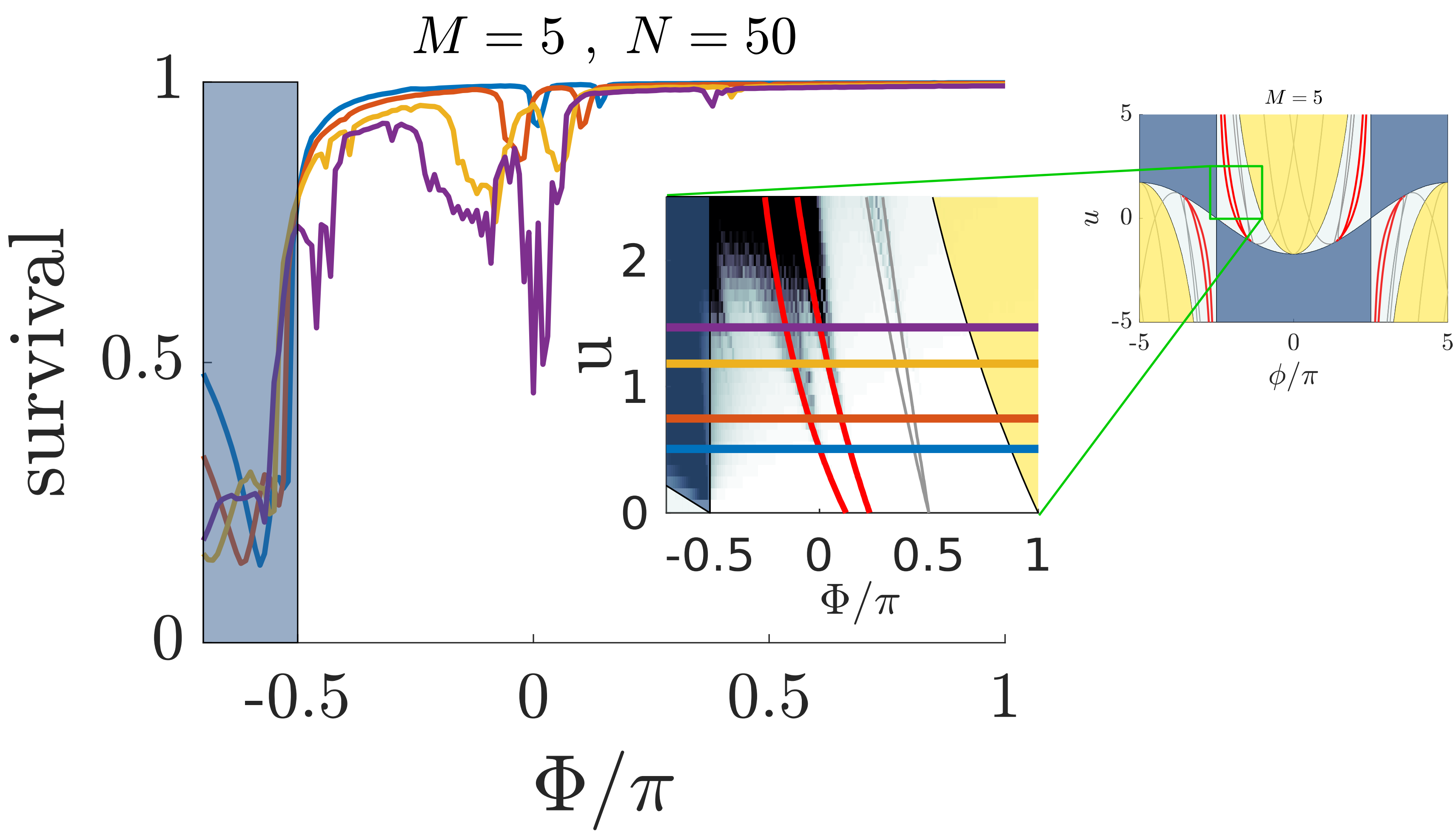

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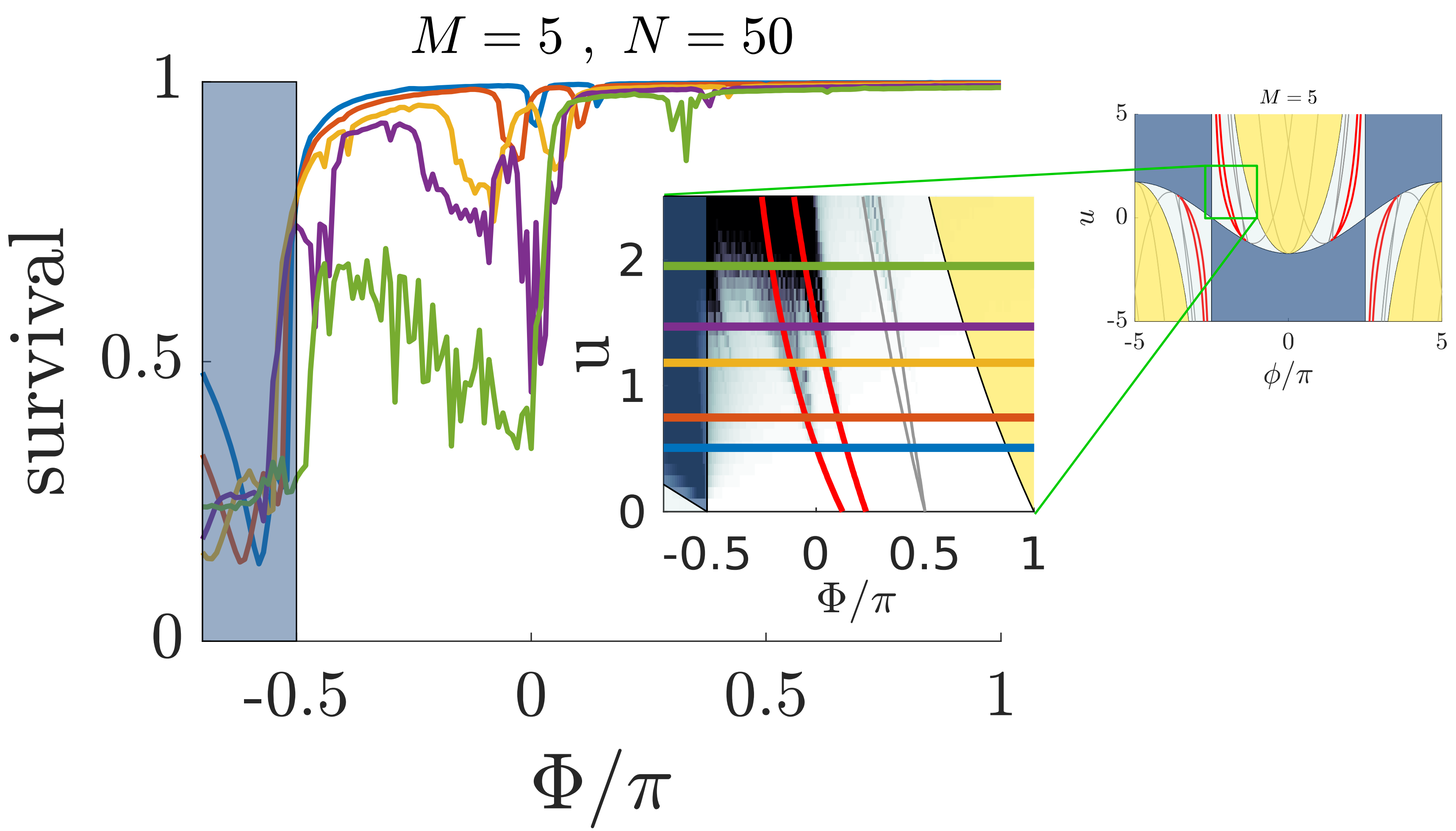

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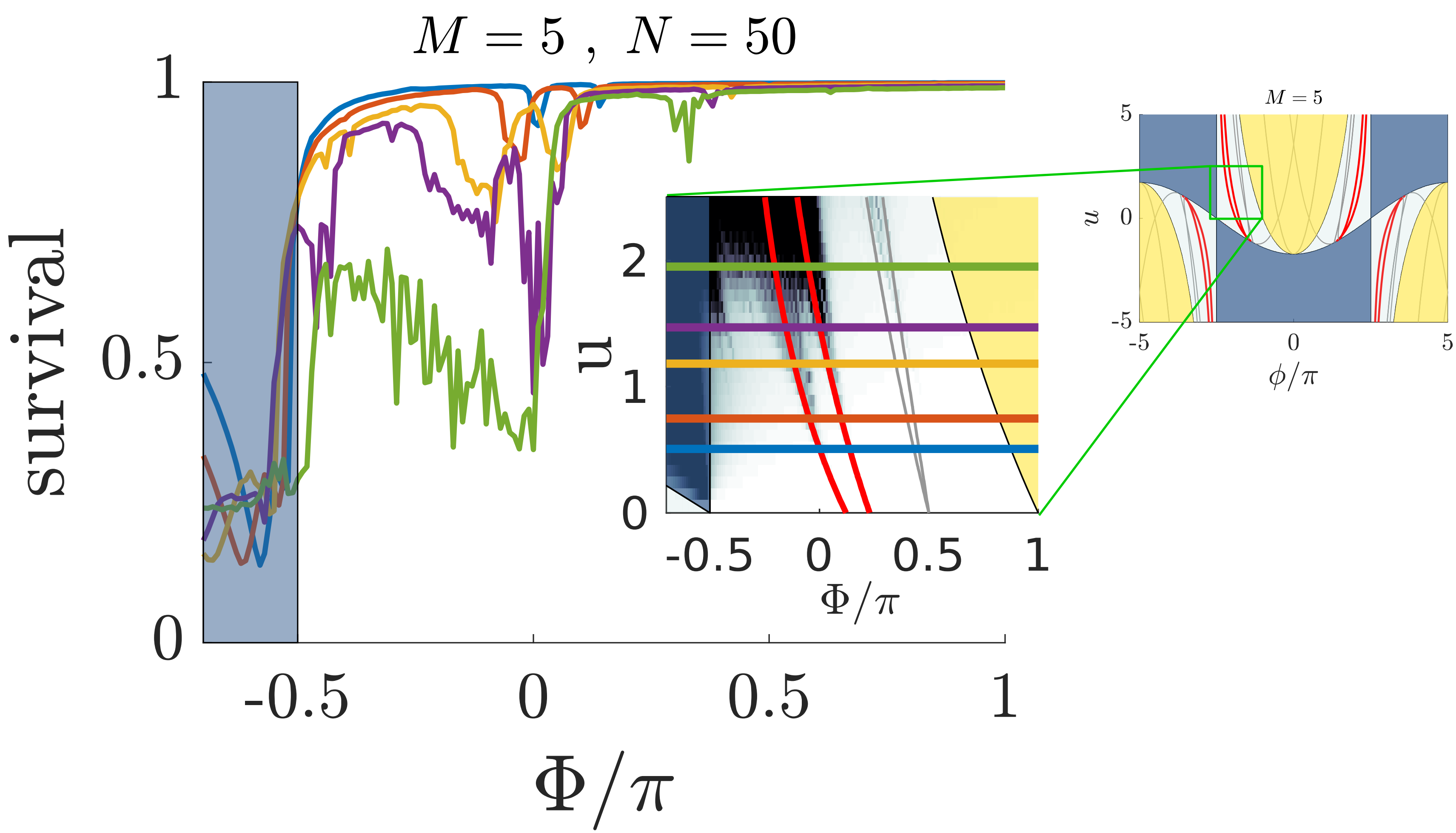


Remarks:

- The “A” terms (red resonances) dominate. The “B” terms (gray resonances) barely affect.
- How will the results change if we have more (say, N$\sim10^4$) particles?

The secular approximation

Near a "1:2" resonance $2\omega_q + \omega_{-2q} = 0$, we keep in the BH only the modes c_q, c_{-2q} coupled by the resonance. In action-angle variables $c_q \to \sqrt{\tilde{n}_q} e^{\varphi_q}$ we obtain:

$$H_q = \omega J + \nu I + \mu I \sqrt{(J/2) + I}\ \cos(\varphi)$$

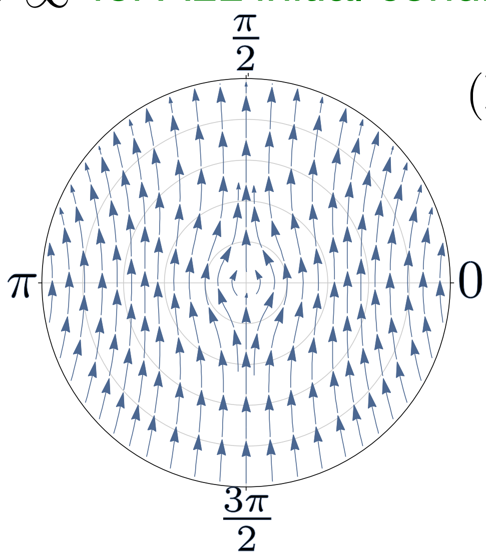
where $I = \tilde{n}_q/(2N)$ conjugate to φ $\qquad \mu = 4(NU/M)A$

and $J = (2\tilde{n}_{-2q} - \tilde{n}_q)/N$ = const $\qquad \nu \equiv 2\omega_q + \omega_{-2q} = \text{detuning}$

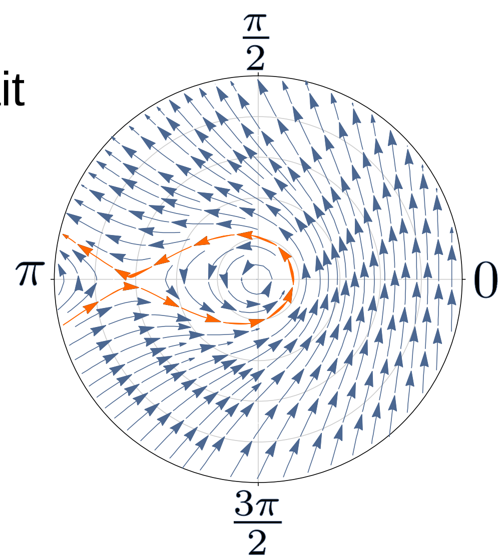
Zero quasi particle occupations $\tilde{n}_q = \tilde{n}_{-2q} = 0$ $\longleftrightarrow$ Linearly stable fixed point at I=J=0

For $\nu=0$ we obtain the so called Cherry Hamiltonian (1928). The action $I \to \infty$ for ALL initial conditions.

For $\nu \neq 0$ a stability island exists.



(I,φ) phase portrait

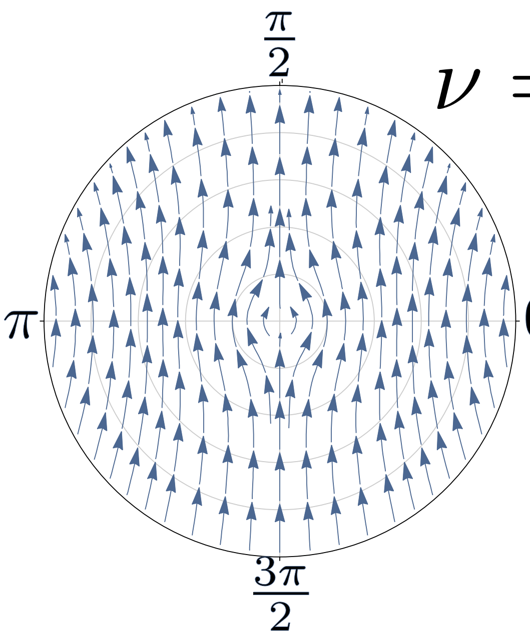


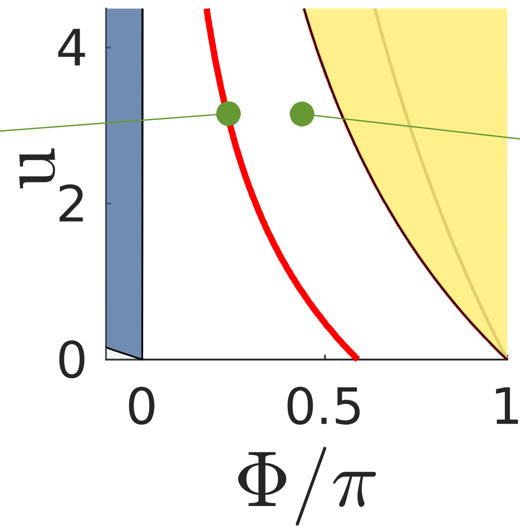
Note: in contrast, the Beliaev and Landau terms do not generate an escape route

Phase Space structure Near a “1:2” resonance

We define $\nu \equiv 2\omega_q + \omega_{-2q}$ as the detuning

$\nu = 0$





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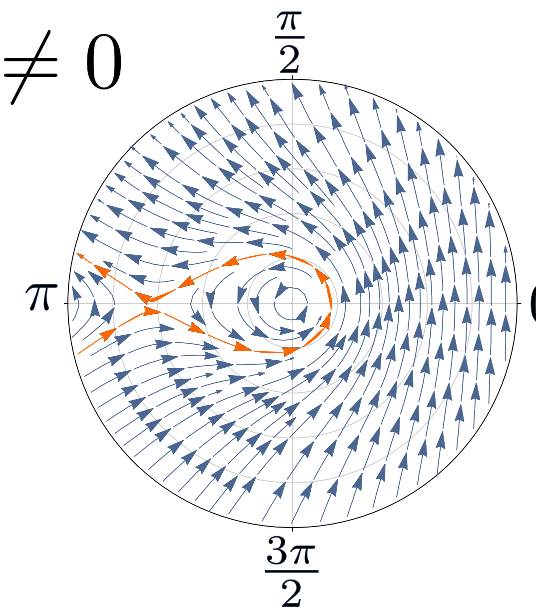


ALL trajectories $\to \infty$

A stability island exists

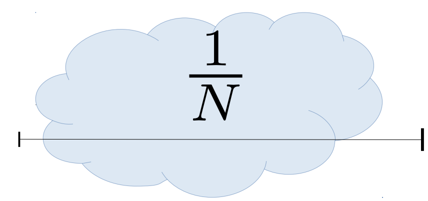
The radial coordinate represents the quasiparticle occupation $\tilde{n}_q$

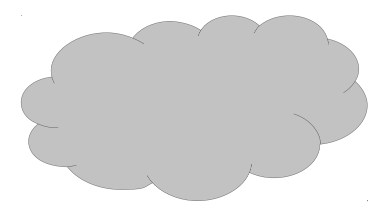
(In action angle variables $\mathrm{c}_q \to \sqrt{\tilde{n}_q}\mathrm{e}^{\varphi_q}$)

Semiclassical theory

The flow state is represented in phase-space by a Gaussian-like ”cloud” of uncertainty width $\frac{1}{N}$

$$|m\rangle \;=\; \left(b_m^\dagger\right)^N |0\rangle \longrightarrow$$





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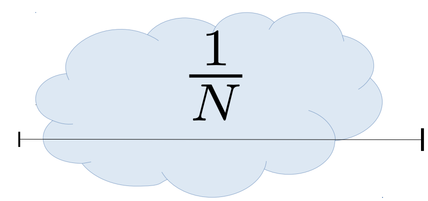

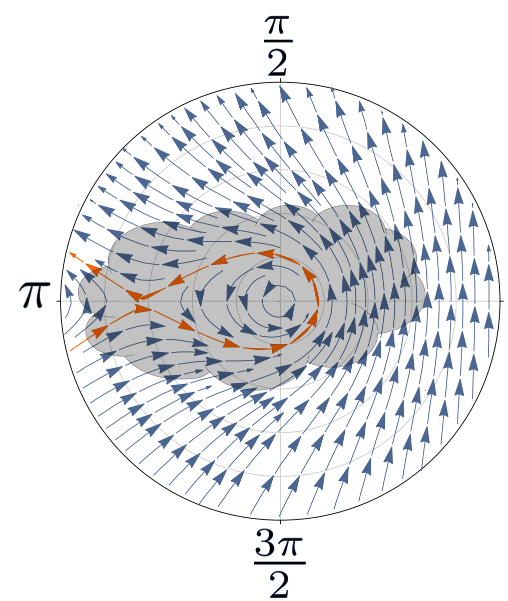

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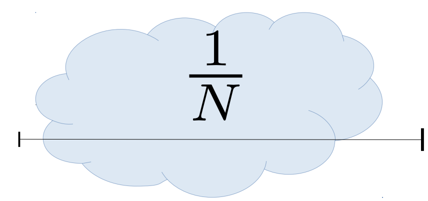
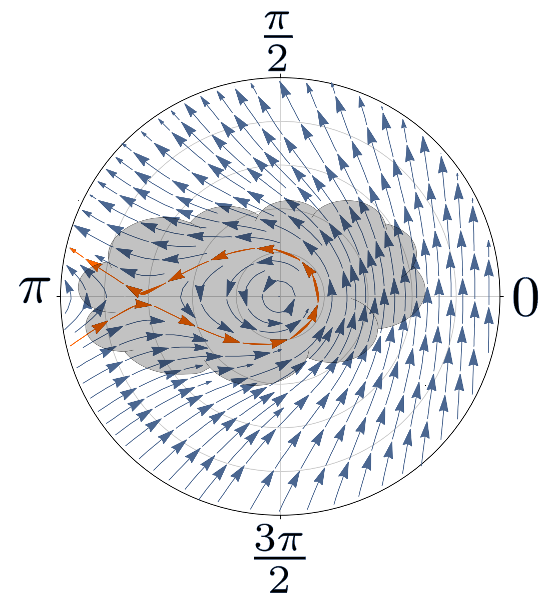


By comparing the size of the stability island and the cloud, we get the width of the resonance region:

$$\left|\nu\right| \;<\; A\left(\frac{1}{N}\right)^{1/2}\frac{u}{M}K$$



$$\nu \neq 0$$

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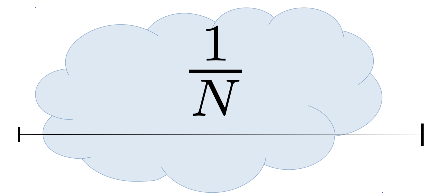

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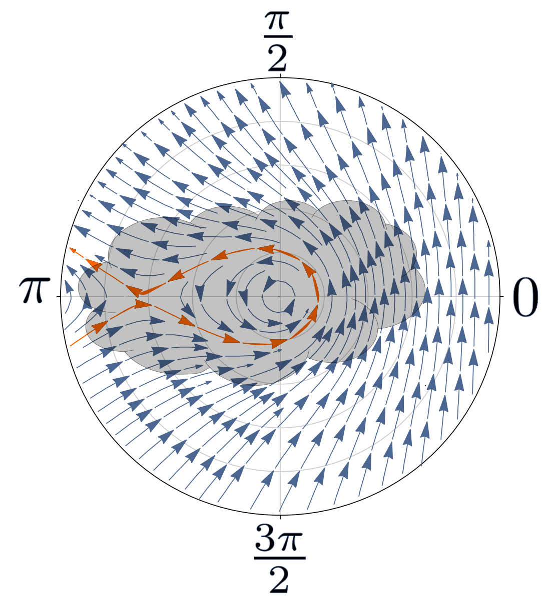
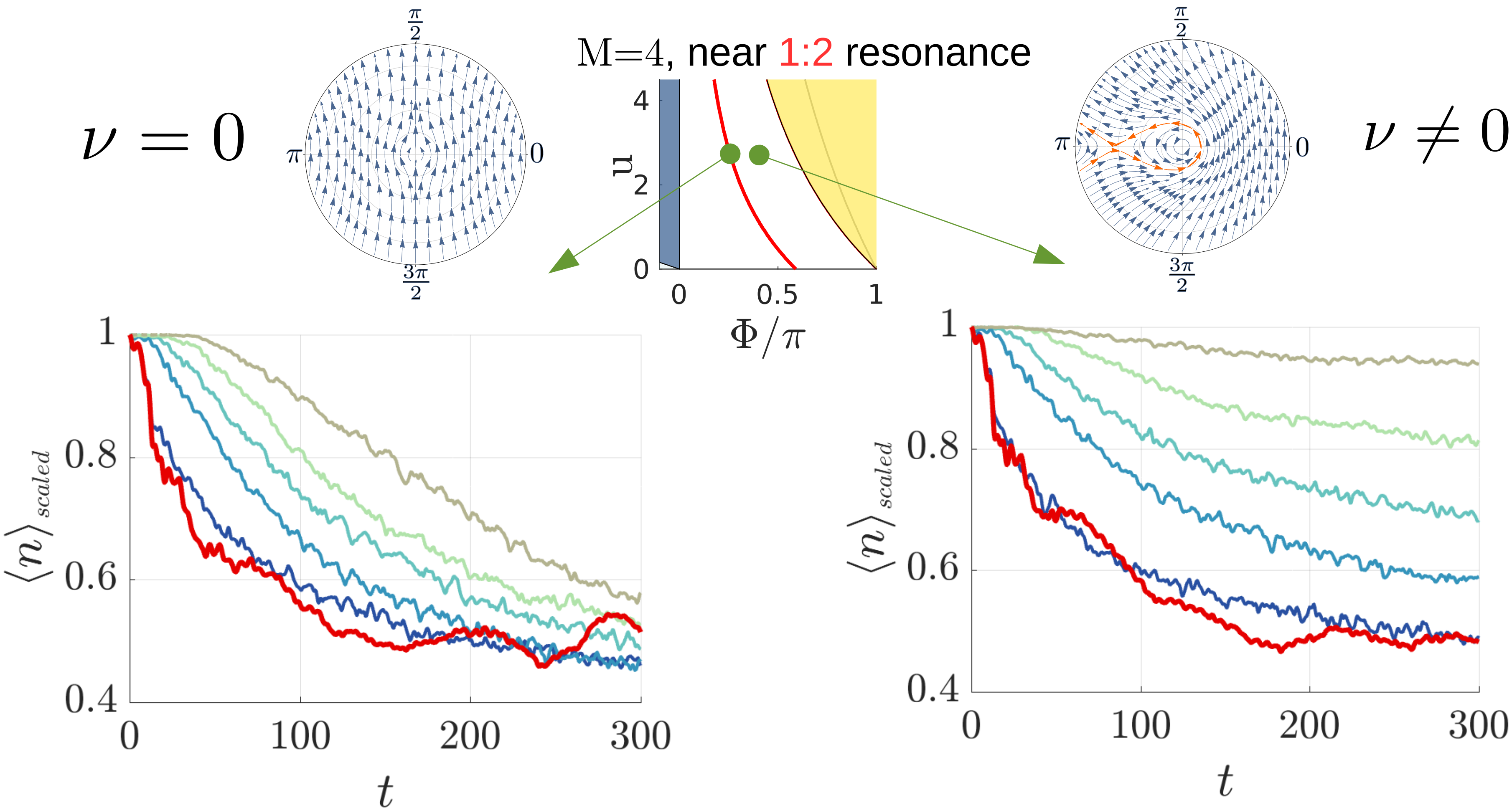


$\nu \neq 0$

The flow state is stable if N is large enough!

The decay of the flow-state

Semiclassical simulation: We launch this cloud of trajectories in phase space and calculate the cloud-averaged $\langle n \rangle$ (in this example $\mathrm{m}=1$ and $n = b_1^\dagger b_1$)



Quantum N=120

Semiclassical N=120,500,1000,2000,4000 (blue to gray)

Hyperbolic escape

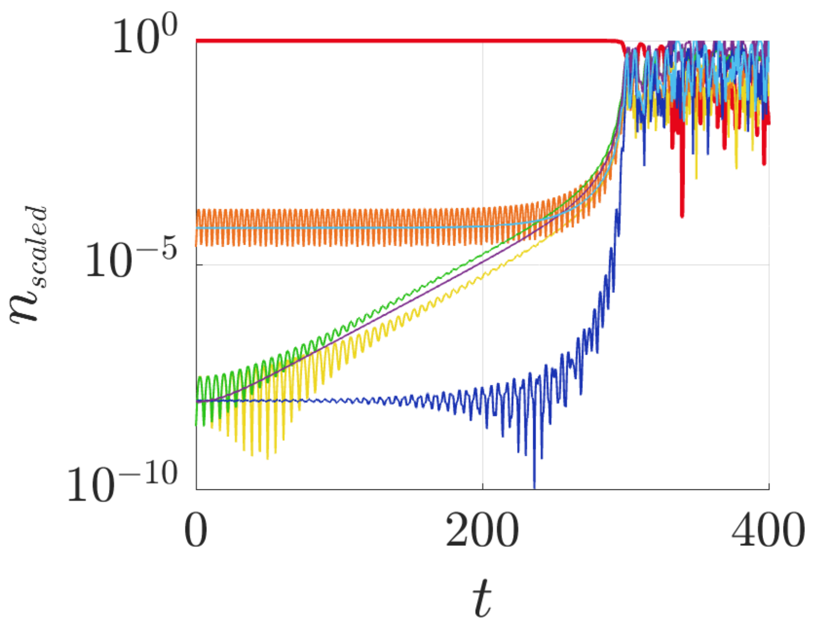
Typically we have either exponential, or parabolic time dependence of the $\tilde{n}_q$, followed by hyperbolic escape:

$$\tilde{n}_q \quad \propto \quad \frac{1}{(t_e - t)^2} \quad \text{for } t < t_e$$

After that transition to chaos. Complete decay as in the linear unstable regime.

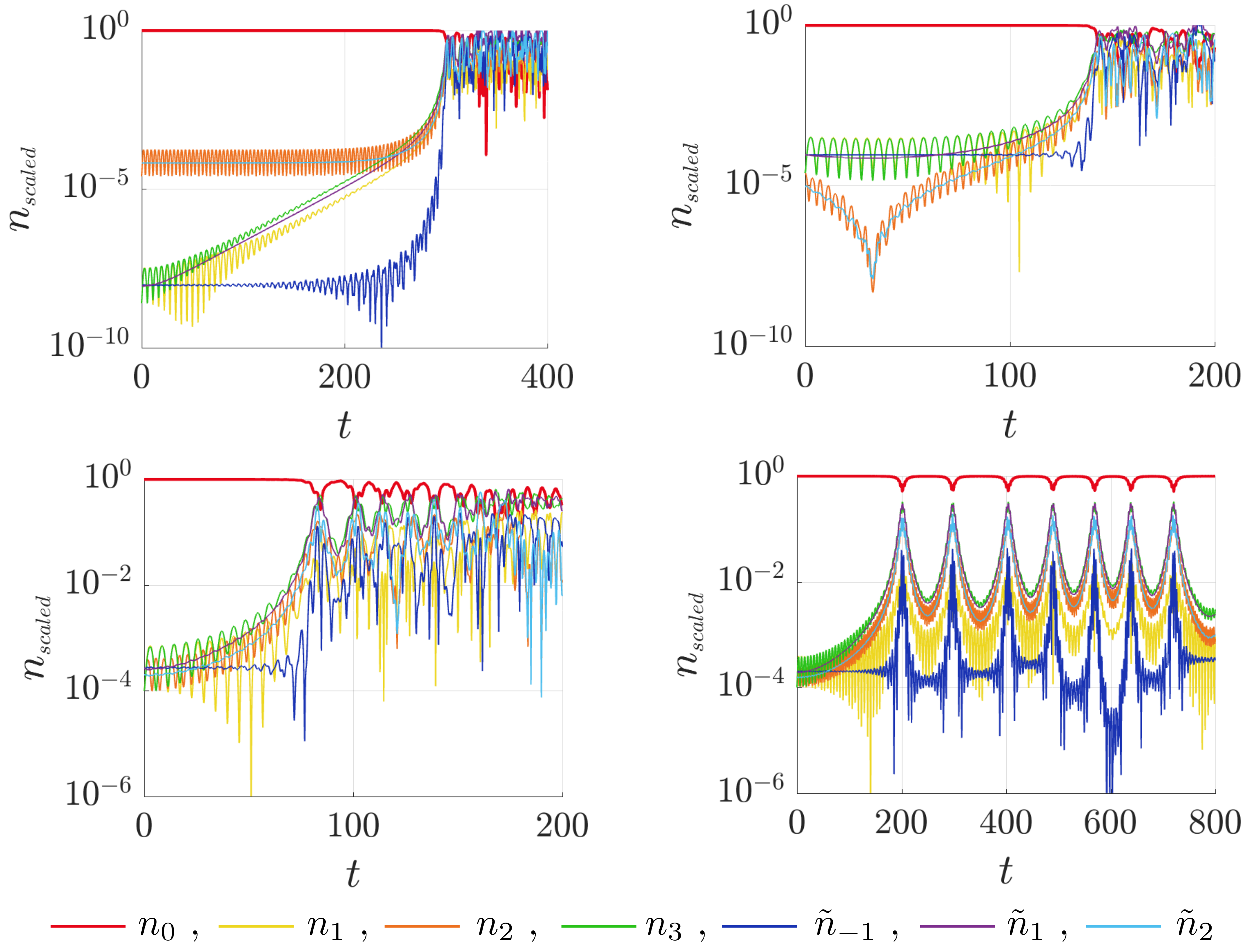
For small u the decay process is suppressed:
Re-injection scenario.
Dynamical localization.

For ν=0 and large u



n_0 (red) flow-state orbital
n_k other momentum orbital
$\tilde{n}_q$ quasi-particle occupations

Hyperbolic escape - the possible scenarios

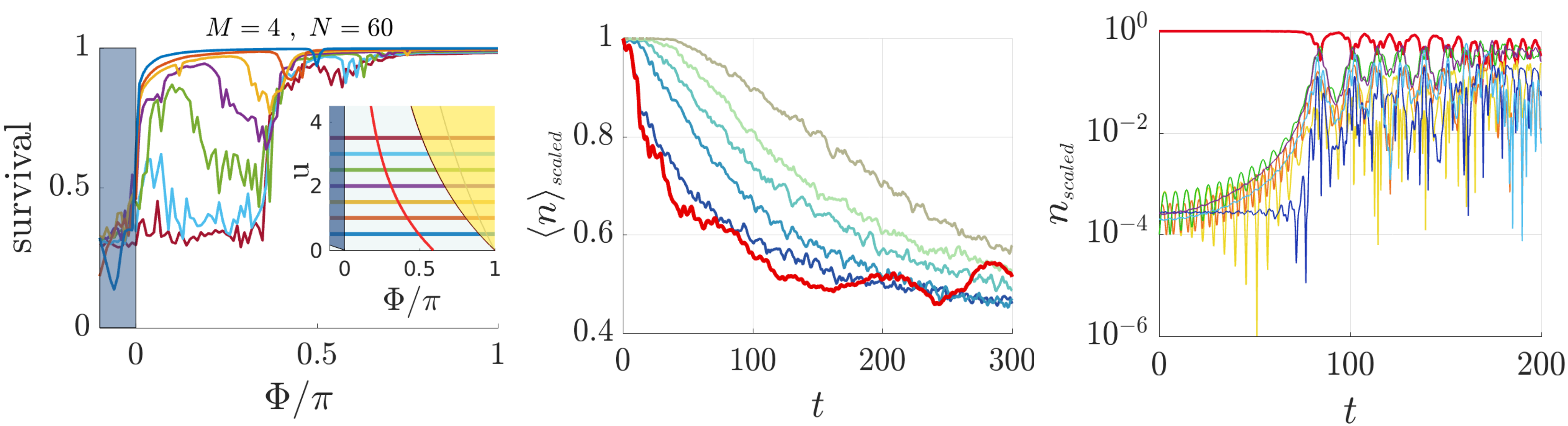


Concluding Remarks

- We have presented a semiclassical theory for the metastability regime-diagram of flow-states in BHH superfluid circuits, taking non-linear resonances into account.

- Contrary to the expectation these resonances do not originate from the familiar Beliaev and Landau damping terms.

- In a broader perspective we would like to demonstrate that tools of semiclassics are extremely advantageous in an arena that is largely dominated by field-theoretical many-body methods.



- G Arwas, D Cohen , arXiv:1612.00251, (2016)