Parametric evolution of eigenstates: Beyond perturbation theory and semiclassics

J. A. Méndez-Bermúdez,[1,2] Tsampikos Kottos,[1,3] and Doron Cohen[2]
[1]*Max-Planck-Institute for Dynamics and Self-Organnization, 37073 Göttingen, Germany*
[2]*Department of Physics, Ben-Gurion University, Beer-Sheva 84105, Israel*
[3]*Department of Physics, Wesleyan University, Middletown, CT 06459–0155, USA*
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Considering a quantized chaotic system, we analyze the evolution of its eigenstates as a result of varying a control parameter. As the induced perturbation becomes larger, there is a crossover from a perturbative to a nonperturbative regime, which is reflected in the structural changes of the local density of states. The *full* scenario is explored for a physical system: an Aharonov-Bohm cylindrical billiard. As we vary the magnetic flux, we discover an intermediate twilight regime where perturbative and semiclassical features coexist. This is in contrast with the *simple* crossover from a Lorentzian to a semicircle line shape which is found in random-matrix models.

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The analysis of the evolution of eigenvalues and of the structural changes that the corresponding eigenstates of a chaotic system exhibit as one varies a parameter ϕ of the Hamiltonian $\mathcal{H}(\phi)$ has sparked a great deal of research activity for many years. Physically the change of ϕ may represent the effect of some externally controlled field (like electric field, magnetic flux, gate voltage) or a change of an effective interaction (as in molecular dynamics). Thus, these studies are relevant for diverse areas of physics ranging from nuclear [1,2] and atomic physics [3,4] to quantum chaos [5–8] and mesoscopics [9,10].

Up to now the majority of this research activity was focused on the study of eigenvalues, where a good understanding has been achieved, while much less is known about eigenstates. The pioneering work in this field has been done by Wigner [2], who studied the parametric evolution of eigenstates of a simplified random-matrix theory (RMT) model of the type $\mathcal{H}=\boldsymbol{E}+\phi\boldsymbol{B}$. The elements of the diagonal matrix $\boldsymbol{E}$ are the ordered energies $\{E_n\}$, with mean level spacing Δ, while $\boldsymbol{B}$ is a banded *random* matrix. Wigner found that as the parameter ϕ increases the eigenstates undergo a transition from a perturbative *Lorentzian-type line shape* to a nonperturbative *semicircle line shape*.

For many years the study of parametric evolution for *canonically quantized systems* was restricted to the exploration of the crossover from integrability to chaos [7,8]. Only later [6] was it realized that a theory is lacking for systems that are chaotic to begin with. Inspired by Wigner theory, the natural prediction was that the local density of states (LDOS) should exhibit a crossover from a regime where a perturbative treatment is applicable to a regime where semiclassical approximation is valid. However, despite a considerable amount of numerical efforts [6], there was no clear-cut demonstration of this crossover, neither has a theory been developed describing how the transition from the perturbative to the nonperturbative regime takes place.

It is the purpose of this Brief Report to present a complete scenario of parametric evolution in case of a physical system that exhibits *hard* chaos. We explore the validity of perturbation theory and semiclassics, and we discover the appearance of an intermediate regime ("twilight zone") where *both perturbative and semiclassical features coexist*. Without loss of generality we consider as an example a billiard system whose classical dynamics is characterized by a correlation time $\tau_{\rm cl}$, which is simply the ballistic time. Associated with $\tau_{\rm cl}$ is the energy scale $\hbar/\tau_{\rm cl}$. Next we look at a *similar* billiard, but with a rough boundary. This roughness is characterized by a length scale which is ℓ times smaller; hence, we can associate with it an energy scale $\delta E_{\rm NU}=(\hbar/\tau_{\rm cl})\ell$. The roughness does not affect the chaoticity: the correlation time $\tau_{\rm cl}$ as well as the whole power spectrum are barely affected. Consequently we explain that $\delta E_{\rm NU}$ is not reflected in the RMT modeling of the Hamiltonian. Still in the LDOS analysis we find that nonuniversal (system-specific) features appear. The appearance of such features is a *generic* phenomenon in quantum chaos studies. It introduces an ingredient in the theory of parametric evolution *which goes beyond RMT*.

The model that we will use in our analysis is a particle confined to an Aharonov-Bohm (AB) cylindrical billiard (see Fig. 1) where one can control the magnetic flux Φ. The cylindrical billiard is constructed by wrapping a two-dimensional (2D) billiard with hard-wall boundaries. The lower boundary at $y=0$ is flat, while the upper boundary $y=L_y+W\xi(x)$ is deformed. The deformation is described by $\xi(x)=\sum_{n=1}^{\ell}A_n\cos(nx)$ where A_n are random numbers in the range $[-1,1]$. The illustration in Fig. 1 assumes a smooth boundary ($\ell=1$). The Hamiltonian of a particle in the cylindrical AB billiard is

$$\mathcal{H}(\phi)=\frac{1}{2\mathsf{m}}\left[\left(p_x-\frac{e}{L_x}\Phi\right)^2+p_y^2\right] \qquad (1)$$

supplemented by L_x periodic boundary conditions in the horizontal direction and hard-wall boundary conditions along the lower and upper boundaries. p_x and p_y are the momenta. Later we shall use the notation $\phi=e\Phi/\hbar$. We consider the chaotic $\mathcal{H}(\phi=0)$ as the unperturbed Hamiltonian.

After conformal transformation [7] the billiard is mapped into a rectangular, with a mass tensor which is space dependent. Then it is possible to compute the matrix representation of the Hamiltonian in the plane-wave basis $|\nu\mu\rangle$ of the rectangular. The result is

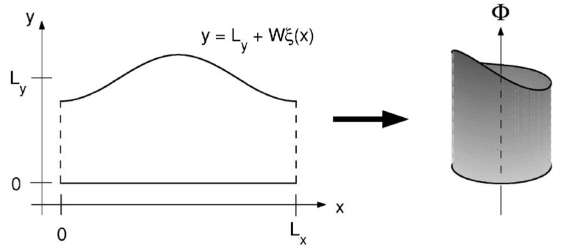

FIG. 1. Left: two-dimensional billiard with $\ell=1$. Right: corresponding Aharonov-Bohm cylinder.

$$\mathcal{H}_{\nu\mu,\nu'\mu'}(\phi) = \frac{\hbar^2}{2\pi \mathsf{m}}\Bigg\{ \pi\left(\nu - \frac{\phi}{2\pi}\right)^2 \delta_{\nu,\nu'}\delta_{\mu,\mu'} + \left[\frac{\mu^2}{8\alpha^2}J^{(0,2)}_{\nu'\nu} + \epsilon^2 J^{(2,2)}_{\nu'\nu}\left(\frac{1}{8}+\frac{\pi^2\mu^2}{6}\right)\right]\delta_{\mu,\mu'} + (-1)^{\mu+\mu'}\mu\mu'\left[\epsilon^2\frac{2(\mu^2+\mu'^2)}{(\mu^2-\mu'^2)^2}J^{(2,2)}_{\nu'\nu} - i\epsilon\frac{\nu+\nu'-\phi/\pi}{\mu^2-\mu'^2}J^{(1,1)}_{\nu'\nu}\right]\Bigg\}, \quad (2)$$

where

$$J^{(l,k)}_{\nu'\nu} = \int_0^{L_x} dx\, e^{i(\nu'-\nu)2\pi x/L_x}\left(\frac{d\xi}{dx}\right)^l \frac{1}{[1+\epsilon\xi(x)]^k}.$$

The classical dimensionless parameters of the model are the aspect ratio $\alpha=L_y/L_x$, the tilt relative amplitude $\epsilon=W/L_y$, and the roughness parameter ℓ. Upon quantization we have $\hbar$, which together with m and E determines the de Broglie wavelength of the particle and, hence, leads to an additional dimensionless parameter $n_E=[L_xL_y/(2\pi\hbar^2)]\mathsf{m}E$. For 2D billiards the mean level spacing Δ is constant, and hence $n_E=E/\Delta \propto 1/\hbar^2$ can be interpreted as either the scaled energy or as the level index. Optionally we define a semiclassical parameter $\hbar_{\rm scaled}=1/\sqrt{n_E}$.

In the numerical study we have taken $\epsilon=0.06$ and $\alpha=1$, for which the classical dynamics is completely chaotic (for any ϕ). We consider either $\ell=1$ for a smooth boundary or $\ell=100$ for a rough boundary. The eigenstates $|n(\phi)\rangle$ of the Hamiltonian $\mathcal{H}(\phi)$ were found numerically for various values of the flux $(0.0006<\phi<60)$. We were interested in the states within an energy window $\delta E\approx 45$ that contains $\delta n_E \sim 200$ levels around the energy $E\approx 400$. Note that the size of the energy window is classically small $(\delta E \ll E)$, but quantum mechanically large $(\delta E \gg \Delta)$.

The object of our interest is the overlaps of the eigenstates $|n(\phi)\rangle$ with a given eigenstate $|m(0)\rangle$ of the unperturbed Hamiltonian:

$$P(n|m) = |\langle n(\phi)|m(0)\rangle|^2 = \int \frac{dx dy dp_x dp_y}{(2\pi\hbar)^2}\rho^{(n)}\rho^{(m)}. \quad (3)$$

The overlaps $P(n|m)$ can be regarded as a distribution with respect to n. Up to some trivial scaling it is essentially the LDOS. The associated dispersion is defined as $\delta E=[\sum P(n|m)(E_n-E_m)^2]^{1/2}$. In practice we plot $P(n|m)$ as a function of $r=n-m$ or as a function of (E_n-E_m) and average over the reference state m. The second equality in Eq. (3) is useful for the semiclassical analysis. It involves the Wigner functions $\rho^{(n)}(x,y,p_x,p_y)$ which are associated with the eigenstates $|n(\phi)\rangle$. The semiclassical approximation is based on the microcanonical approximation $\rho^{(n)}\propto \delta(E_n-\mathcal{H}(x,y,p_x,p_y))$. With this approximation the integral can be calculated analytically, leading to

$$P_{\rm cl}(n|m) = \frac{\Delta}{\pi\sqrt{2(\delta E_{\rm cl})^2-[(E_n-E_m)-\delta E_{\rm cl}^2/(2E_m)]^2}}, \quad (4)$$

where $\delta E_{\rm cl}=(\hbar v_E/L_x)\phi$ with $v_E=(2E/\mathsf{m})^{1/2}$. It is implicit in Eq. (4) that $P_{\rm cl}(n|m)=0$ outside of the allowed range, which is where the expression under the square root is negative: For large $|E_n-E_m|$ there is no intersection of the corresponding energy surfaces and, hence, no classical overlap.

A few words are in order regarding the quantum to classical correspondence (QCC). Whenever $P(n|m)\approx P_{\rm cl}(n|m)$ we call it a "detailed QCC," while $\delta E\approx \delta E_{\rm cl}$ is referred to as a "restricted QCC" [6]. It is remarkable that the (robust) restricted QCC holds even if the (fragile) detailed QCC fails completely. We have verified that also in the present system δE is numerically indistinguishable from $\delta E_{\rm cl}$.

A fixed assumption of this work is that ϕ is classically small. But quantum mechanically it can be either "small" or "large." Quantum mechanically *small* ϕ means that perturbation theory does provide a valid approximation for $P(n|m)$. What is the *border* between the perturbative regime and nonperturbative regime, we discuss later. First we would like to show that the prediction which is based on perturbation theory, to be denoted as $P_{\rm prt}(n|m)$, is very different from the semiclassical approximation.

In order to write the expression for $P_{\rm prt}(n|m)$ we have first to clarify how to apply perturbation theory in the context of the present model. To this end, we write the perturbed Hamiltonian $\mathcal{H}(\phi)$ in the basis of $\mathcal{H}(\phi=0)$. Since we assume that the perturbation is classically small, it follows that we can linearize the Hamiltonian with respect to ϕ. Consequently the perturbed Hamiltonian is written as $\mathcal{H}=\boldsymbol{E}+\phi\boldsymbol{B}$, where $\boldsymbol{E}=\text{diag}\{E_n\}$ is a diagonal matrix, while $\boldsymbol{B}=\{-(\hbar/e)\mathcal{I}_{nm}\}$. The current operator is conventionally defined as

$$\mathcal{I} \equiv -\partial\mathcal{H}/\partial\Phi = [e/(\mathsf{m}L_x)]p_x.$$

Its matrix elements can be found using a semiclassical recipe [11]: namely, $|\mathcal{I}_{nm}|^2\approx(\Delta/(2\pi\hbar))\tilde{C}((E_n-E_m)/\hbar)$, where $\tilde{C}(\omega)$ is the Fourier transform of the current-current correlation function $C(\tau)$. Conventional condensed-matter calculations are done for *disordered* rings where one assumes $C(\tau)$ to be exponential, with time constant $\tau_{\rm cl}$ which is essentially the ballistic time. Hence $\tilde{C}(\omega)\propto 1/[\omega^2+(1/\tau_{\rm cl})^2]$ is a Lorentzian. This Lorentzian approximation works well also for the chaotic ring that we consider. In fact we can do better by exploiting a relation between $\mathcal{I}(t)$ and the force $\mathcal{F}(t)=-\dot{p}_x$, leading to $\tilde{C}(\omega)=(e/(\mathsf{m}L_x))^2\tilde{C}_{\mathcal{F}}(\omega)/\omega^2$. The force $\mathcal{F}(t)$ is a train of spikes corresponding to collisions with the boundaries. Assuming that the collisions are uncorrelated on short times we have $\tilde{C}_{\mathcal{F}}(\omega)\approx(8/3\pi)\mathsf{m}^2v_E^3/L_y$, for $\omega\gg(1/\tau_{\rm cl})$. This is known as the "white noise" approximation [12]. We

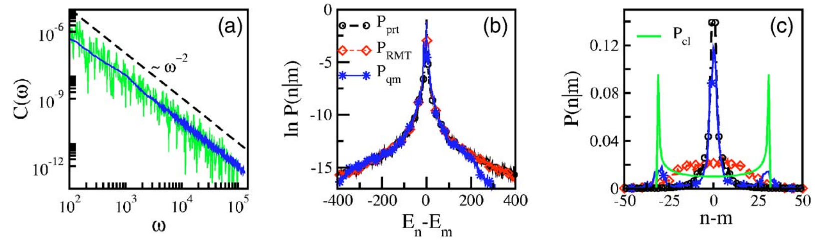

FIG. 2. (Color online) (a) The classical power spectrum $C(\omega)$ plotted (as green/gray curves) together with the quantum mechanical band profile (the dark curve) $(2\pi e^2/\Delta\hbar)|\mathbf{B}_{nm}|^2$ for $\ell=1$ and $\hbar_{\text{scaled}}\approx 0.018$. (b) The LDOS kernel $P(n|m)$ in the perturbative regime for a billiard with $\ell=100$ and perturbation $\phi=2.7$. (c) Same as (b) but zoomed-in normal scale. The width of the nonperturbative component is $\Gamma/\Delta=36$. Note that in this regime the variance $\Delta E/\Delta\approx 58$ is still dominated by the (perturbative) tails. For comparison we display the calculated P_{prt}, P_{cl}, and P_{RMT}.

have checked the validity of this approximation in the present context by a direct numerical evaluation of $\tilde{C}(\omega)$ and also verified the validity of the above recipe by direct evaluation of the matrix elements of $\boldsymbol{B}$ via Eq. (2); see Fig. 2(a). The classical $\tilde{C}(\omega)$ was numerically evaluated by Fourier analysis of the fluctuating current $\mathcal{I}(t)$ for a very long ergodic trajectory that covers densely the whole energy surface $\mathcal{H}(0)=E$.

Perturbation theory to infinite order with the Hamiltonian $\mathcal{H}=\boldsymbol{E}+\phi\boldsymbol{B}$ leads to a Lorentzian-type approximation for the LDOS [2] [see also Sec. 18 of [6(c)]]. It is an approximation because all the higher orders are treated within a Markovian-like approach (by iterating the first-order result) and convergence of the expansion is preassumed, leading to $P_{\text{prt}}(n|m)=\phi^2|\boldsymbol{B}_{nm}|^2/[\Gamma^2+(E_n-E_m)^2]$. In practice the parameter $\Gamma(\phi)$ can be determined (for a given ϕ) by imposing the requirement of having $P_{\text{prt}}(r)$ normalized to unity. Substituting the expression for the matrix elements we get

$$P_{\text{prt}}(n|m)=\frac{8\hbar^2(\hbar v_E)^3/(3\pi\mathsf{m}L_y^2L_x^3)}{(E_n-E_m)^2+(\hbar/\tau_{\text{cl}})^2}\frac{\phi^2}{(E_n-E_m)^2+\Gamma^2}. \quad (5)$$

By comparing the exact $P(r)$ to the approximation, Eq. (5), we can determine the regime $\phi<\phi_{\text{prt}}$ for which the approximation $P(r)\approx P_{\text{prt}}(r)$ makes sense. The practical procedure to determine ϕ_{prt} is to plot δE_{prt} and to see where it departs from δE_{cl}. The latter is a linear function of ϕ while the former becomes sublinear for large enough ϕ (and even would exhibit saturation if we had a finite bandwidth). In case of Eq. (5) this reasoning leads to a crossover when $\delta E_{\text{cl}}(\phi)\sim\hbar/\tau_{\text{cl}}$. Hence we get that the border of the perturbative regime[1] is $\phi_{\text{prt}}=L_x/(v_E\tau_{\text{cl}})\sim 1$.

What happens to $P(r)$ in practice? If we take the Wigner RMT model as an inspiration, we expect to have at $\phi\sim\phi_{\text{prt}}$ a simple crossover from a P_{prt} line shape to a P_{cl} line shape. The latter is regarded as the semiclassical analog of the (artificial) semicircle line shape. Indeed for the smooth billiard ($\ell=1$) we have verified that this naive expectation is realized [13]. But for the rough billiard ($\ell=100$) we witness a more complicated scenario. In Figs. 2(b) and 2(c) we show the LDOS for $\phi<\phi_{\text{prt}}$, where it (still) agrees quite well with P_{prt}. In Fig. 3 we show the LDOS for $\phi>\phi_{\text{prt}}$, where we would naively expect agreement with P_{cl}. Rather we witness a three-peak structure, where the $r\sim 0$ peak is of perturbative nature, while the others are the fingerprint of semiclassics. For sake of comparison we show the corresponding results for a smooth billiard ($\ell=1$) and otherwise the same parameters. There we have detailed the QCC as is naively expected. The coexistence of perturbative and semiclassical features persists within an intermediate regime of ϕ values, to which we refer as the "twilight zone."

Before we adopt a phase-space picture in order to explain the above observations, we would like to verify that indeed random-matrix modeling does not lead to a similar effect: After all the standard Wigner model, which gives rise to a simple crossover from a Lorentzian to a semicircle line shape, assumes a simple banded matrix, which is *not* the case in our model. As argued above the matrix elements of $\boldsymbol{B}$ decay as $1/|n-m|^2$ from the diagonal. This implies that $P_{\text{prt}}(r)$ is in fact not a Lorentzian and also may imply that the crossover to the nonperturbative regime is more complicated. In order to resolve this subtlety we have taken a randomized version of the Hamiltonian $\mathcal{H}=\boldsymbol{E}+\phi\boldsymbol{B}$. Namely, we have randomized the signs of the off-diagonal elements of the $\boldsymbol{B}$ matrix. Thus we get an RMT model with the same band profile as in the physical model. This means that P_{prt} is the same for both models (the physical and the randomized), but still they can differ in the nonperturbative regime. Indeed, looking at the LDOS of the randomized model we observe that the semiclassical features are absent: $P_{\text{RMT}}(r)$ unlike $P(r)$ exhibits a simple crossover from perturbative to nonperturbative line shape.

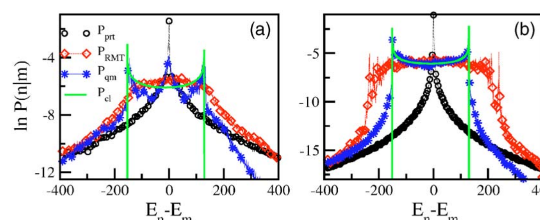

FIG. 3. (Color online) (a) The LDOS kernel $P(n|m)$ for $\phi=31.4$, where $\ell=100$. (b) The same parameters but $\ell=1$. In panel (a) we observe coexistence of perturbative and SC structures while in panel (b) we witness detailed QCC.

[1] Optionally ϕ_{prt} is determined by $\Gamma(\phi)\sim\hbar/\tau_{\text{cl}}$. It should be distinguished from the border of the first-order perturbative regime which is determined by $\Gamma(\phi)\sim\Delta$, leading to $\phi_{\text{FOPT}}\sim\phi_{\text{prt}}/\sqrt{b}$, where $b=(\hbar/\tau_{\text{cl}})/\Delta\gg 1$. In other words, ϕ_{FOPT} is the perturbation which is needed to mix neighboring levels.

In what follows we would like to argue that the structure of $P(r)$, both perturbative and nonperturbative components, can be explained using a *phase-space picture*. (For phrasing purposes we find the "Wigner function language" most convenient; still the reader should notice that we do not need or use this representation in practice.) We recall that the $P(n|m)$ is determined by the overlap of two Wigner functions. In the present context the Wigner functions $\rho^{(n)}$ are supported by shifted circles $[p_x-(\phi/2\pi)]^2+p_y^2=2\mathsf{m}E_n$. We are looking for their overlap with a reference Wigner function which is supported by the circle $p_x^2+p_y^2=2\mathsf{m}E_m$. The question is whether the overlaps of the Wigner functions $\rho^{(n)}$ and $\rho^{(m)}$ can be approximated by a classical calculation and under what circumstances we need perturbation theory.

Generically the Wigner function has a transverse Airy-type structure. If the "thickness" of the Wigner function is much smaller compared with the separation $|E_n-E_m|$ of the energy surfaces, then we can trust the semiclassical approximation. This will always be the case if $\hbar$ is small enough or, equivalently, if we can make ϕ large enough. In such case the dominant contribution comes from the intersection of the energy surfaces, which is the phase-space analog of the stationary-phase approximation. The other extreme is the case where the "thickness" of Wigner function is larger compared with the separation of the energy surfaces [namely, $\delta E_{\text{cl}}(\phi)<\hbar/\tau_{\text{cl}}$]. Then the contribution to the overlap comes "collectively" from all the regions of the Wigner (quasi)distribution, not just from the intersections. In such case we expect perturbation theory to work.

The above reasoning assumes that the wave function is concentrated in an ergodiclike fashion in the vicinity of the energy surface. This is known as the "Berry conjecture" [13]. In case of billiards it implies that the wave function looks like a random superposition of plane waves with $|p|=(2\mathsf{m}E)^{1/2}$. We find (see Fig. 4) that this does not hold in case of a rough billiard (unless $\hbar$ were extremely small, so as to make the de Broglie wavelength very short). Namely, in the case of a rough billiard there are eigenstates that have a lot of weight in the region $|p|<(2\mathsf{m}E)^{1/2}$. Consequently there are both semiclassical and nonsemiclassical overlaps. Specifically, if we have nonsemiclassical wave functions and

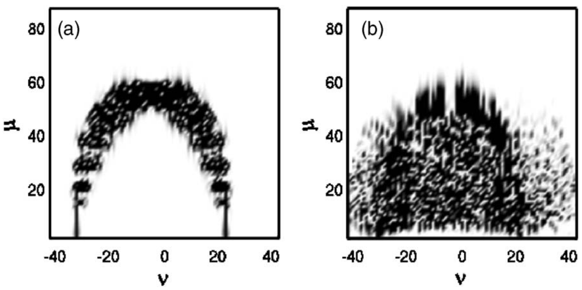

FIG. 4. The probability distribution $|\langle \nu,\mu|n\rangle|^2$ for (a) the $n=2423$ eigenstate of the smooth ($\ell=1$) billiard and (b) the $n=1000$ eigenstate of the rough ($\ell=100$) billiard. Note that this is essentially the (p_x,p_y) momentum distribution. The state in panel (a), unlike the state in panel (b), is a typical semiclassical state. Namely it is well concentrated on the energy shell.

$|E_n-E_m|\sim 0$, then the *collective* contribution dominates, which gives rise to the perturbativelike peak in the LDOS.

Our findings apply to systems, such as the rough billiard, where there is an additional (large) *nonuniversal* energy scale δE_{NU}. This is defined as an energy scale which is *not related* to the band profile and, hence, does not emerge in the RMT modeling. Hence in general there is a distinct twilight regime $\hbar/\tau_{\text{cl}}<\delta E_{\text{cl}}(\phi)<\delta E_{\text{NU}}$, which is neither "perturbative" nor "semiclassical." (In our numerics $\ell=100$ is so large that $\delta E_{\text{NU}}\sim E$.)

We have analyzed the parametric evolution of the eigenstates of an Aharonov-Bohm cylindrical billiard, as the flux is changed. The full crossover from the perturbative to the nonperturbative regime is demonstrated. Random-matrix theory suggests a *simple* crossover. Instead, we discover an intermediate twilight regime where perturbative and semiclassical features coexist. This can be understood by adopting a phase-space picture and taking into account the inapplicability of the Berry conjecture regarding the semiclassical structure of the wave functions.

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