Quasistatic transfer protocols for atomtronic superfluid circuits

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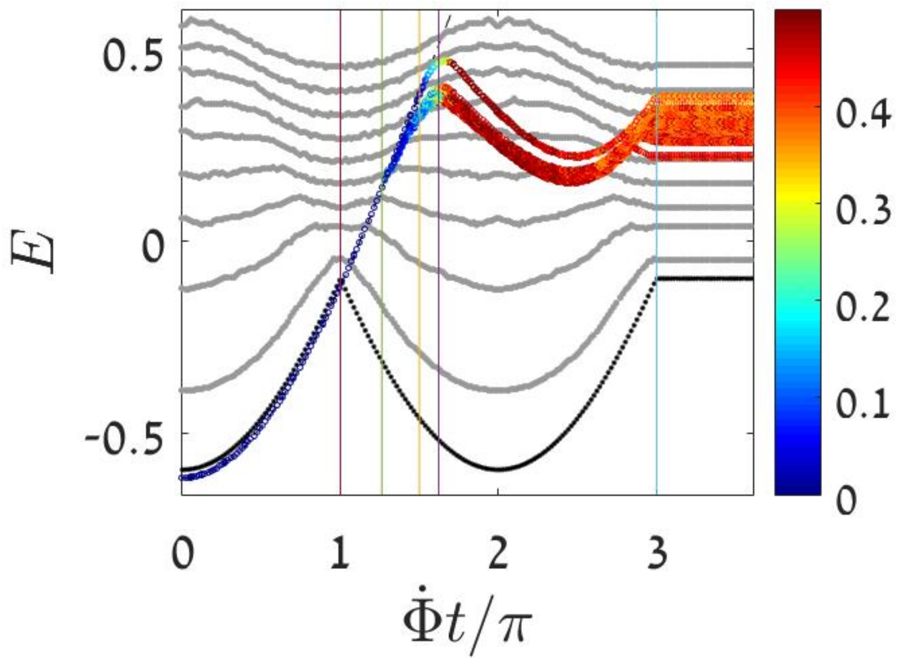
Circuits with condensed bosons are the building blocks for quantum Atomtronics. Such circuits will be used as QUBITs (for quantum computation) or as SQUIDs (for sensing of acceleration or gravitation). We study the feasibility and the design considerations for devices that are described by the Bose-Hubbard Hamiltonian. It is essential to realize that the theory involves "Quantum chaos" considerations.

- The Bose-Hubbard Hamiltonian.
- Optimal, faster and slow sweeps.
- Phase space landscape.
- Thresholds and Bogolyubov frequencies.
- Dynamic in Phase space.
- Efficiency of the sweep process.

[1] Y. Winsten, D. Cohen, Sci Rep 11, 3136 (2021).

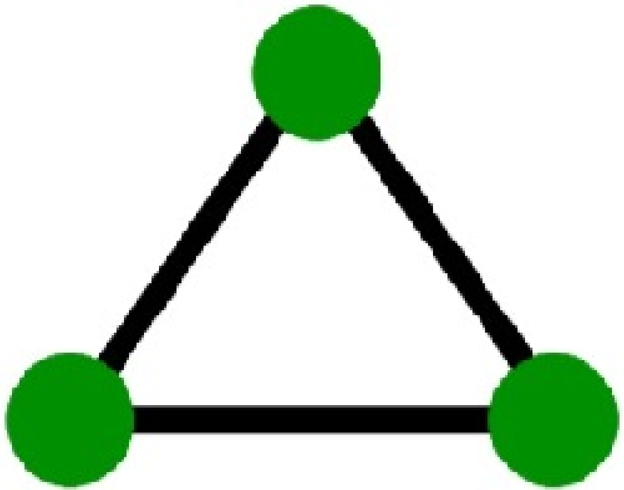


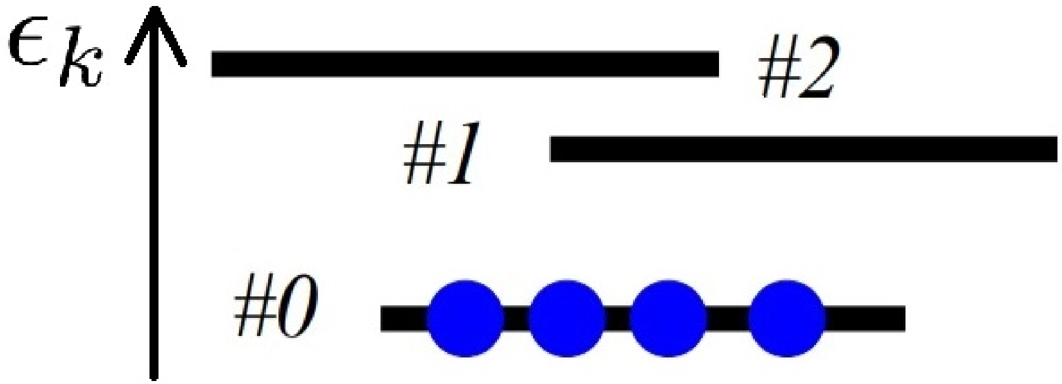
The Bose Hubbard Ring Circuit

In the rotating reference frame we have a Coriolis force,
which is like magntic field $\mathcal{B} = 2\mathrm{m}\Omega$. Hence it is like having flux $\Phi = \text{area} \times \mathcal{B}$

$$\mathcal{H} \;=\; \sum_{j=1}^{L} \left[\frac{U}{2} a_j^\dagger a_j^\dagger a_j a_j - \frac{K}{2} \left(\mathrm{e}^{i(\Phi/L)} a_{j+1}^\dagger a_j + \mathrm{e}^{-i(\Phi/L)} a_j^\dagger a_{j+1} \right) \right] \qquad L=3$$

$$\mathcal{H} \;=\; \sum_k \epsilon_k(\Phi)\, b_k^\dagger b_k \;+\; \frac{U}{2L} \sum' b_{k_4}^\dagger b_{k_3}^\dagger b_{k_2} b_{k_1}$$





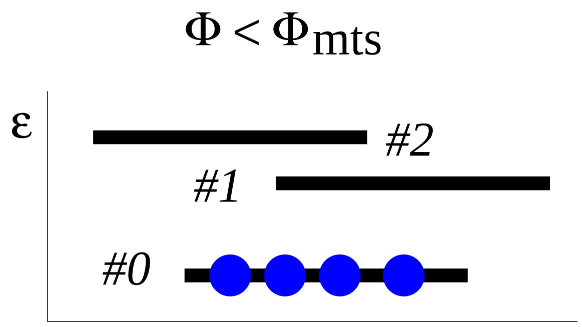
The Protocol

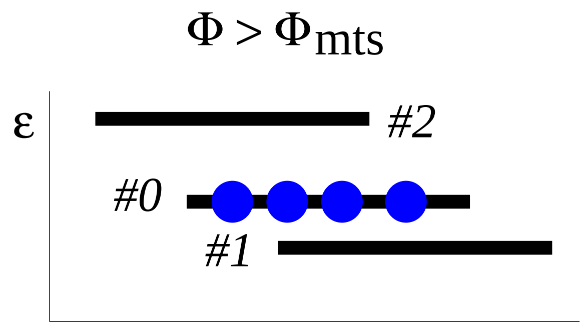
(1) Initially, at the preparation stage, all the particles are condensed into the lowest momentum orbital that has a zero winding number.

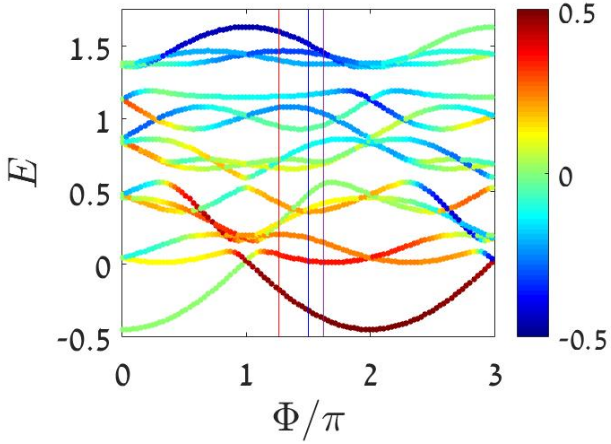
(2) The rotation frequency Φ of the ring is gradually changed, aka sweep process.

(3) The final state of the system is probed, and the momentum distribution is measured.

What is the fate of the evolving cloud at the end of the sweep?
Is it going to ergodize, or is it going to maintain some coherence?







Coordinate System

For L=3 sites, the Hamiltonian can be rewritten using $b_k = \sqrt{n_k}e^{i\varphi_k}$, where n_0, n_1, n_2 are the occupations in the momentum states. We use the coordinates

$$n = \frac{n_1+n_2}{2}$$ is the depletion coordinate

Initially $n = 0$

$$M = \frac{n_1-n_2}{2}$$ is the population imbalance

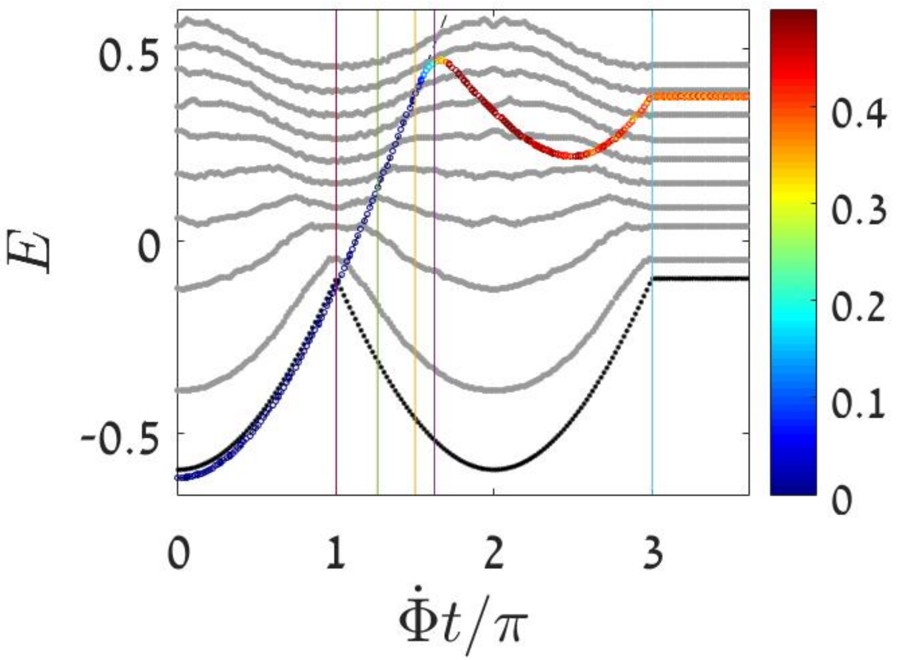
$$\mathcal{H}(\varphi, n; \phi, M) \;=\; \mathcal{H}^{(0)}(\varphi, n; M) \;+\; \left[\mathcal{H}^{(+)} + \mathcal{H}^{(-)}\right]$$

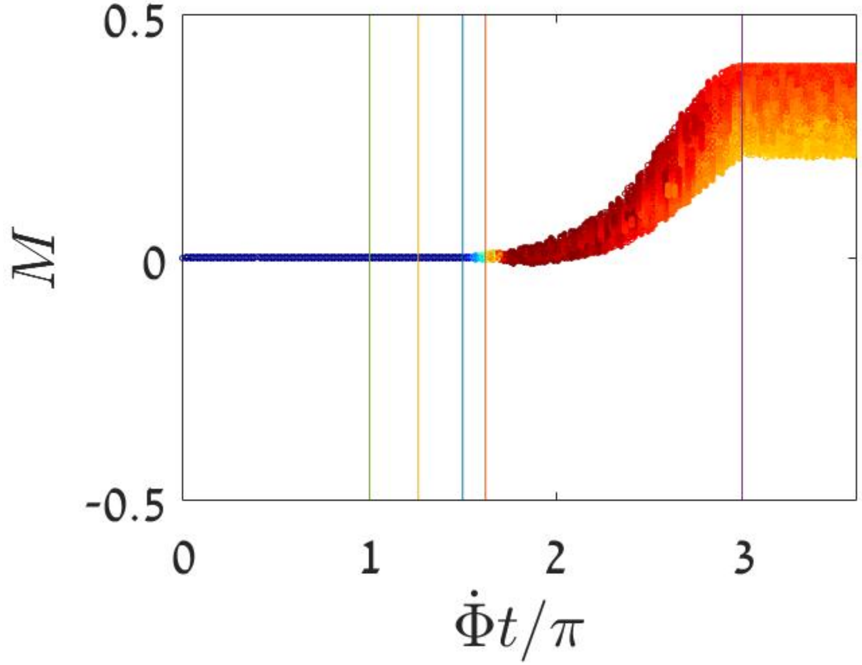
$$\begin{aligned}
\mathcal{H}^{(0)}(\varphi, n; M) &= \mathcal{E}n + \mathcal{E}_\perp M - \frac{U}{3}M^2 \;+\; \frac{2U}{3}(N-2n)\left[\frac{3}{4}n + \sqrt{n^2-M^2}\cos(\varphi)\right] \\
\mathcal{H}^{(\pm)} &= \frac{2U}{3}\sqrt{(N-2n)(n\pm M)}(n\mp M)\cos\left(\frac{3\phi\mp\varphi}{2}\right)
\end{aligned}$$

The current has the following expression in terms of (n, M):

$$I \;=\; -\frac{\partial\mathcal{H}}{\partial\Phi} \;=\; \left(n-\frac{N}{3}\right)K\sin\frac{\Phi}{3} \;+\; \frac{M}{\sqrt{3}}K\cos\frac{\Phi}{3}$$

Optimal sweep

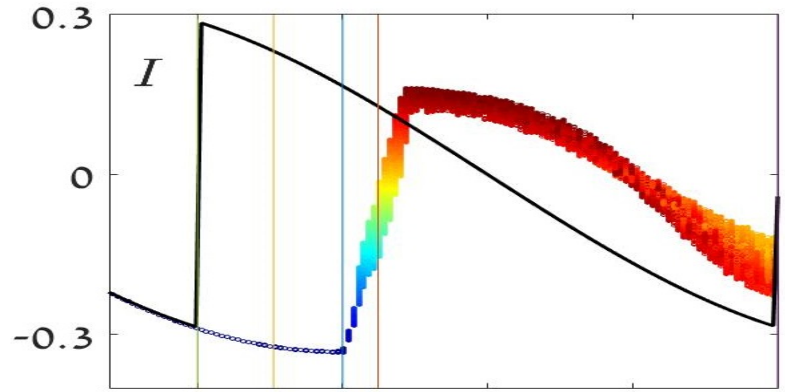




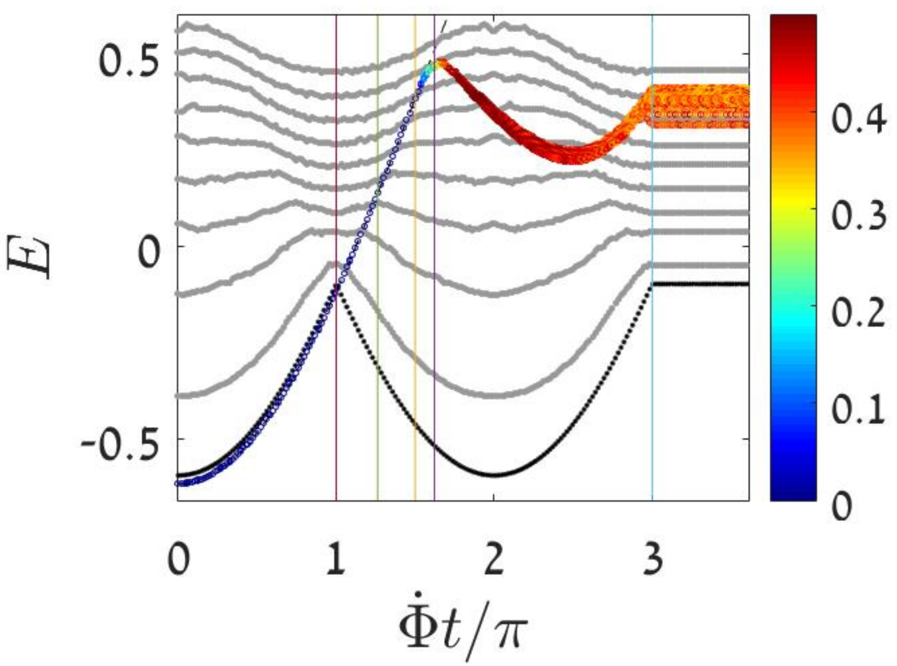
$n = (n_1 + n_2)/2$ - depletion (color-coded)
$M = (n_1 - n_2)/2$ - population imbalance

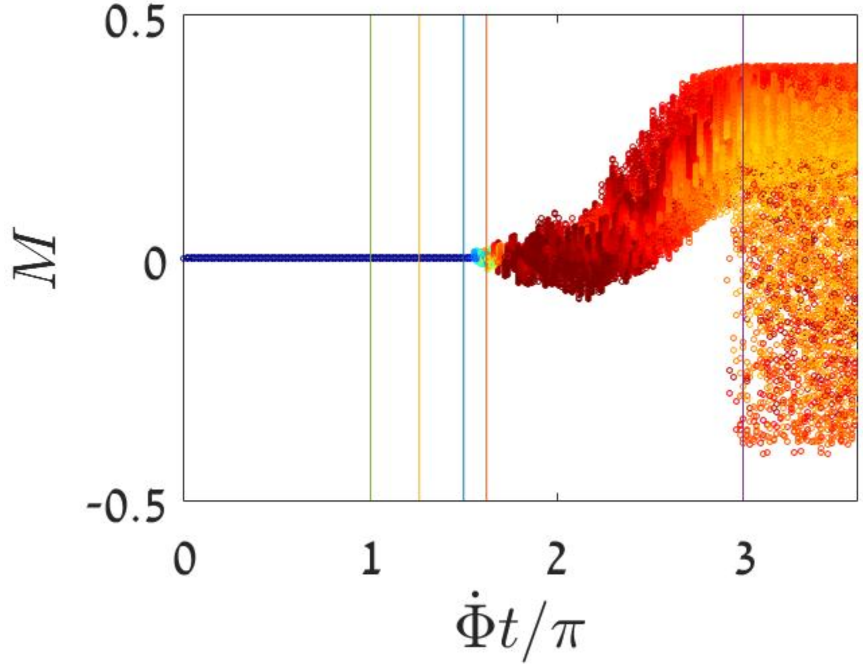
Thresholds: $(\Phi_{\mathrm{mts}}, \Phi_{\mathrm{stb}}, \Phi_{\mathrm{dyn}}, \Phi_{\mathrm{swp}})$

The depletion occurs for a small range around $\Phi \sim \Phi_{\mathrm{swp}}$



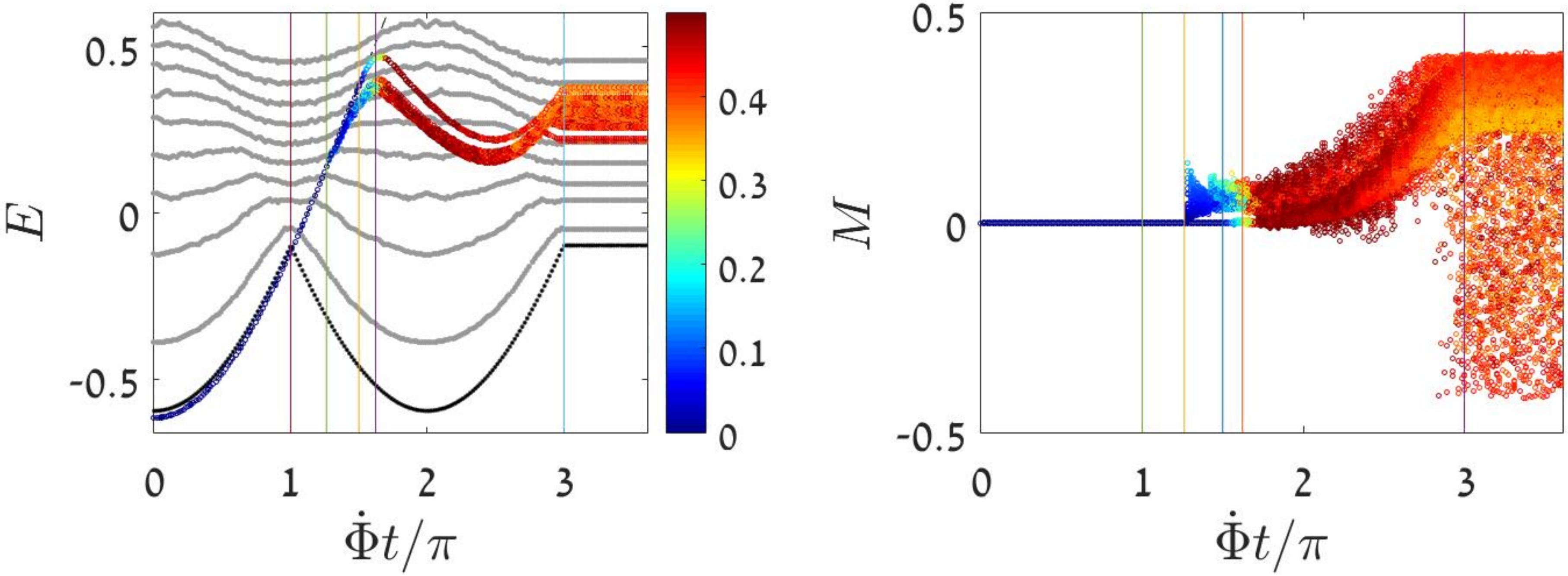
Faster sweep





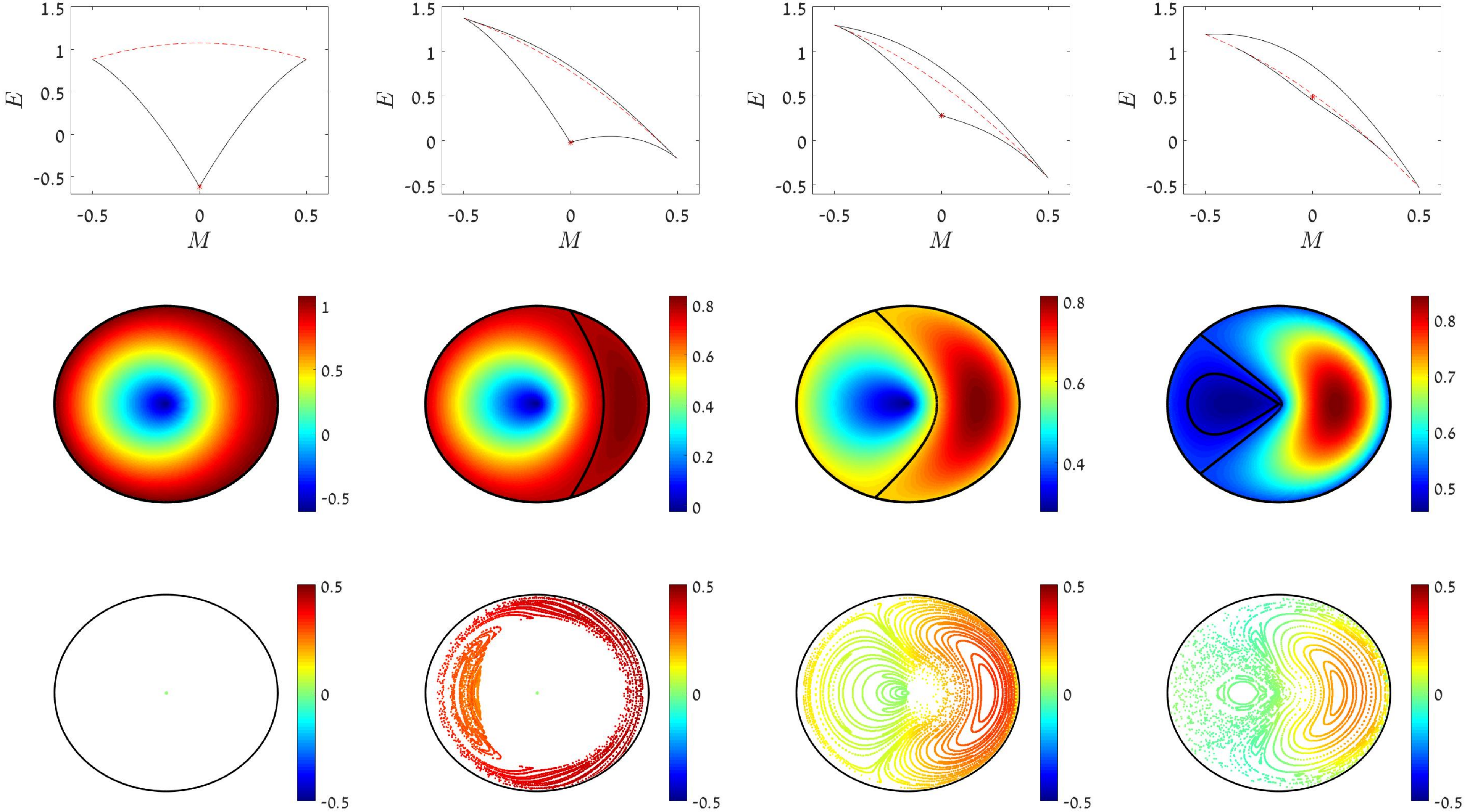
For faster sweep the cloud fails to follow the evolving energy landscape, and therefore an ergodicity is observed.

Slower sweep

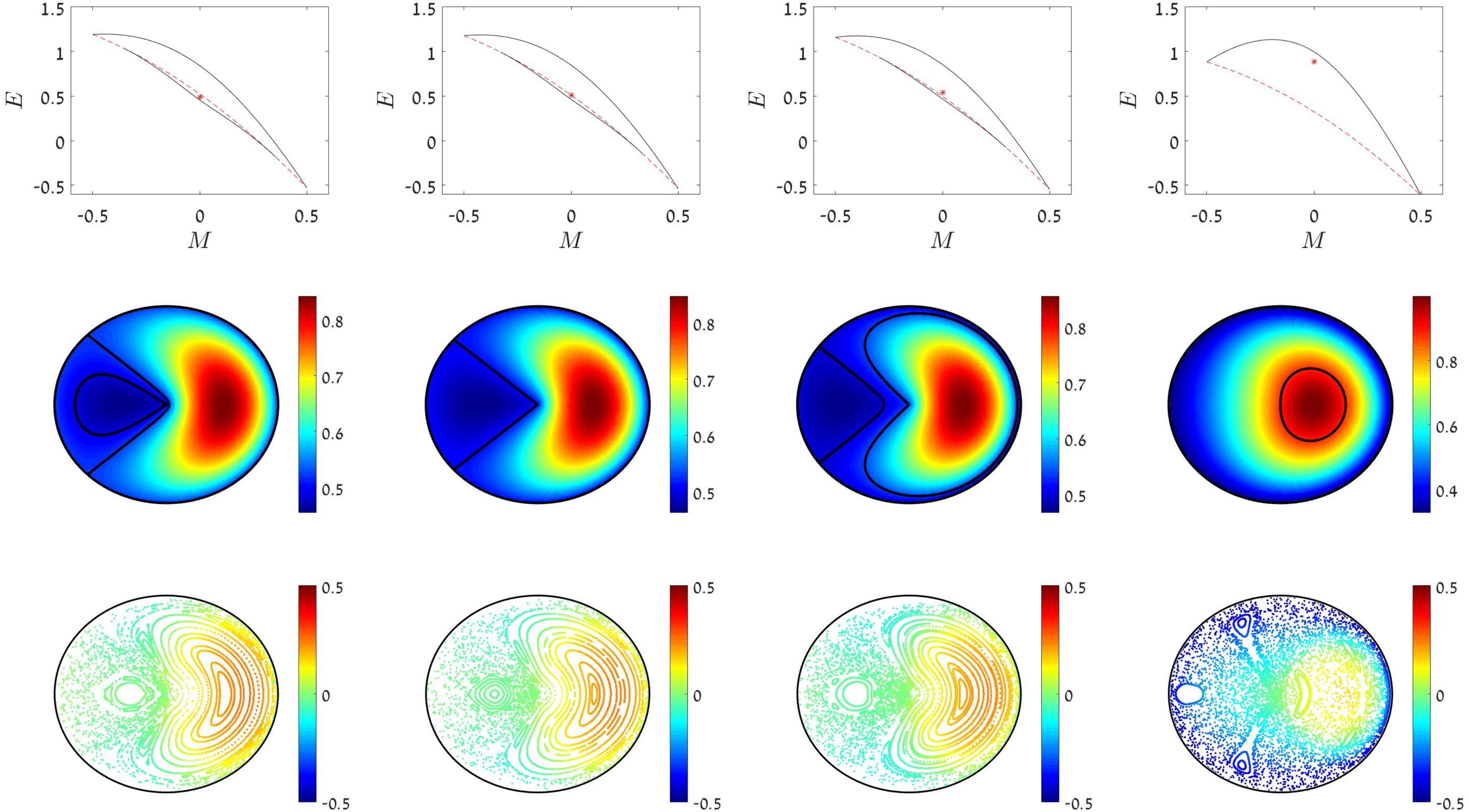


The cloud splits to two sub-clouds, one spreads as the optimal sweep and the other escapes at $\Phi \sim \Phi_{\mathrm{stb}}$ and an ergodicity is observed.

Phase space landscape ($E = \mathcal{H}^{(0)}(\varphi, n; M)$)



Phase space landscape ($E = \mathcal{H}^{(0)}(\varphi, n; M)$)



Thresholds

The central SP is the global minimum of the energy landscape up to

$$\Phi_{\mathrm{mts}} = \pi$$

It is still a local minimum up to

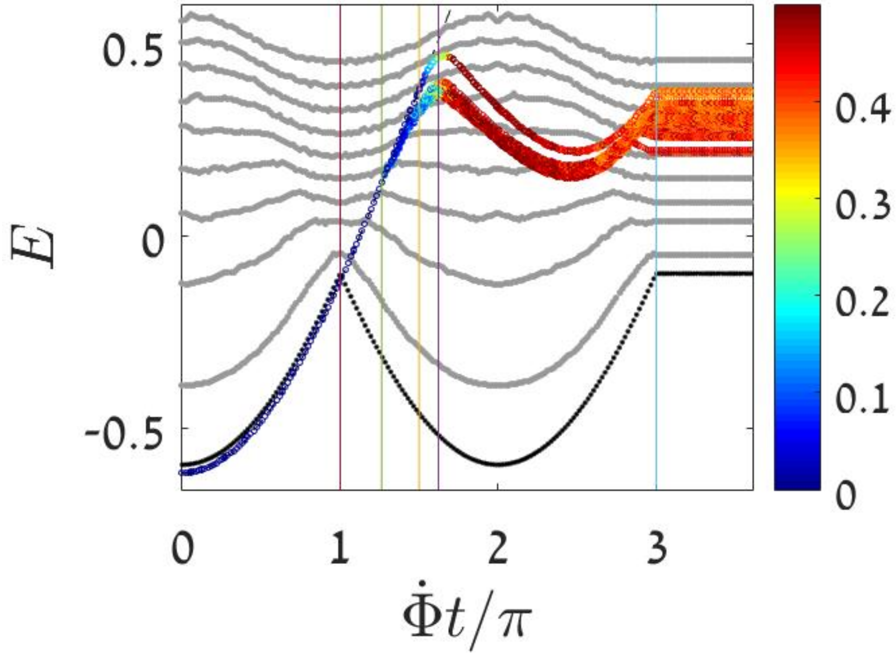
$$\Phi_{\mathrm{stb}} = 3\arccos\left(\frac{1}{6}\left(\sqrt{u^2+9}-u\right)\right)$$

It becomes an unstable saddle at

$$\Phi_{\mathrm{dyn}} = \frac{3}{2}\pi$$

The transition is available for a small range of Φ around

$$\Phi_{\mathrm{swp}} = 3\arccos\left(-\frac{1}{18}u\right)$$



where $u = \frac{NU}{K}$

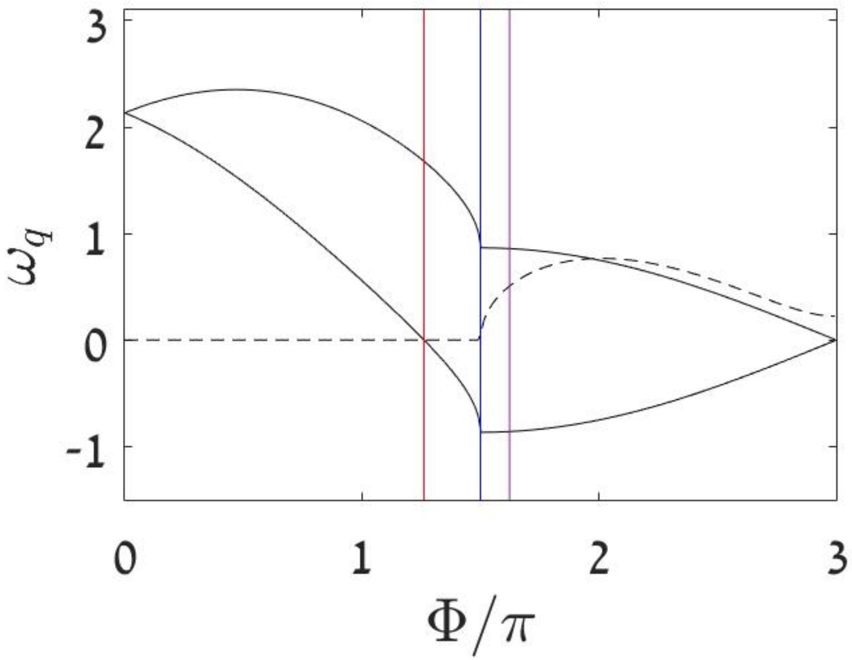
Bogolyubov frequencies

The Bogolyubov procedure brings the Hamiltonian in the vicinity of the SP to a diagonalized form

$$\mathcal{H} \approx E_0 + \sum_q \omega_q c_q^\dagger c_q$$

and the frequencies are

$$\omega_\pm = \pm \frac{\sqrt{3}}{2} \sin \frac{\Phi}{3} + \sqrt{\left(\frac{3}{2} \cos \frac{\Phi}{3}\right)^2 + u \cos \frac{\Phi}{3}}$$



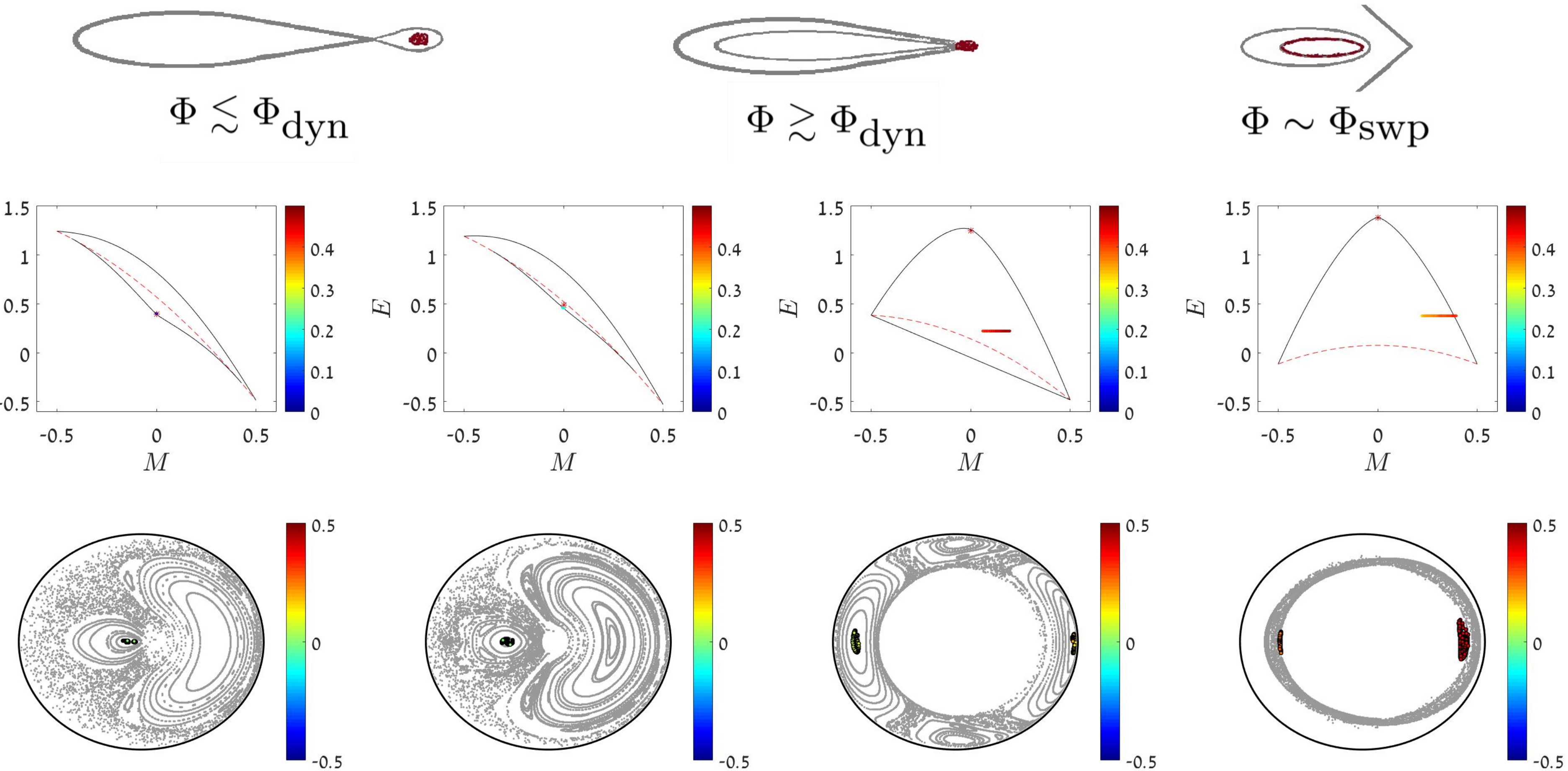
the SP becomes a saddle where ω_- changes sign and becomes negative.

$$\Phi_{\text{stb}} = 3 \arccos \left(\frac{1}{6} \left(\sqrt{u^2 + 9} - u \right) \right)$$

The SP becomes dynamically unstable where $\omega_\pm$ become complex.

$$\Phi_{\text{dyn}} = \frac{3}{2}\pi$$

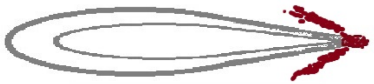
Dynamic in Phase space (optimal sweep)



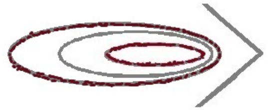
Dynamic in Phase space (slower sweep)

For slow sweep, at $\Phi \sim \Phi_{\text{stb}}$, the central piece of the cloud follows the SP, and the wings of the cloud spread into the chaotic region.

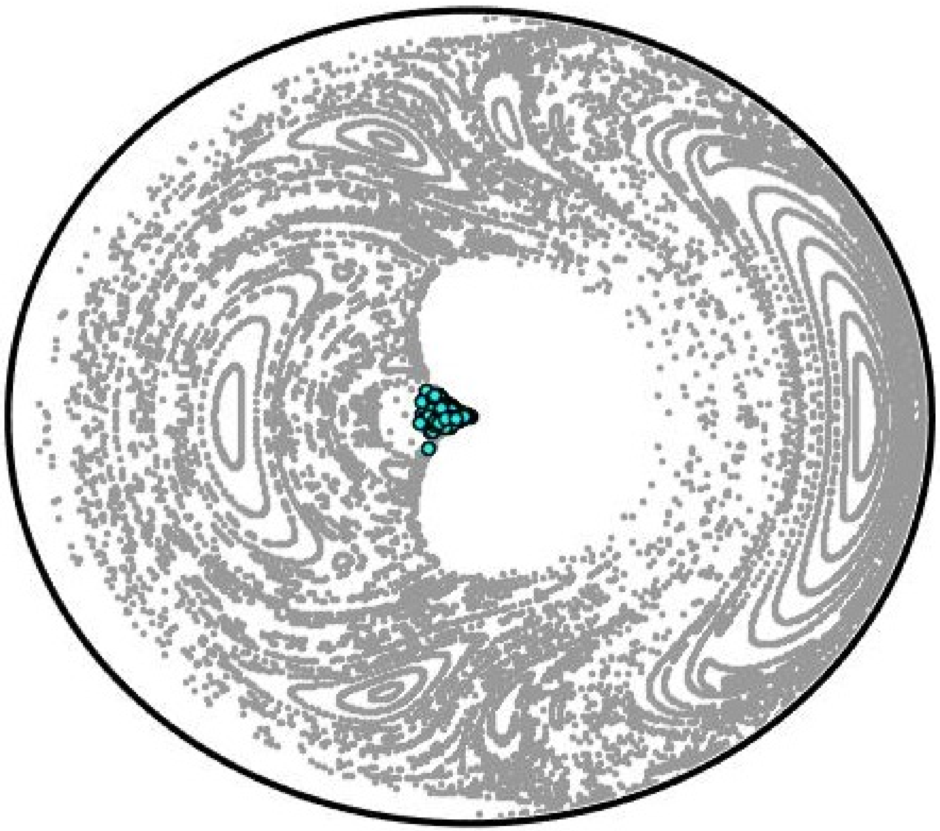
$\Phi \lesssim \Phi_{\text{dyn}}$

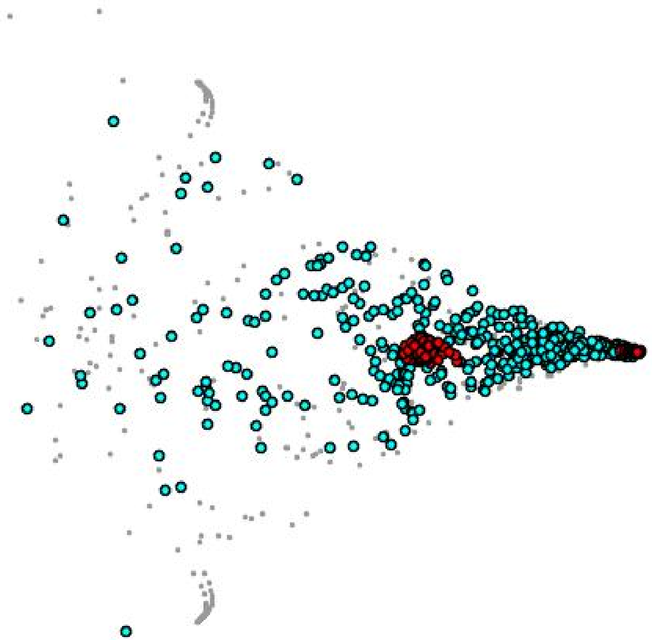


$\Phi \gtrsim \Phi_{\text{dyn}}$

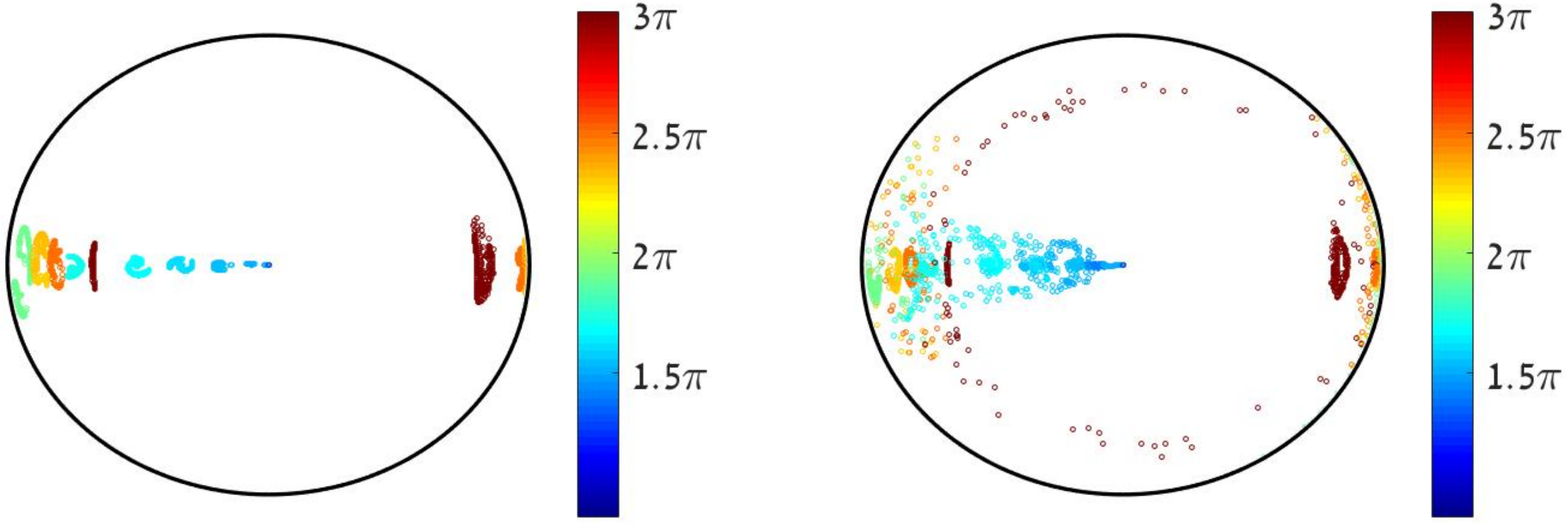


$\Phi \sim \Phi_{\text{swp}}$



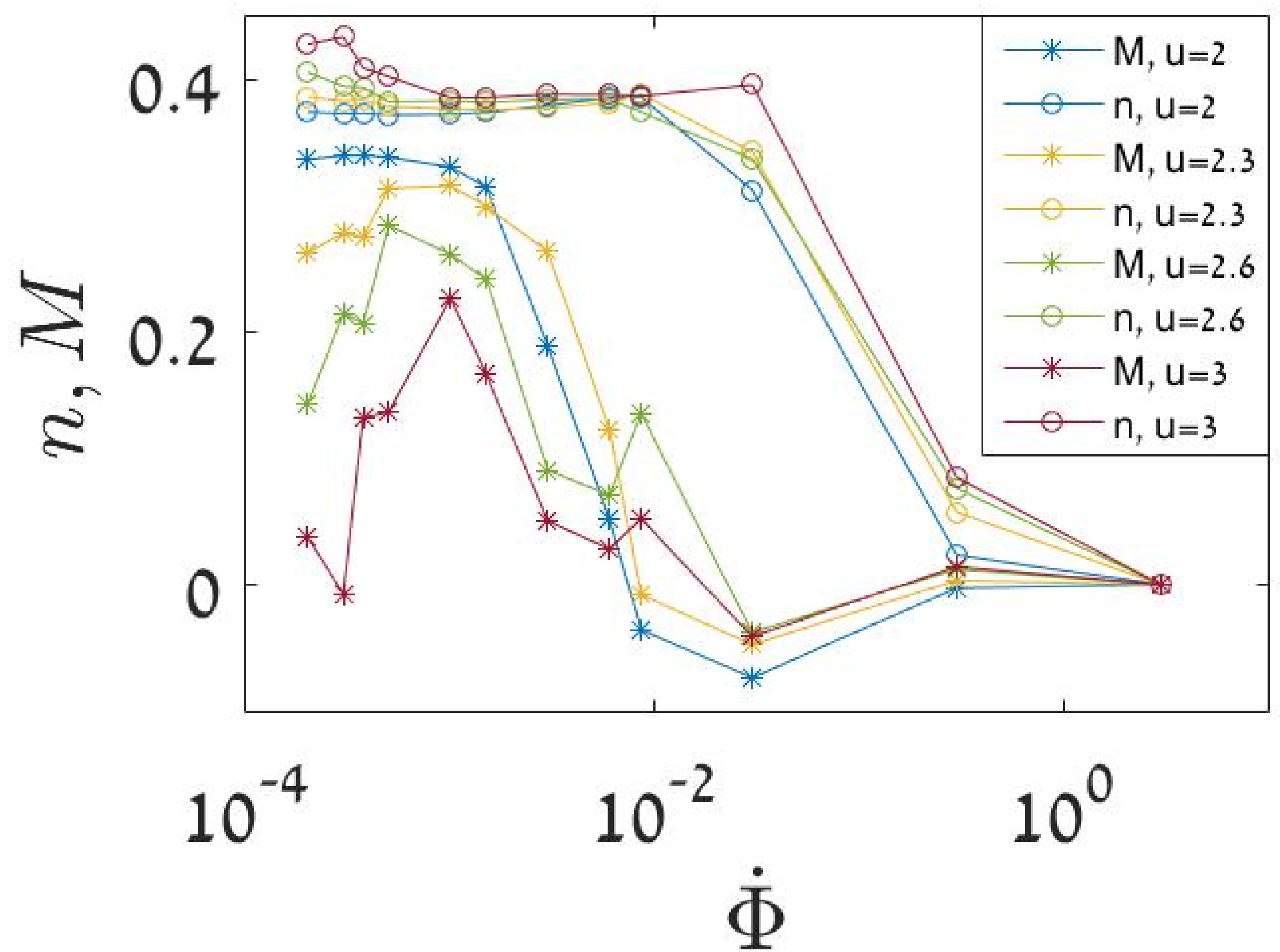


Dynamic in Phase space (optimal vs slower sweep)

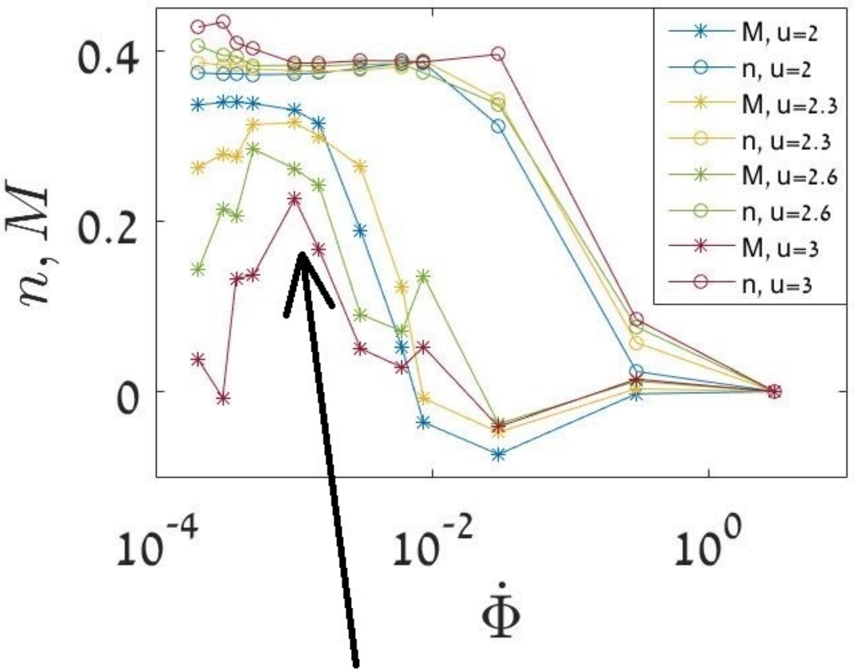


Color-coded by Φ. One spreading cloud in the optimal sweep vs two different spreading sub-clouds in the slower sweep.

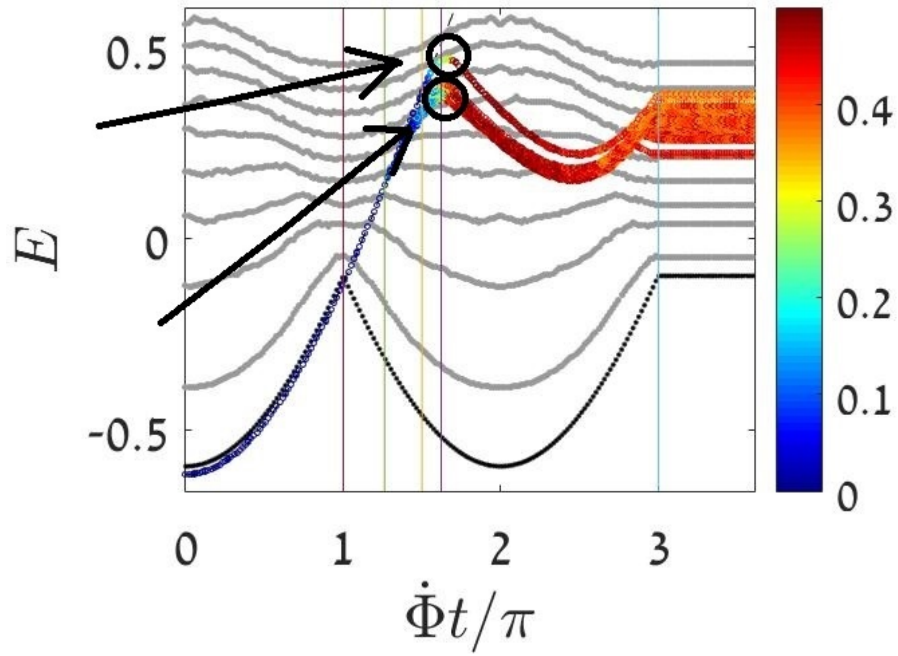
Efficiency of the sweep process



Thank you



Optimal rate



Too slow rate