Breakdown of adiabaticity in the quasi-static limit

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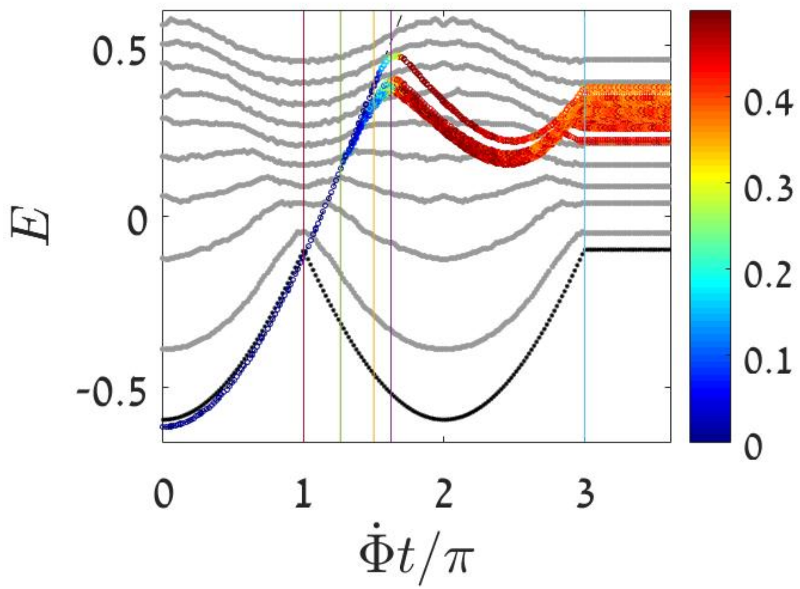
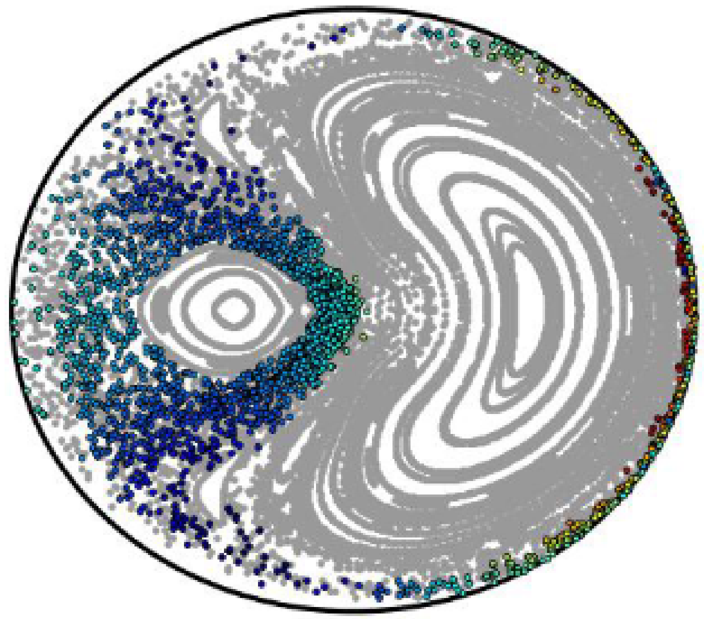




Yehoshua Winsten, DC,
Quasistatic transfer protocols for atomtronic superfluid circuits,
Scientific Reports 11, 3136 (2021).

The Bose Hubbard Hamiltonian

The system consists of N bosons in L sites.
Optionally we can add a gauge-field Φ.

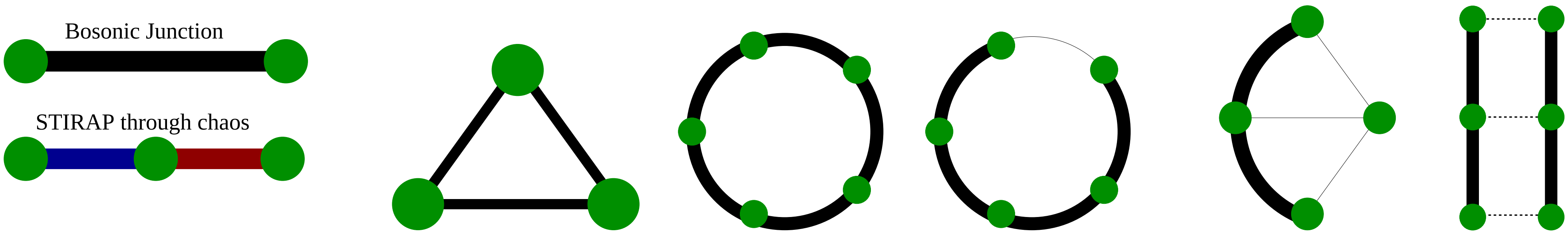
$$\mathcal{H}_{\text{BHH}} = \frac{U}{2}\sum_{j=1}^{L} a_j^\dagger a_j^\dagger a_j a_j - \sum_{\langle i,j \rangle} \frac{K_{ij}}{2} a_i^\dagger a_j$$

$$u \equiv L\frac{NU}{K} \qquad \text{[classical, stability, supefluidity, self-trapping]}$$

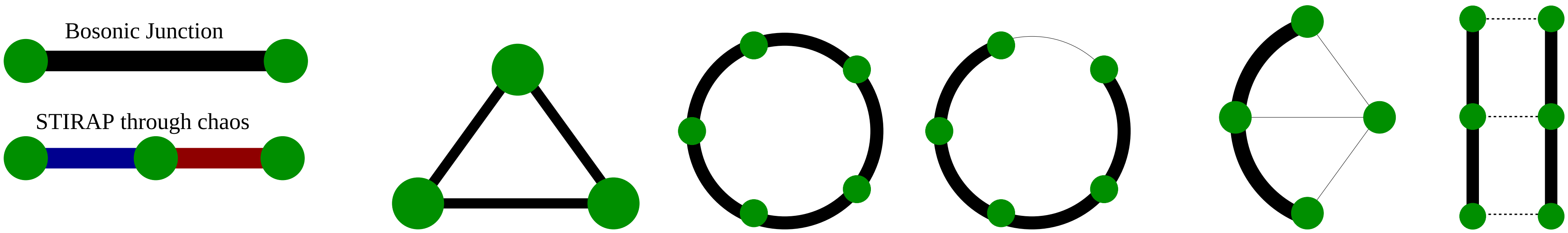
$$\gamma \equiv \frac{LU}{NK} = \frac{u}{N^2} \qquad \text{[quantum, Mott-regime]}$$

The two dimensionless parameters have a well defined value also in the GP/continuum limit.

$L = 2, 3, 4, 5, 6$



Minimal configurations



Dimer (L=2): Bosonic Josephson junction; Pendulum physics [1a].
Driven dimer: Landau-Zener dynamics [1b]; Kapitza effect [1c]; Zeno effect [1d]; Scars [1e].
Rings (L>2): Superfluidity [2a]; SF-Mott transition [2b].
Driven trimer: Many body STIRAP [3a]; Hamiltonian Hysteresis [3b]; Quasistatic transfer protocols [3c].
Coupled subsystems (L>3): Minimal model for Thermalization [4a,4b].

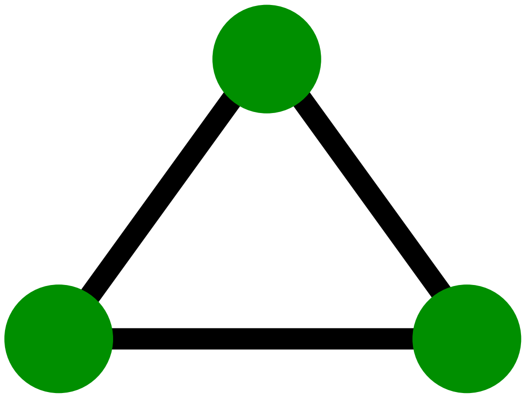
[1a] Chuchem, Smith-Mannschott, Hiller, Kottos, Vardi, DC (PRA 2010).
[1b] Smith-Mannschott, Chuchem, Hiller, Kottos, DC (PRL 2009).
[1c] Boukobza, Moore, DC, Vardi (PRL 2010).
[1d] Khripkov, Vardi, DC (PRA 2012)
[1e] Khripkov, DC, Vardi (JPA 2013, PRE 2013).
[2a] Arwas, DC (SREP 2015, NJP 2016, PRB 2017, PRA 2019).
[2b] Arwas, DC, Hekking, Minguzzi (PRA 2017).
[3a] Dey, DC, Vardi (PRL 2018, PRA 2019).
[3b] Burkle, Vardi, DC, Anglin (PRL 2019).
[3c] Winsten, DC (SREP 2021).
[4a] Tikhonenkov, Vardi, Anglin, DC, (PRL 2013).
[4b] Khripkov, Vardi, DC (NJP 2015, PRE 2018, PRA 2020).

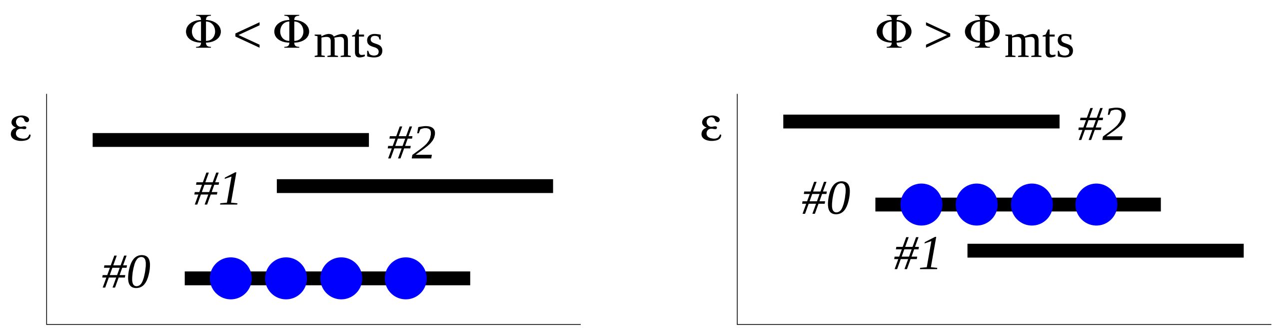
The Bose Hubbard Ring Circuit

In the rotating reference frame we have a Coriolis force,
which is like magnetic field $\mathcal{B} = 2\mathrm{m}\Omega$.
which implies an effective flux $\Phi = \text{area} \times \mathcal{B}$

$$\mathcal{H} \;=\; \sum_{j=1}^{L} \left[\frac{U}{2} a_j^\dagger a_j^\dagger a_j a_j - \frac{K}{2} \left(\mathrm{e}^{i(\Phi/L)} a_{j+1}^\dagger a_j + \mathrm{e}^{-i(\Phi/L)} a_j^\dagger a_{j+1} \right) \right] \qquad L = 3$$

$$\mathcal{H} \;=\; \sum_k \epsilon_k(\Phi)\, b_k^\dagger b_k \;+\; \frac{U}{2L} {\sum}' b_{k_4}^\dagger b_{k_3}^\dagger b_{k_2} b_{k_1}$$





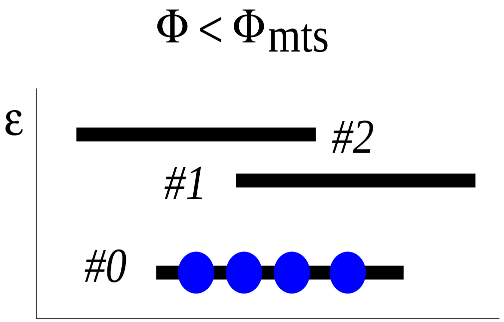
Quasistatic transfer protocols for atomtronic superfluid circuits

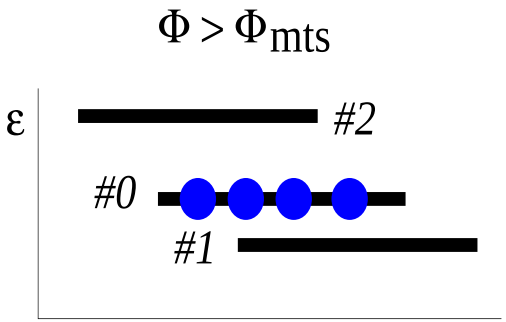
(1) All the particles are condensed into the lowest momentum orbital that has a zero winding number.
(2) The rotation frequency Φ is gradually changed, aka sweep process.
(3) The final state of the system is probed; the momentum distribution is measured.

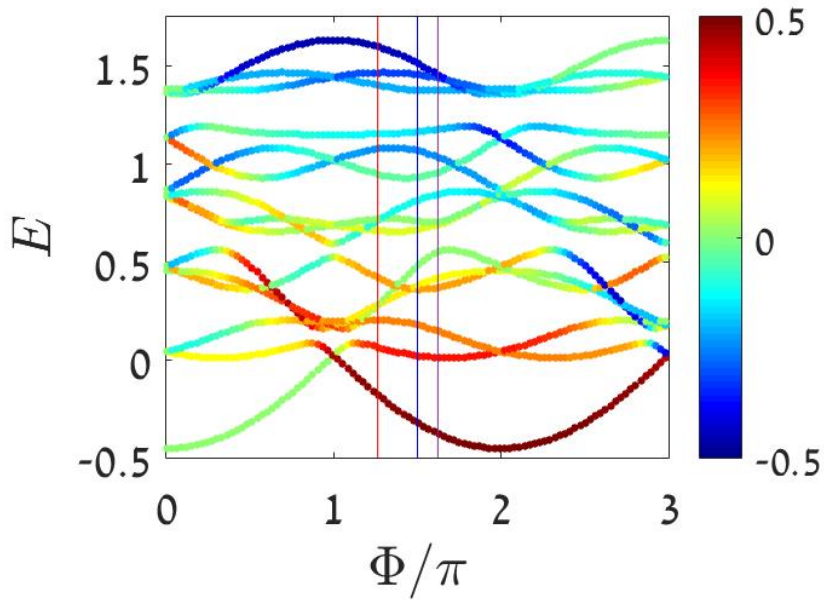
What is the fate of the evolving many-body state?

$n = \frac{1}{2}(n_1 + n_2)$ = depletion coordinate

$M = \frac{1}{2}(n_1 - n_2)$ = population imbalance







Semiclassical Hamiltonian

We set $b_k = \sqrt{n_k} e^{i\varphi_k}$, where $n_0 + n_1 + n_2 = N$.

$$n = \frac{n_1+n_2}{2} \quad = \text{the depletion coordinate} \qquad [\text{Initially } n = 0]$$

$$M = \frac{n_1-n_2}{2} \quad = \text{the population imbalance}$$

$$\mathcal{H}(\varphi, n; \phi, M) \; = \; \mathcal{H}^{(0)}(\varphi, n; M) \; + \; \left[\mathcal{H}^{(+)} + \mathcal{H}^{(-)}\right]$$

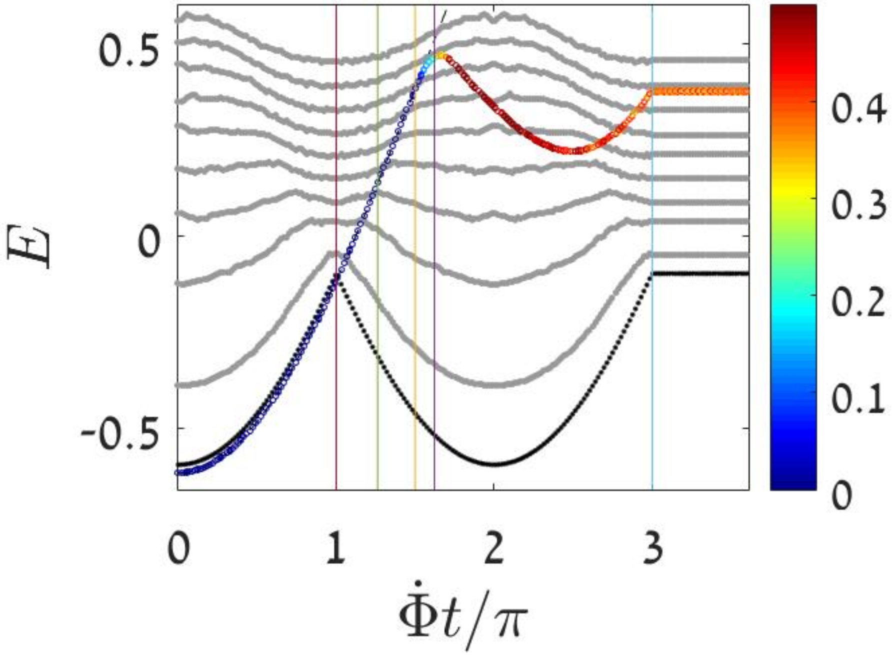
$$\mathcal{H}^{(0)}(\varphi, n; M) \quad = \quad \mathcal{E}n + \mathcal{E}_\perp M - \frac{U}{3}M^2 \; + \; \frac{2U}{3}(N-2n)\left[\frac{3}{4}n + \sqrt{n^2 - M^2}\cos(\varphi)\right]$$

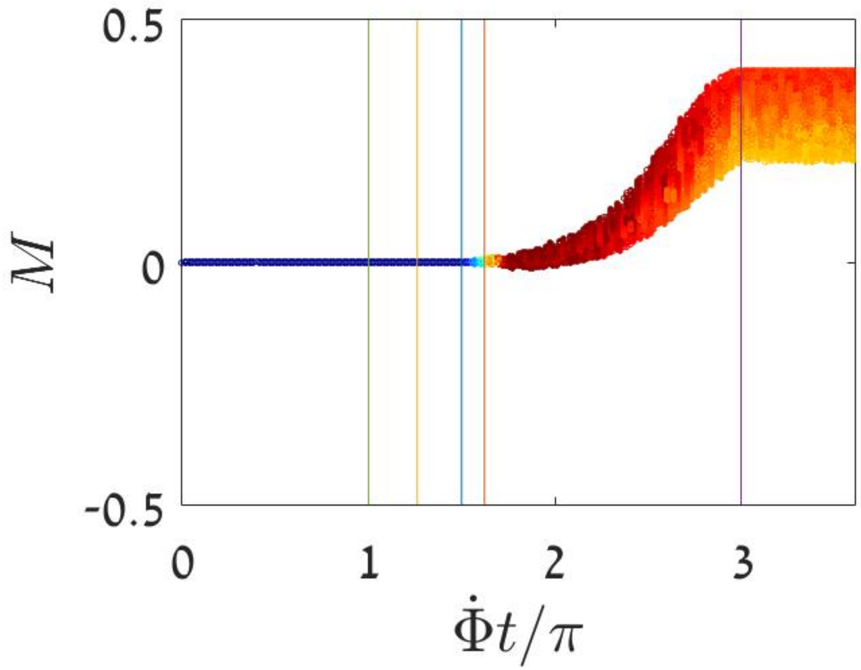
$$\mathcal{H}^{(\pm)} \quad = \quad \frac{2U}{3}\sqrt{(N-2n)(n\pm M)}(n\mp M)\cos\left(\frac{3\phi\mp\varphi}{2}\right)$$

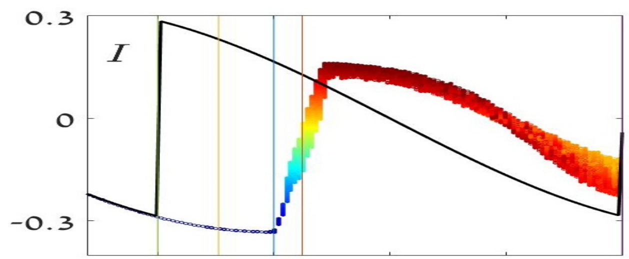
The current has the following expression in terms of (n, M):

$$I \; = \; -\frac{\partial \mathcal{H}}{\partial \Phi} \; = \; \left(n - \frac{N}{3}\right) K \sin\frac{\Phi}{3} \; + \; \frac{M}{\sqrt{3}} K \cos\frac{\Phi}{3}$$

Semiclassical simulation of a sweep process



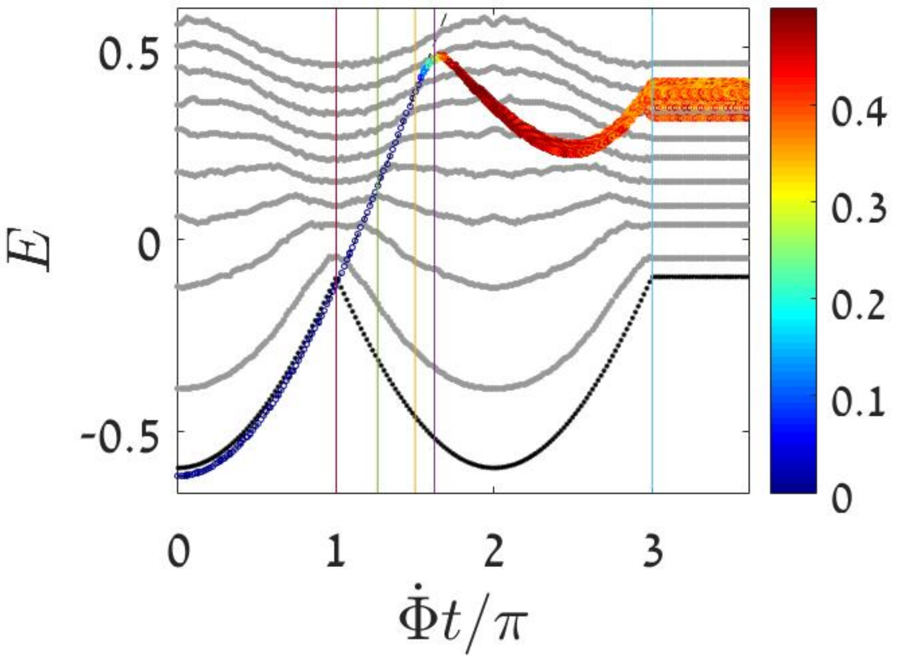


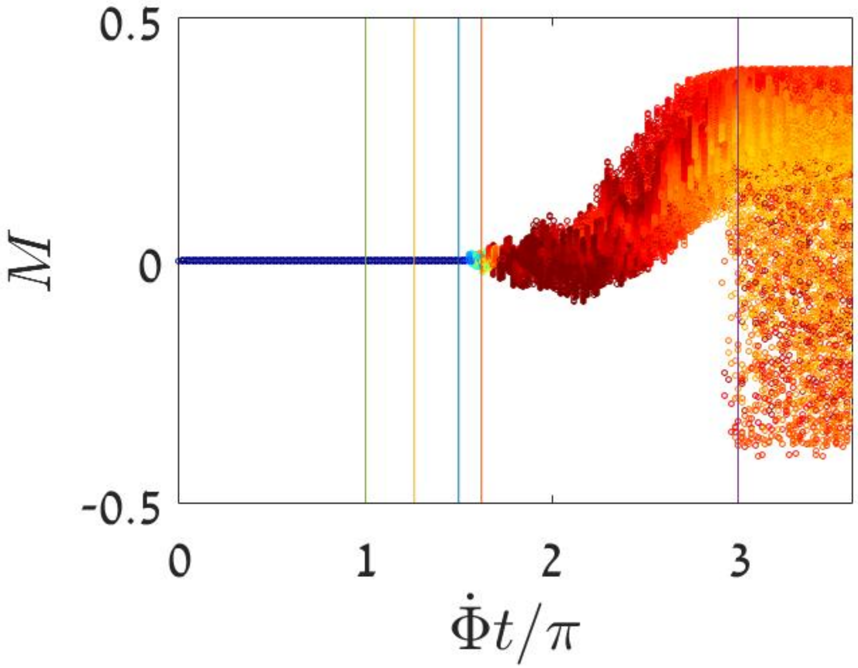


$n = (n_1 + n_2)/2 =$ depletion (color-coded)
$M = (n_1 - n_2)/2 =$ population imbalance

Thresholds: $(\Phi_{\rm mts}, \Phi_{\rm stb}, \Phi_{\rm dyn}, \Phi_{\rm swp})$

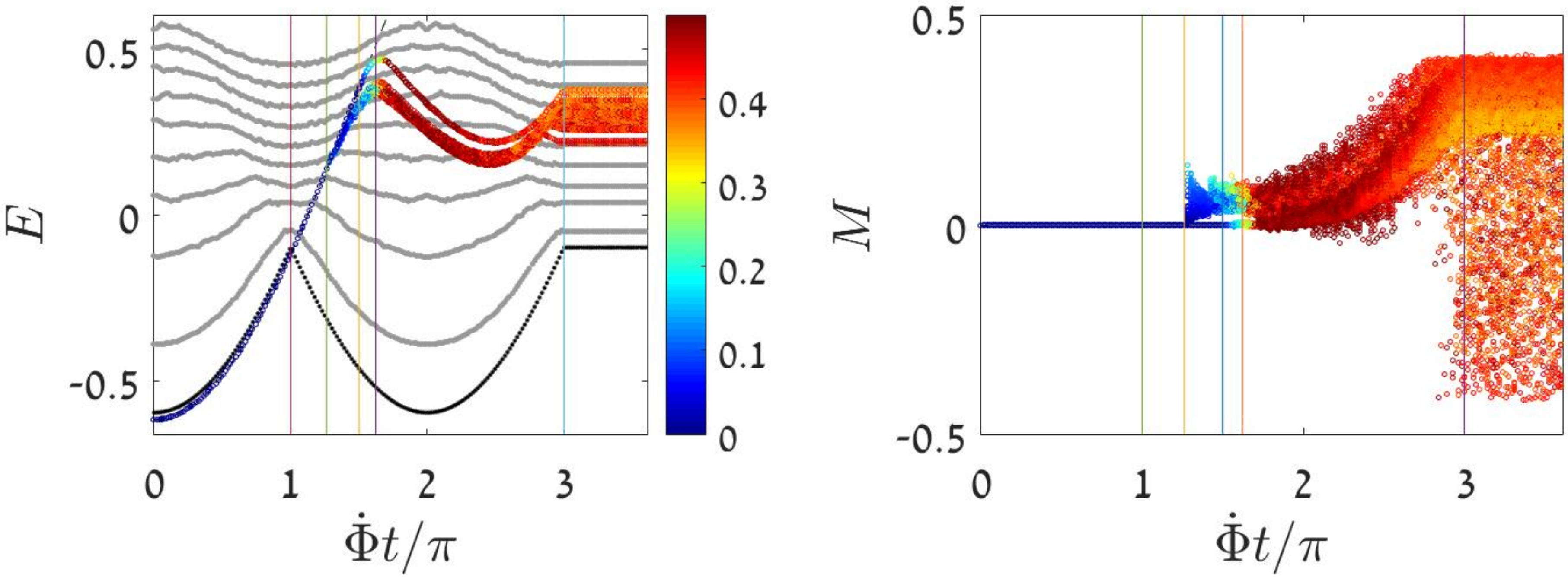
Faster sweep





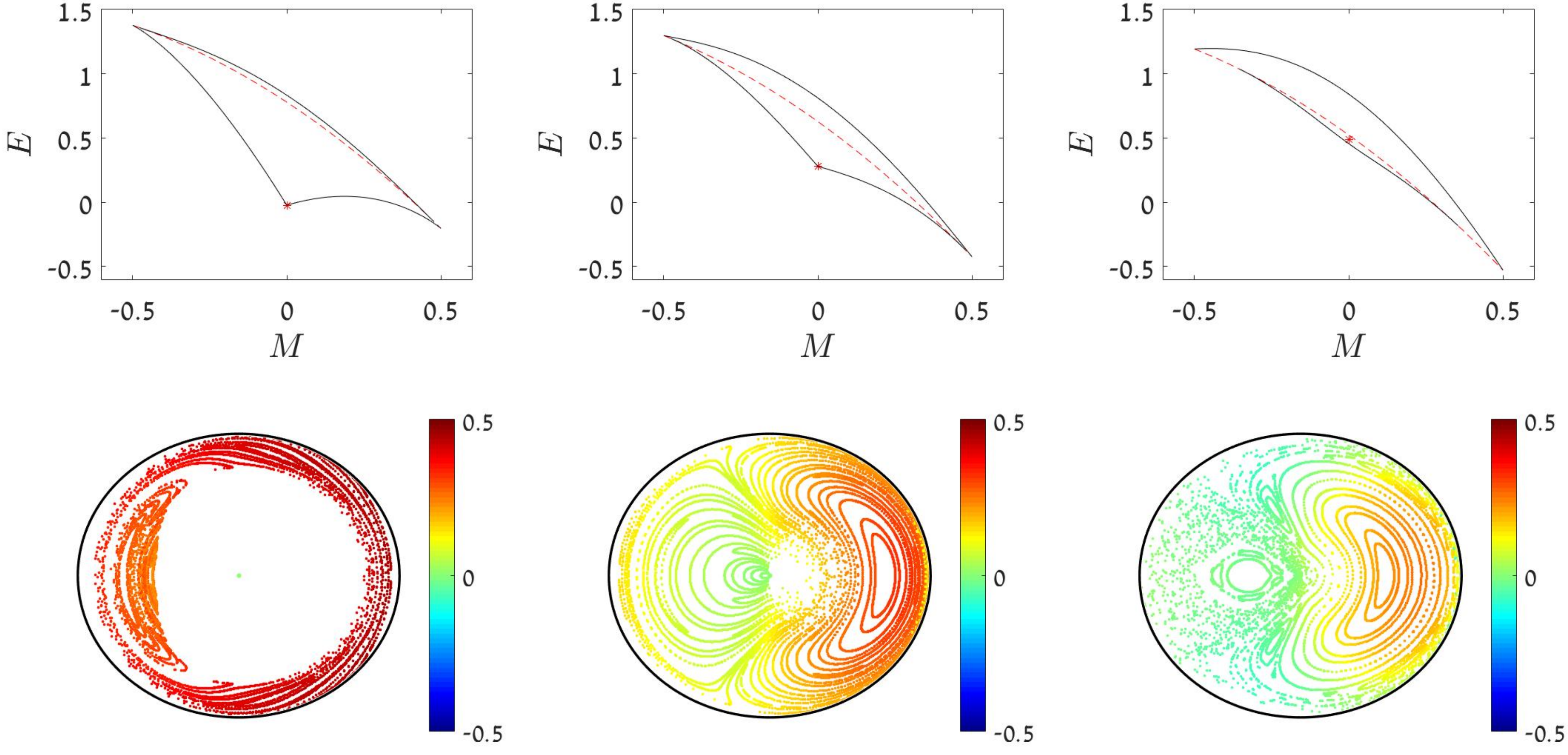
As expected - larger spreading

Slower sweep



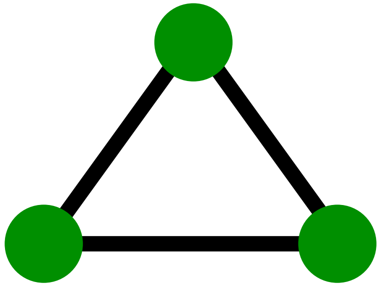
Breakdown of adiabaticity, and implied irreversibility, in the quasi-static limit.

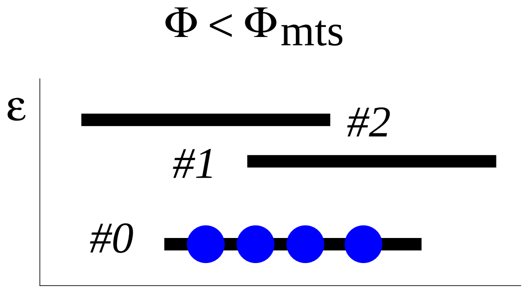
Phase space structure

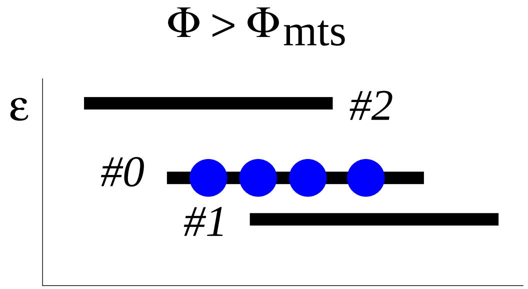


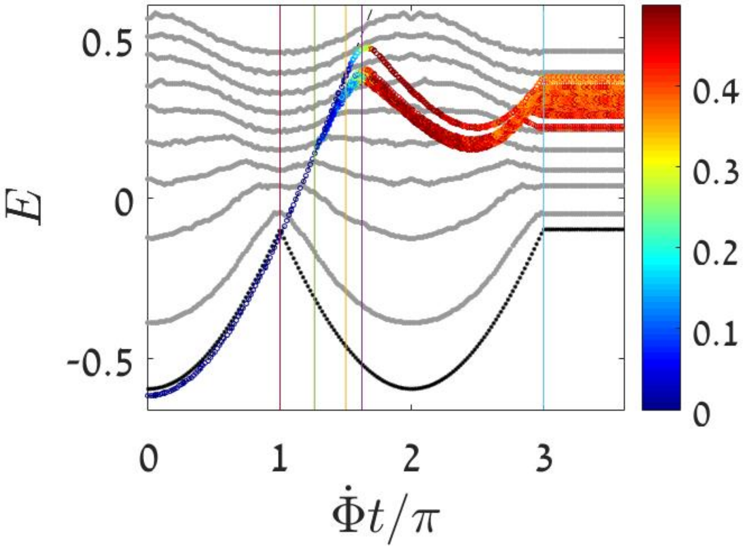
Thresholds: $(\Phi_{\mathrm{mts}}, \Phi_{\mathrm{stb}}, \Phi_{\mathrm{dyn}}, \Phi_{\mathrm{swp}})$

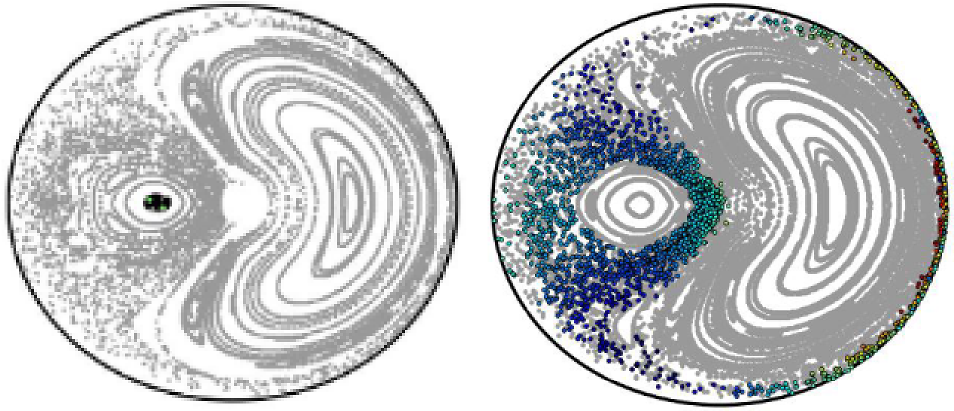
Chaos-assisted depletion











One should not under-estimate the importance of having mixed-chaotic phase-space...

Thresholds

The central SP is the global minimum of the energy landscape up to

$$\Phi_{\mathrm{mts}} = \pi$$

The SP is still a local minimum up to

$$\Phi_{\mathrm{stb}} = 3\arccos\left(\frac{1}{6}\left(\sqrt{u^2+9}-u\right)\right)$$

The SP becomes an unstable saddle at

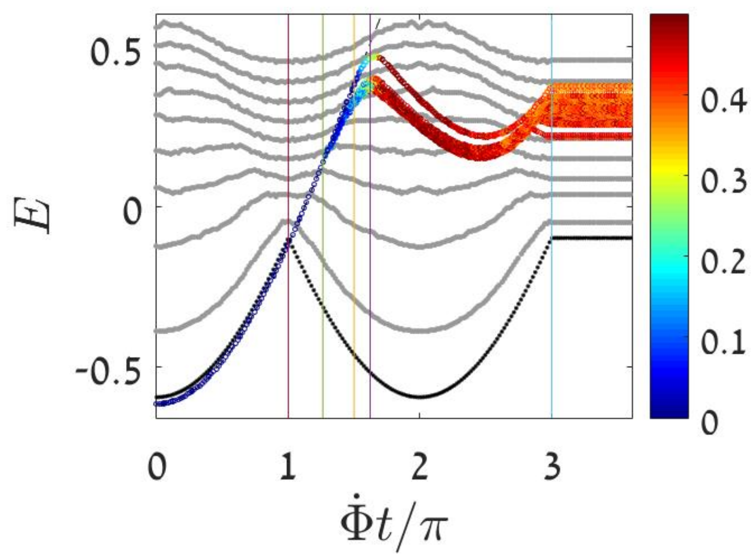
$$\Phi_{\mathrm{dyn}} = \frac{3}{2}\pi$$

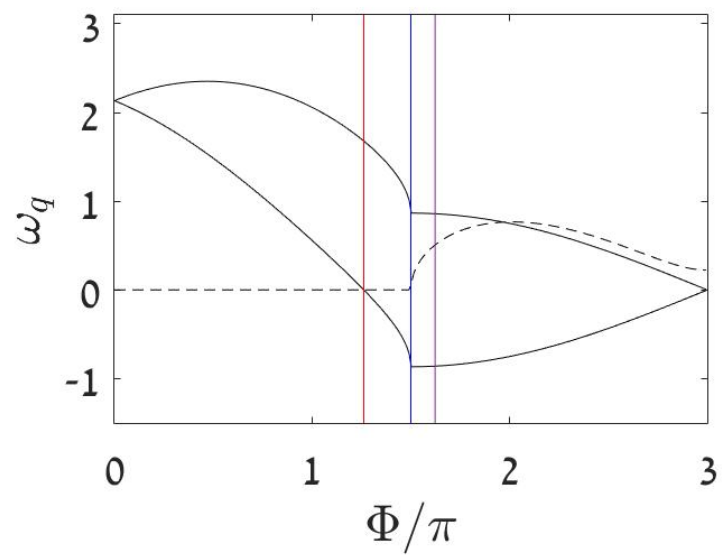
The SP becomes connected to the periphery at the swap transition

$$\Phi_{\mathrm{swp}} = 3\arccos\left(-\frac{1}{18}u\right)$$

The Bogolyubov frequencies:

$$\omega_{\pm} = \pm\frac{\sqrt{3}}{2}\sin\frac{\Phi}{3} + \sqrt{\left(\frac{3}{2}\cos\frac{\Phi}{3}\right)^2 + u\cos\frac{\Phi}{3}}$$



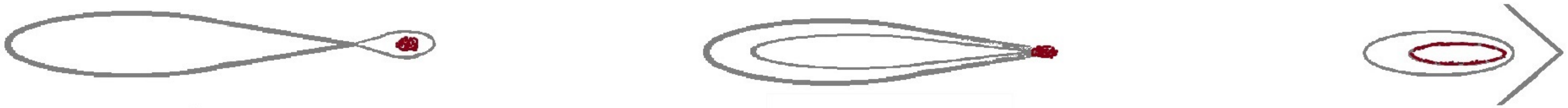


$$\mathcal{H} \approx E_0 + \sum_q \omega_q c_q^\dagger c_q$$

Dynamic in Phase space: slow vs optimal sweep

Optimal sweep:

The cloud is shuttled by a fixed-point (PO) that has bifurcated from the center.



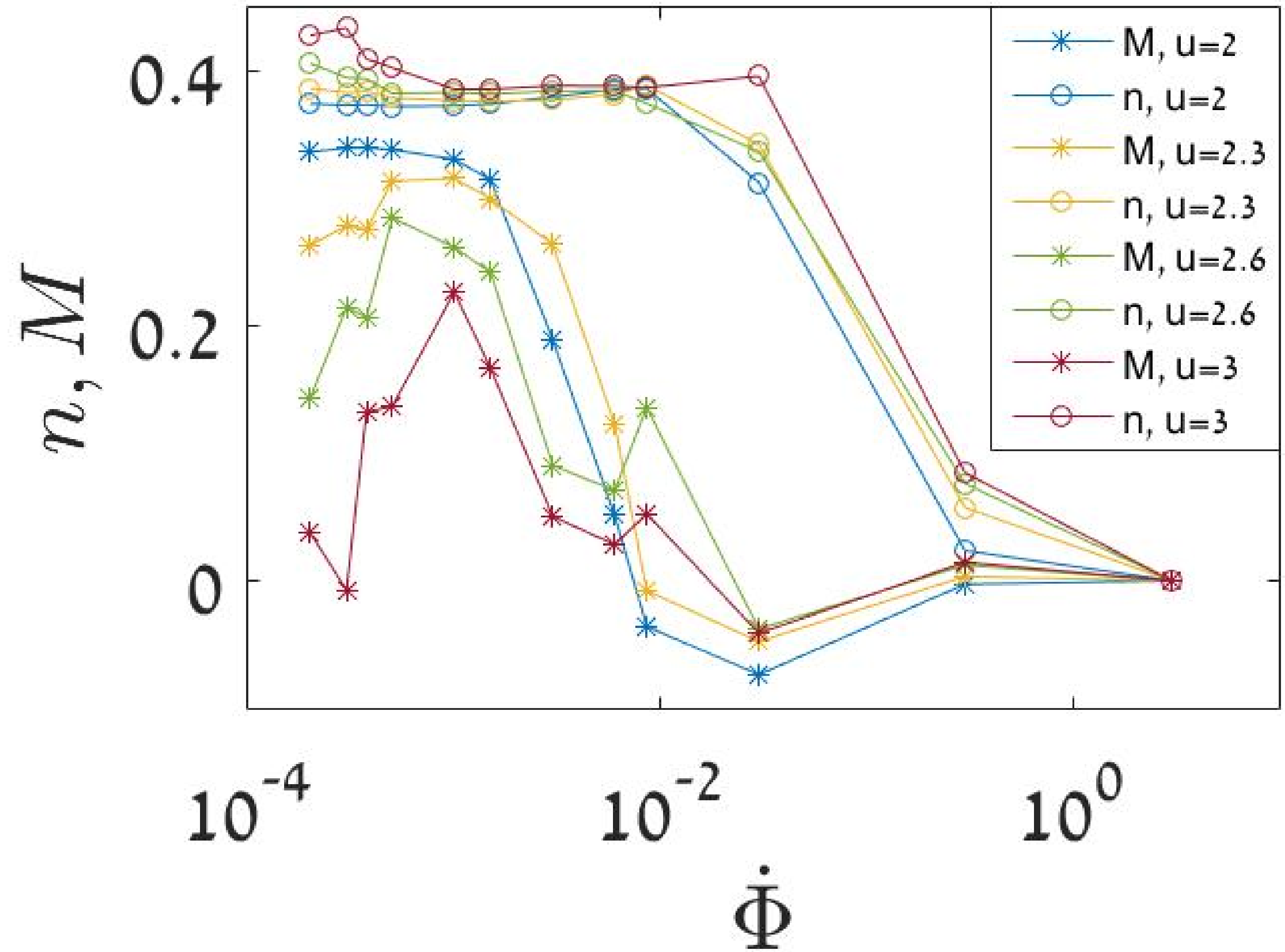
$\Phi \lesssim \Phi_{\rm dyn}$ $\Phi \gtrsim \Phi_{\rm dyn}$ $\Phi \sim \Phi_{\rm swp}$

Slow sweep:

The cloud spreads through the corridor, and after that shuttled by an outer torus.

$\Phi \lesssim \Phi_{\rm dyn}$ $\Phi \gtrsim \Phi_{\rm dyn}$ $\Phi \sim \Phi_{\rm swp}$

Efficiency of the sweep process



Optimal rate maximizes the transfer efficiency

Main messages

- A quasi-static protocol is in general not adiabatic,
  and hence not reversible, due to mixed-chaotic dynamics.
- It is implied that slowness is bad for adiabaticity.
- We have considered a protocol whose aim is to transfer condensed particles
  from a source orbital to a target orbital.
- Two competing mechanisms: adiabatic shuttling versus chaos-assisted depletion.
- The irreversible chaos-assisted depletion mechanism dominates in the quasi-static limit.
- An implied optimal sweep rate for the performance of the transfer protocol.