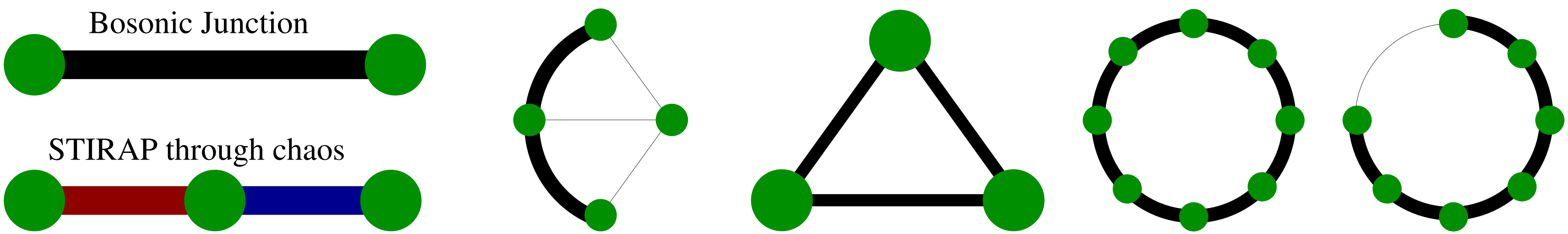
Atomtronics and Quantum Chaos

Doron Cohen and Ami Vardi

Circuits with condensed bosons are the building blocks for quantum Atomtronics. Such circuits will be used as QUBITs (for quantum computation) or as SQUIDs (for sensing of acceleration or gravitation). We study the feasibility and the design considerations for devices that are described by the Bose-Hubbard Hamiltonian. It is essential to realize that the theory involves "Quantum chaos" considerations.

- Why Bose-Hubbard (discrete) circuits?
- Why "Quantum chaos" relevant?
- Themes addressed: Stabilization; Population transfer; Thermalization; Superfluidity...



The Bose Hubbard Hamiltonian

The system consists of N bosons in M sites. Later we add a gauge-field Φ.

$$\mathcal{H}_{\text{BHH}} \;=\; \frac{U}{2}\sum_{j=1}^{M} a_j^{\dagger}a_j^{\dagger}a_j a_j \;-\; \frac{K}{2}\sum_{j=1}^{M}\left(a_{j+1}^{\dagger}a_j + a_j^{\dagger}a_{j+1}\right), \qquad u \equiv \frac{NU}{K}, \qquad \gamma \equiv \frac{Mu}{N^2}$$

Dimer (M=2): Minimal BHH; Bosonic Josephson junction; Pendulum physics [1].
Driven dimer: Landau-Zener dynamics [2], Kapitza effect [3], Zeno effect [4], Standard-map physics [5].

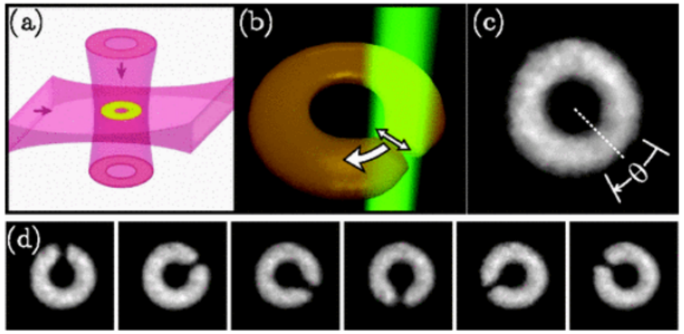
[1] Chuchem, Smith-Mannschott, Hiller, Kottos, Vardi, Cohen (PRA 2010).
[2] Smith-Mannschott, Chuchem, Hiller, Kottos, Cohen (PRL 2009).
[3] Boukobza, Moore, Cohen, Vardi (PRL 2010).
[4] Khripkov, Vardi, Cohen (PRA 2012); Shapira, Cohen (PRE 2017).
[5] Khripkov, Cohen, Vardi (JPA 2013, PRE 2013).

Trimer (M=3): Minimal model for low-dimensional chaos; Coupled pendula physics.
Triangular trimer (M=3): Minimal model with topology; Stirring; Superfluidity [6].
Larger rings (M>3) High-dimensional chaos; web of non-linear resonances; quantum dissipation [7].
Coupled subsystems (M>3): Minimal model for Thermalization [8,9].

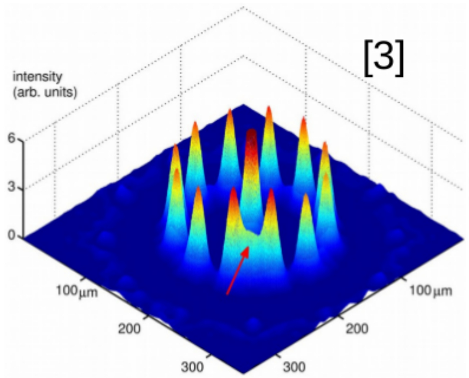
[6] Arwas, Vardi, Cohen (PRA 2014, SREP 2015).
[7] Arwas, Cohen (NJP 2016, PRB 2017); Arwas, Cohen, Hekking, Minguzzi (PRA 2017).
[8] Tikhonenkov, Vardi, Anglin, Cohen, (PRL 2013).
[9] Khripkov, Vardi, Cohen (NJP 2015, arXiv 2017).

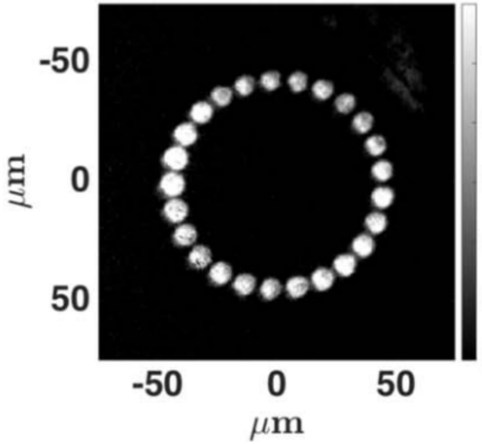
Motivating the interest in Atomtronic circuits

Toroidal ring [1,2]



Discrete ring (optical lattice) [4]





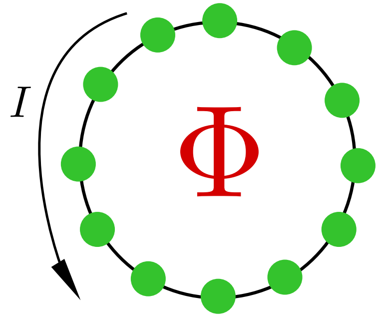
- Recent experiments [1,2,3] have opened a new arena: superfluidity in low dimensional circuits.
- The hallmark of superfluidity is a metastable persistent current: flow-state.
- A stability regime diagram of the flow states in the toroidal ring has been explained by following the reasoning of the Landau superfluidity criterion.
- We claim that a theory for the stability of the flow states in a discrete ring that are described by the Bose-Hubbard Hamiltonian requires a quantum chaos perspective.
- We demonstrate how the stability is affected by non-linear resonances, in regimes where the dynamics is traditionally considered to be stable.

[1] Wright, Blakestad, Lobb, Phillips, Campbell (PRL 2013)
[2] Ekel, Lee, Jendrzeejewski, Murray, Clark, Lobb, Phillips, Edwards, Campbell (Nature 2014)
[3] Amico, Aghamalyan, Auksztol, Crepaz, Dumke, Kwek (Sci. Rep. 2014)
[4] Gauthier, Lenton, Parry, Baker, Davis, Rubinsztein-Dunlop, Neely (Optica 2016)

Superfluidity in an M site ring

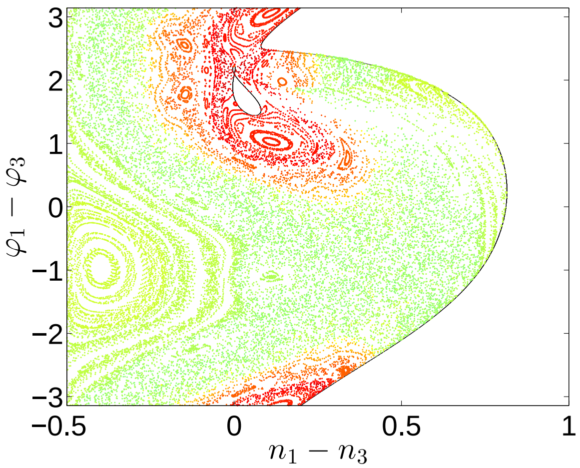
The system consists of N bosons in M sites. Later we add a gauge-field Φ.

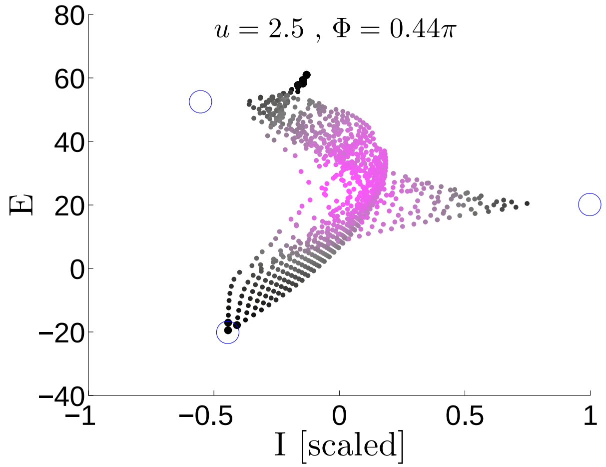
$$\mathcal{H} \;=\; \sum_{j=1}^{M}\left[\frac{U}{2}a_j^\dagger a_j^\dagger a_j a_j - \frac{K}{2}\left(a_{j+1}^\dagger a_j + a_j^\dagger a_{j+1}\right)\right]$$

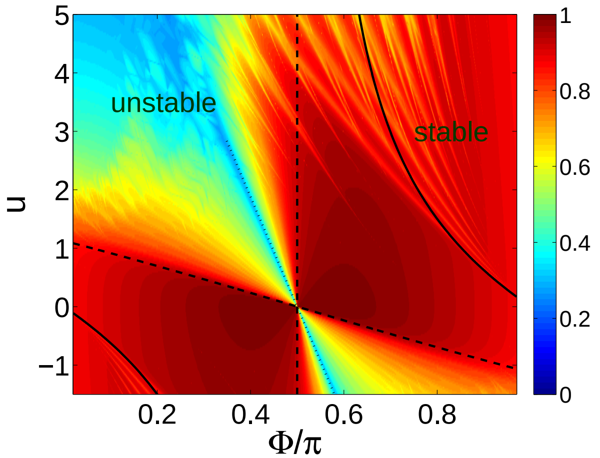


This is like M coupled oscillators with

$$H \;=\; \sum_{j=1}^{M}\left[\frac{U}{2}\boldsymbol{n}_j^2 - K\sqrt{\boldsymbol{n}_{j+1}\boldsymbol{n}_j}\,\cos\left(\boldsymbol{\varphi}_{j+1}-\boldsymbol{\varphi}_j\right)\right]$$



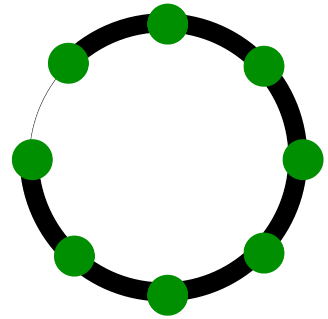




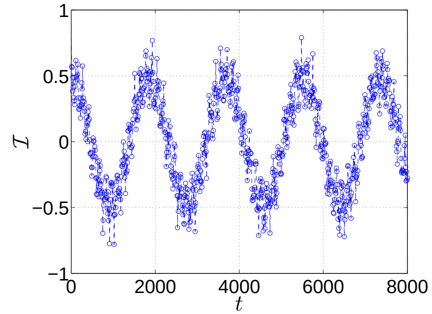
- Arwas, Cohen [**Physical Review B 2017**]
- Arwas, Vardi, Cohen [**Scientific Reports 2015**]

SQUID-like geometry (weak link or barrier)

Coherent Rabi oscillations:



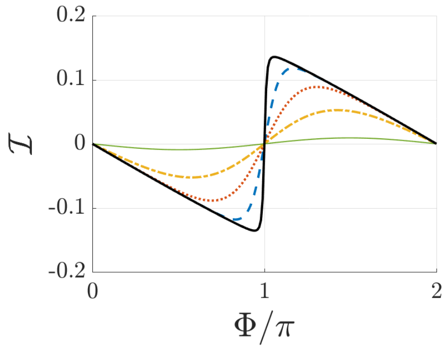
- The hallmark of coherence is Rabi oscillation between flow-states.
- Ohmic-bath perspective $\rightsquigarrow$ $\eta = (\pi/\sqrt{\gamma})$
- Feasibility of Rabi oscillation for $M < 6$ devices.
- Feasibility of of chaos-assisted Rabi oscillation.

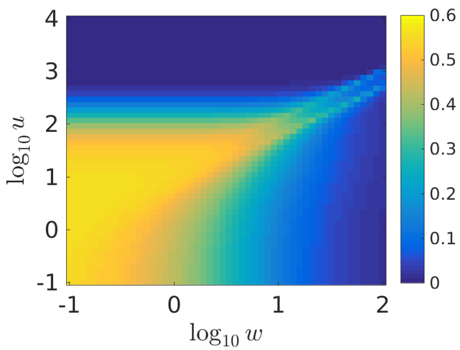


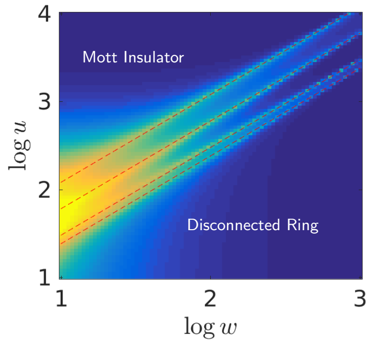
Resonant persistent currents:

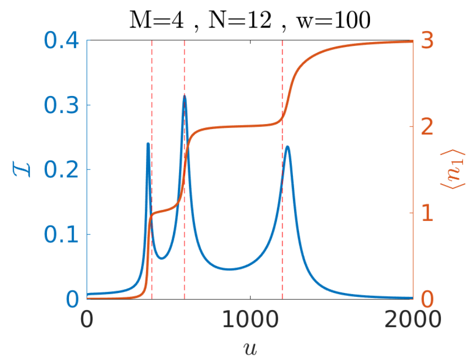
- Without optical lattice - non monotonic dependence on u
- With optical lattice - Mott transition; Quantum resonances

$$\max\left[I(\Phi)\right] \quad = \quad 2J\frac{N}{M}\;\alpha(w,u)$$







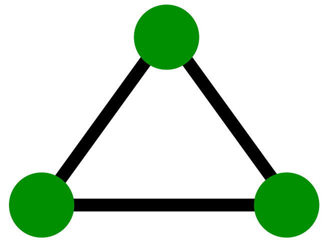


- Arwas, Cohen [**New Journal of Physics 2016**]
- Arwas, Cohen, Hekking, Minguzzi [**PRA 2017**] - **Editors' Suggestion**

Quantum Chaos perspective on Metastability and Ergodicity

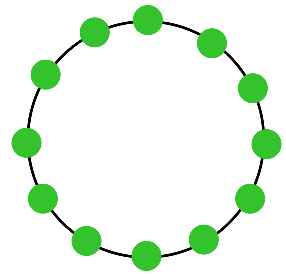
Stability of flow-states (I):

- Landau stability of flow-states ("Landau criterion")
- Bogoliubov perspective of dynamical stability
- KAM perspective of dynamical stability



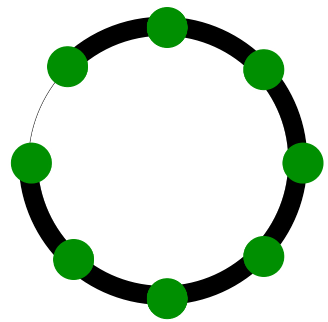
Stability of flow-states (II):

- Considering high dimensional chaos ($M > 3$).
- Web of non-linear resonances.
- Irrelevance of the the familiar Beliaev and Landau damping terms.
- Analysis of the quench scenario.



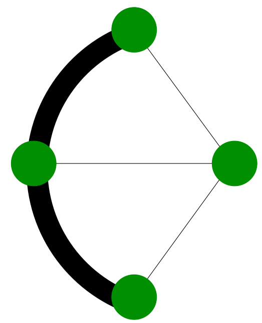
Coherent Rabi oscillations:

- The hallmark of coherence is Rabi oscillation between flow-states.
- Ohmic-bath perspective $\rightsquigarrow$ $\eta = (\pi/\sqrt{\gamma})$
- Feasibility of Rabi oscillation for $M < 6$ devices.
- Feasibility of of chaos-assisted Rabi oscillation.



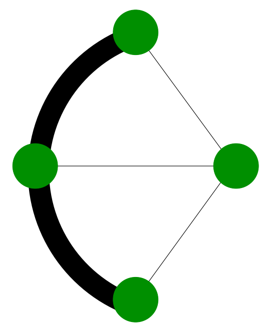
Thermalization:

- Spreading in phase space is similar to Percolation.
- Resistor-Network calculation of the diffusion coefficient.
- Observing regions with Semiclassical Localization.
- Observing regions with Dynamical Localization.

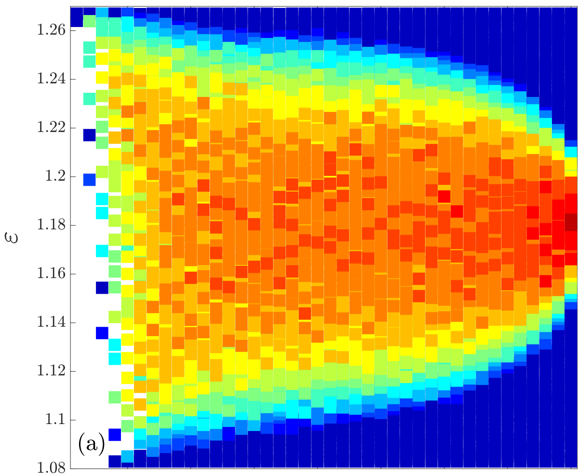


Dynamical localization

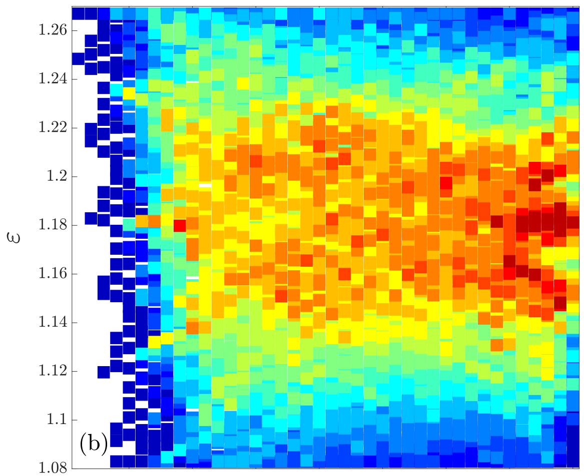
N = total number of particles = 60.
x = trimer population.



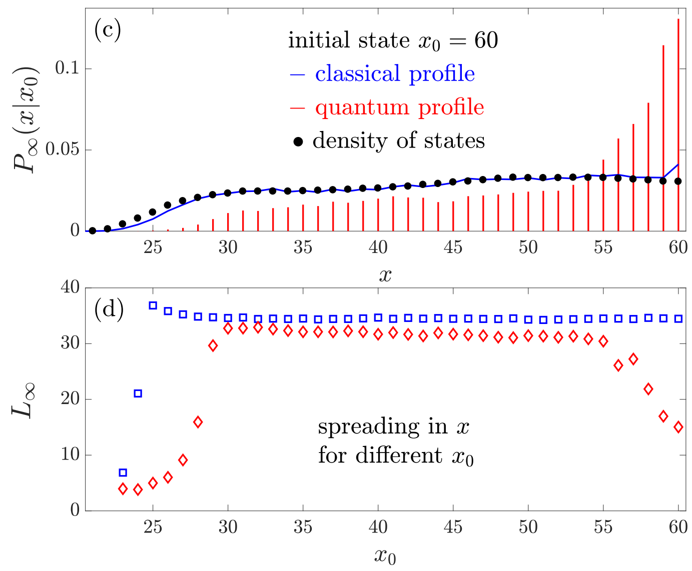
Here we start the simulation with $x_0 = 60$,
meaning that initially all the particles are in the trimer.
We plot the saturation profile $P_\infty(x, \varepsilon)$.



Classical



Quantum

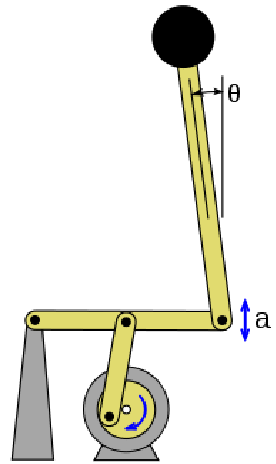


- Khripkov, Vardi, Cohen [**arXiv 2017**]
- Khripkov, Vardi, Cohen [**New Journal of Physics 2016**]

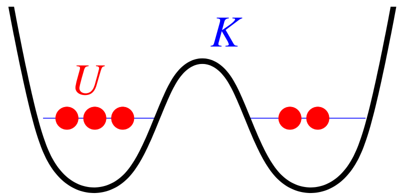
Stability and stabilization of unstable condensates

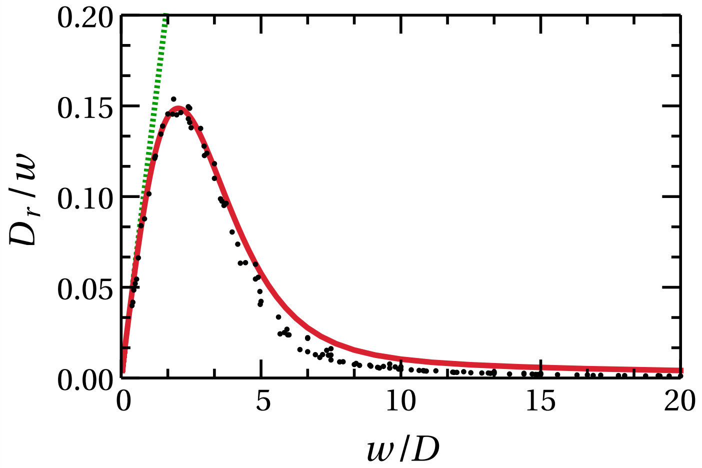
The BHH for a dimer is like a pendulum.
Options to stabilize BEC in the upper orbital:

- Kapitza: Introduce periodic driving $f(t) \propto \sin(\Omega t)$
- Watch the pendulum...
  successive collapses of the wavefunction...
  quantum(?) Zeno effect
- Zeno: Introduce noisy driving $\overline{f(t)f(t')} = 2D\delta(t-t')$



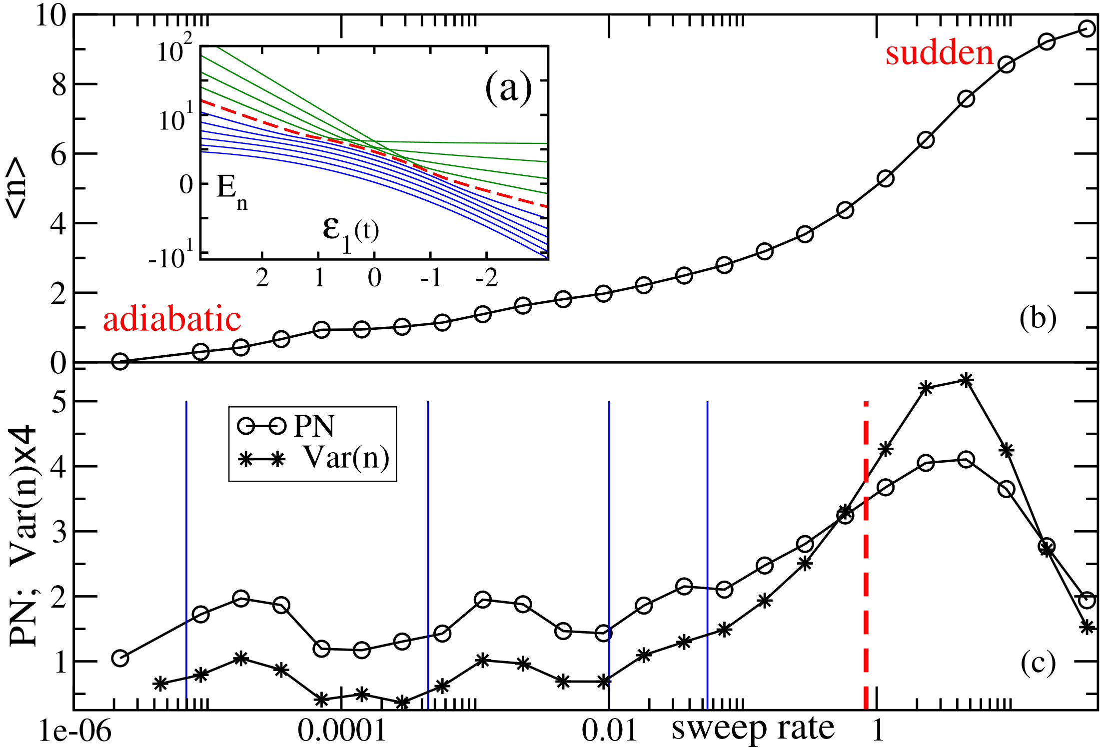
To watch the pendulum is formally like introducing noise.
Using a semiclassical perspective we show that it is a noisy squeeze process.
Mathematical analysis of the induced lognormal-like spreading.

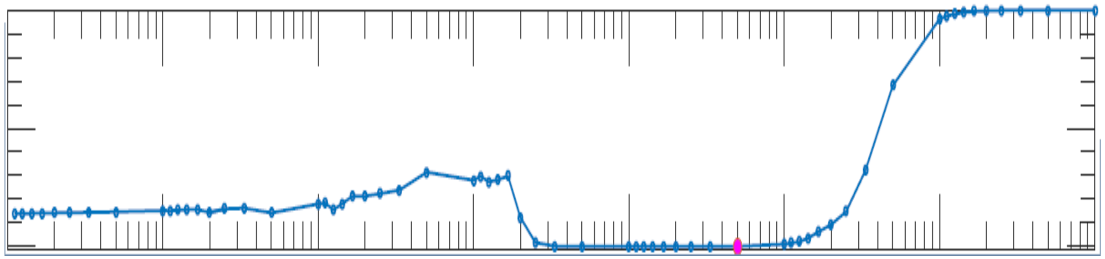


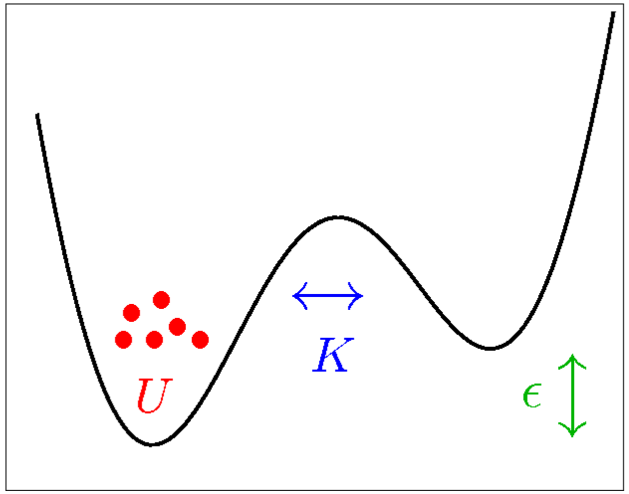


Boukobza, Moore, Cohen, Vardi [**PRL 2010**] - Kapitza
Khripkov, Vardi, Cohen [**PRA 2012**] - Zeno
Shapira, Cohen [**PRE 2017**] - Analytic solution

Sweep through chaos

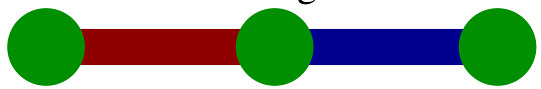






Bosonic Junction

STIRAP through chaos



- Smith-Mannschott, Chuchem, Hiller, Kottos, Cohen [**PRL 2009**] - Many body LZ crossing
- Dey, Cohen, Vardi [**in preparation 2017**] - Stimulated Raman adiabatic passage through chaos