Quantum pumping and dissipation in closed systems

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cond-mat archive

$GIF, $ISF

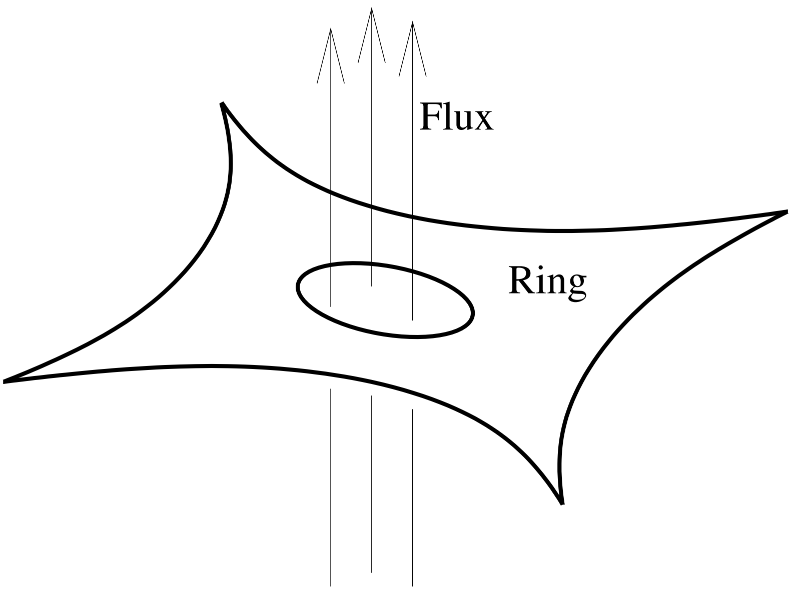
Driven Systems

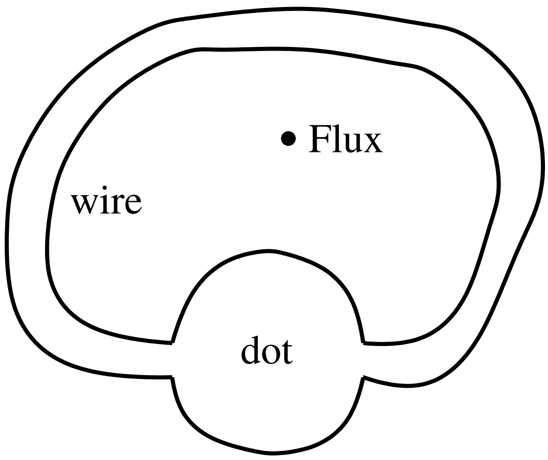
Non interacting "spinless" electrons.

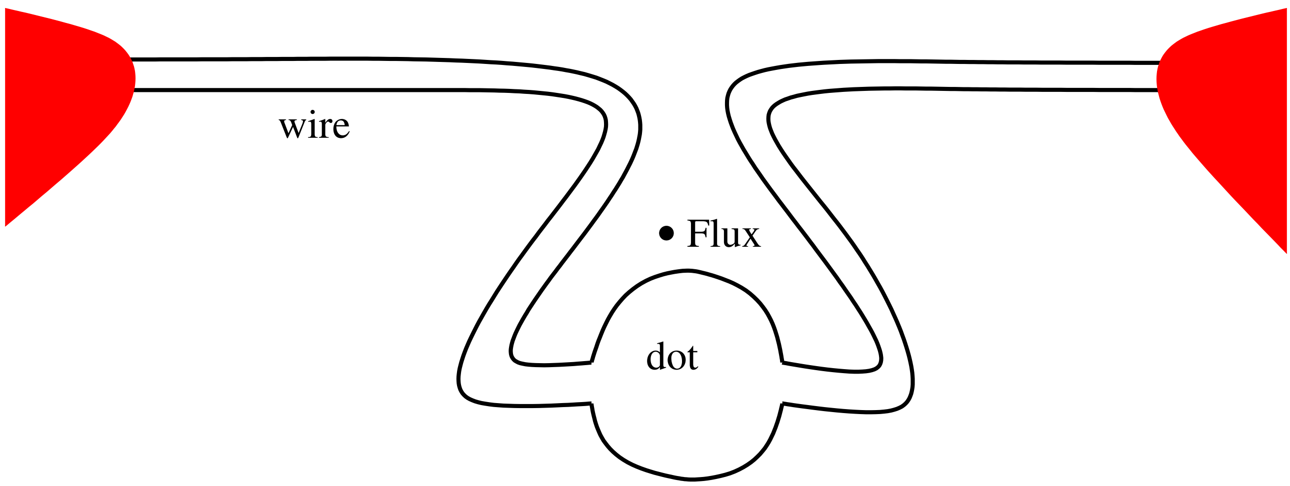
Held by a potential (e.g. AB ring geometry).

$x_1,\ x_2 \ = \ $ shape parameters

$x_3 \ = \ \Phi \ = \ (\hbar/e)\phi \ = \ $ magnetic flux







"Ohm law"

For one parameter driving by EMF

$$I = \boldsymbol{G}^{33} \times (-\dot{x}_3)$$
$$dQ = -\boldsymbol{G}^{33}\ dx_3$$

For driving by changing another parameter

$$I = -\boldsymbol{G}^{31}\ \dot{x}_1$$
$$dQ = -\boldsymbol{G}^{31}\ dx_1$$

For two parameter driving

$$I = -\boldsymbol{G}^{31}\dot{x}_1 - \boldsymbol{G}^{32}\dot{x}_2$$
$$dQ = -\boldsymbol{G}^{31}\ dx_1 - \boldsymbol{G}^{32}\ dx_2$$
$$Q = -\oint \boldsymbol{G} \cdot dx$$

and in general

$$\langle \mathcal{F}^k \rangle = -\sum_j \boldsymbol{G}^{kj}\ \dot{x}_j$$

What is the problem?

From Kubo formula we get
a formal expression for $\boldsymbol{G}^{kj}$.

Can we trust this expression? Conditions?
Quantum chaos!

How to use this expression?
The bare Kubo formula gives no dissipation!

To define an energy scale Γ
Beyond first order perturbation theory!

Γ in case of isolated system is due to
non-adiabaticity.

Γ affects both the dissipative and the
non-dissipative (geometric) part of the response.

Some references

Adiabatic transport

Thouless (PRL 1983) - Periodic arrays

Avron, Sadun, Raveh, Zur (1988) - Networks with fluxes

Berry, Robbins (JPA 1993) - Geometric magnetism

Linear response theory and Mesoscopics

Imry, Shiren (PRB 1986) - Using Kubo for a closed ring

Wilkinson, Austin (JPA 1995) - Challenging the validity

DC (PRL 1999) - The "quantum chaos" identification of regimes

DC and Kottos (PRL 2000) - The (A, Ω) regimes diagram

DC (PRB+Rapid 2003) - the Kubo approach to pumping

DC, Kottos, Schanz (cond-mat) - pumping on networks

Sela, DC (in preperation) - pumping on a ring

Open systems, S matrix formalism

The Landauer / Landauer-Buttiker formula (1970,1986)

Fisher, Lee, Baranger, Stone (1981,1989)

The Buttiker Pretre Thomas [BPT] formula (1994)

Brouwer (1998)

Avron, Elgart, Graf, Sadun

Buttiker, Texier, Moskalets

Marcus - experiments

Shutenko, Aleiner, Altshuler (PRB 2000) - quantization?

Entin-Wohlman, Aharony, Levinson (2002) - two delta functions

Linear response theory

$$\mathcal{H} \quad = \quad \mathcal{H}(\boldsymbol{r},\boldsymbol{p};\ x_1(t), x_2(t), x_3(t))$$

$$\mathcal{F}^k \quad = \quad -\frac{\partial \mathcal{H}}{\partial x_k}$$

$$\langle \boldsymbol{F} \rangle_t \ = \int \boldsymbol{\alpha}(t-t')\ \delta\boldsymbol{x}(t')\ dt'$$

$$\alpha^{kj}(t-t')$$

$$\downarrow$$

$$\chi^{kj}(\omega)$$

$$\swarrow \qquad \searrow$$

$$\mathrm{Re}[\chi^{kj}(\omega)] \qquad\qquad (1/\omega) \times \ \mathrm{Im}[\chi^{kj}(\omega)]$$

$$\downarrow$$

$$\boldsymbol{G}^{kj}$$

$$\swarrow \qquad \searrow$$

$$\boldsymbol{\eta}^{kj} \qquad\qquad \boldsymbol{B}^{kj}$$

(dissipative) (non-dissipative)

$$\langle \mathcal{F}^k \rangle \quad = \quad -\sum_j \boldsymbol{G}^{kj}\ \dot{x}_j$$

From Kubo to a "FD relation"

$$\mathcal{H} \;=\; \mathcal{H}(\boldsymbol{r},\boldsymbol{p};\; x_1(t), x_2(t), x_3(t))$$

$$\mathcal{F}^k \;=\; -\frac{\partial \mathcal{H}}{\partial x_k}$$

Generalized Ohm law:

$$\langle \mathcal{F}^k \rangle = -\sum_j \boldsymbol{G}^{kj} \; \dot{x}_j$$

$$K^{kj}(\tau) \;=\; \frac{i}{\hbar}\langle[\mathcal{F}^k(\tau), \mathcal{F}^j(0)]\rangle$$

$$C^{kj}(\tau) \;=\; \frac{1}{2}\Big(\langle \mathcal{F}^k(\tau)\mathcal{F}^j(0)\rangle + cc\Big)$$

$$\boldsymbol{G}^{kj} \;=\; \lim_{\omega\rightarrow 0}\frac{\mathrm{Im}[\chi^{kj}(\omega)]}{\omega} \;=\; \int_0^\infty K_F^{kj}(\tau)\tau d\tau$$

$$=\; \mathsf{g}(E_F)\; \int_0^\infty C_{E_F}^{kj}(\tau)d\tau$$

DC, PRB(R) 2003

BPT versus Geometric magnetism

$$\boldsymbol{G}^{kj} \;=\; \mathsf{g}(E_F)\int_0^\infty C^{kj}_{E_F}(\tau)d\tau$$

from now on k=3, j=1,2

Due to non-adiabaticity

$$C^{kj}_E(\tau) \;\mapsto\; C^{kj}_E(\tau)\,\mathrm{e}^{-(\Gamma/\hbar)t}$$

$$\Gamma \;=\; \left(\frac{\hbar\sigma}{\Delta^2}|\dot{x}|\right)^{2/3} \times\ \Delta \quad\sim\quad \left(L\,|\dot{x}|\right)^{2/3}\frac{1}{L}$$

In the absence of magnetic field one obtains

$$\boldsymbol{G}^{3j} \;=\; 2\hbar\sum_n f(E_n)\sum_{m(\neq n)}\frac{\mathrm{Im}\left[\mathcal{I}_{nm}\mathcal{F}_{mn}\right]}{(E_m-E_n)^2+(\Gamma/2)^2}$$

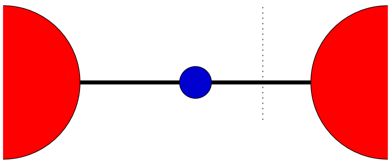
For dot-wire system in the $L\rightarrow\infty$ limit

we have $\Delta \ll \Gamma \rightarrow 0$ leading to

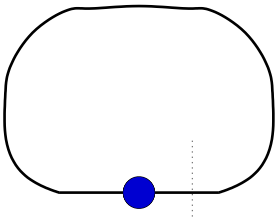
$$\boldsymbol{G}^{3j} \;=\; \frac{e}{2\pi i}\mathrm{trace}\left(P_{\mathrm{A}}\frac{\partial S}{\partial x_j}S^\dagger\right)$$

[BPT]

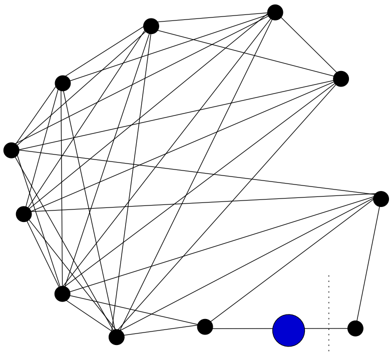
Simple model systems - networks



$$dQ = (1-g_0) \times \frac{e}{\pi} k_F \times dX$$



$$dQ = 1 \times \frac{e}{\pi} k_F \times dX$$



$$dQ = \left[\frac{g_T}{1-g_T}\right] \left[\frac{1-g_0}{g_0}\right] \times \frac{e}{\pi} k_F \times dX$$

Adiabatic versus non-adiabatic result

DC, Kottos, Schanz, cond-mat 2004

The network Hamiltonian

$$\mathcal{H} \quad = \quad \text{network} \quad + \quad \lambda \frac{\hbar^2}{2\mathsf{m}}\, \delta(x - X_0)$$

$$\mathsf{g}(E) \quad = \quad \text{density of states}$$

$$g_0 \quad = \quad \tfrac{1}{1+(\lambda/(2k_F))^2} \quad = \quad \text{transmission}$$

$$\mathcal{I} \quad = \quad \frac{e}{2\mathsf{m}} \left(\delta(x - X_1)p + p\delta(x - X_1)\right)$$

$$\mathcal{F} \quad = \quad -\frac{\partial \mathcal{H}}{\partial X_0} \quad = \quad \lambda \frac{\hbar^2}{2\mathsf{m}} \delta'(x - X_0)$$

$$C(\tau) \quad = \quad \text{cross correlation function of } \mathcal{I} \text{ and } \mathcal{F}$$

$$\mathcal{I} \quad \mapsto \quad -i\frac{e\hbar}{2\mathsf{m}} \left(\overrightarrow{\partial} - \overleftarrow{\partial}\right)_{x=X_1}$$

$$\mathcal{F} \quad \mapsto \quad -\lambda \frac{\hbar^2}{2\mathsf{m}} \left(\overrightarrow{\partial} + \overleftarrow{\partial} - \lambda\right)_{x=X_0+0}$$

Kubo - "classical" calculation

$$\boldsymbol{G}^{IF} = \mathsf{g}(E_F) \int_0^\infty C(\tau) d\tau$$

$$C(\tau) = e\frac{v_{\mathrm{F}}}{2L} 2\mathsf{m} v_{\mathrm{F}} \left[(1-g_0) \sum_{\pm} \pm \delta(\tau \pm \tau_1) \right] + \ldots.$$

$$\tau_1 = (X_1 - X_0) \;/\; v_{\mathrm{F}}$$

$$\int_0^{\mathrm{short}} C(\tau) d\tau = -e\frac{\mathsf{m} v_{\mathrm{F}}^2}{L} [1 - g_0]$$

$$\int_0^\infty C(\tau) d\tau = -e\frac{\mathsf{m} v_{\mathrm{F}}^2}{L} \left[\frac{1-g_0}{g_0} \right] \left[\frac{g_T}{1-g_T} \right]$$

$$\boldsymbol{G}^{IF} = -(1-g_0) \times \frac{e}{\pi} k_{\mathrm{F}}$$

$$\boldsymbol{G}^{IF} = -\frac{e}{\pi} k_{\mathrm{F}} \left[\frac{1-g_0}{g_0} \right] \left[\frac{g_T}{1-g_T} \right]$$

Kubo - "diagonal" quantum calculation

$$\boldsymbol{G}^{IF} = \mathsf{g}(E_F) \int_0^{\infty} C(\tau) d\tau$$

$$C(\tau) \mapsto C(\tau)\, \mathrm{e}^{-(\Gamma/\hbar)t}$$

$$\boldsymbol{G}^{IF} = \hbar \mathsf{g}(E_F) \int_{-\infty}^{\infty} \frac{-i\tilde{C}(\omega)}{\hbar\omega - i(\Gamma/2)} \frac{d\omega}{2\pi}$$

$$= \frac{-i\hbar}{4\pi} \int_{-\infty}^{\infty} \frac{C(E_F+\hbar\omega, E_F)+C(E_F-\hbar\omega, E_F)}{\omega + i(\Gamma/2)} d\omega$$

$$C(E', E) = \frac{2}{\pi} \operatorname{trace}\left[\mathcal{I}\ \mathrm{Im}[\mathsf{G}(E')]\ \mathcal{F}\ \mathrm{Im}[\mathsf{G}(E)]\right]$$

$$\langle x|\mathsf{G}(E)|x_0\rangle = -\frac{i}{\hbar v_F} \sum_p A_p \mathrm{e}^{ik_E L_p}$$

$$\boldsymbol{G}^{IF} = -\frac{e}{\pi} k_F \frac{1-g_0}{g_0} \sum_p s_p^+ s_p^- |A_p|^2 \mathrm{e}^{L_p/L_\Gamma}$$

Kubo - numerical quantum calculation

$$\boldsymbol{G}^{IF} = g(E_F)\int_0^{\infty} C(\tau)d\tau$$

same steps as before...

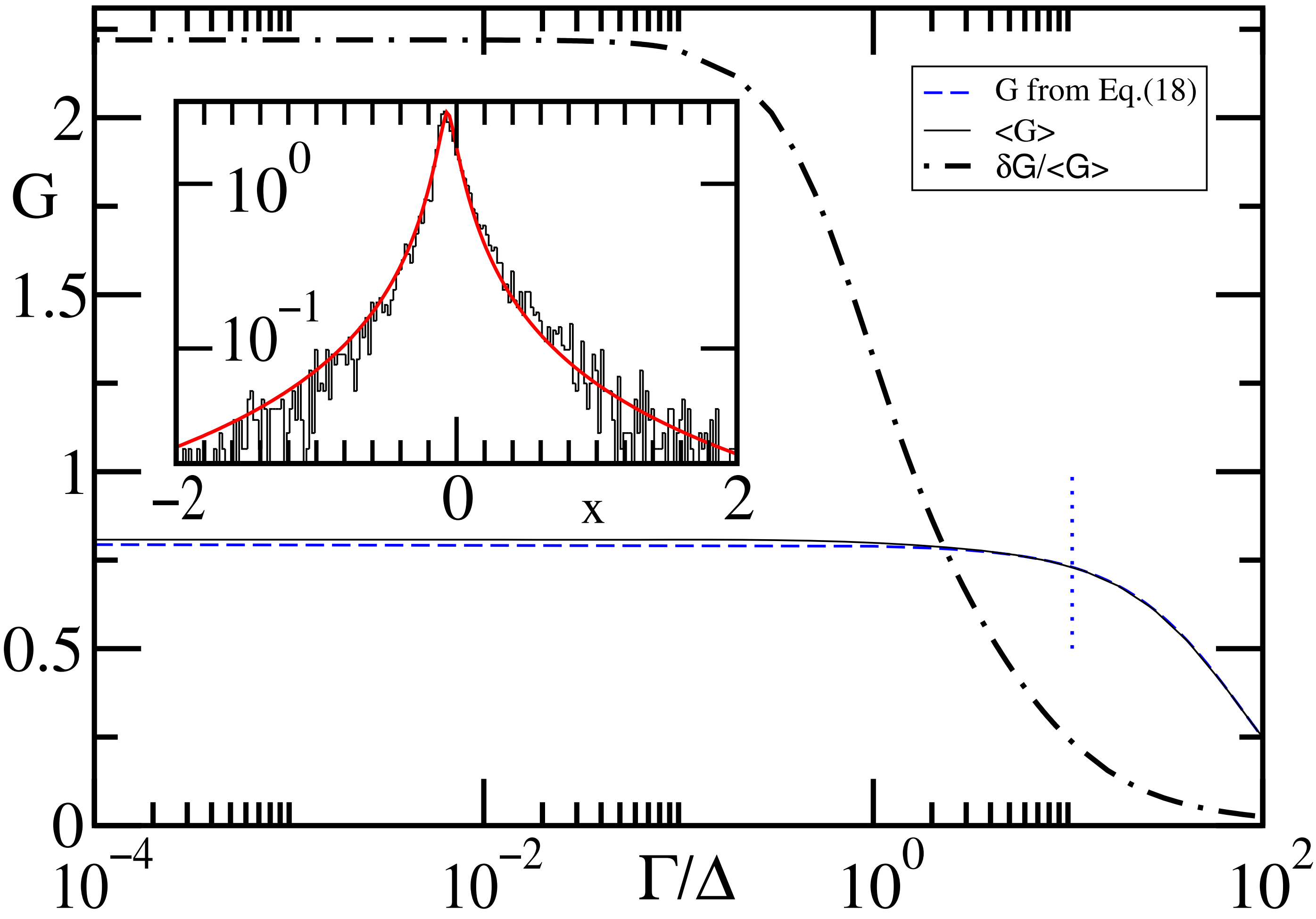
$$C(E',E) \;=\; 2\pi\sum_{nm}\mathcal{I}_{nm}\delta(E'-E_m)\mathcal{F}_{mn}\delta(E-E_n)$$

$$\boldsymbol{G}^{IF} \;=\; 2\hbar\sum_{n} f(E_n)\sum_{m(\neq n)}\frac{\mathrm{Im}\left[\mathcal{I}_{nm}\mathcal{F}_{mn}\right]}{(E_m-E_n)^2+(\Gamma/2)^2}$$

$$\mathcal{I}_{nm} = -i\frac{e\hbar}{2\mathsf{m}}\left(\psi^n\partial\psi^m - \partial\psi^n\psi^m\right)_{x=X_1}$$

$$\mathcal{F}_{nm} = -\lambda\frac{\hbar^2}{2\mathsf{m}}\left(\psi^n\partial\psi^m + \partial\psi^n\psi^m - \lambda\psi^n\psi^m\right)_{x=X_0+0}$$

numerics



Summary

- LRT gives a unified framework for the theory of quantum pumping.
- Quantum chaos considerations are essential.
- Distinction between adiabatic, non-adiabatic and non-prt regimes.
- We have a generalized FD relation for pumping calculation.

$$\boldsymbol{G}^{kj} = \mathrm{g}(E_F) \int_0^{\infty} C^{kj}(\tau)\ \mathrm{e}^{-(\Gamma/2\hbar)\tau} d\tau$$

- It reduces to geometric magnetism for $\Gamma = 0$.

  $\Gamma = 0$ means strict adiabaticity and coherence

- It reduces to the BPT formula in case of a dot-wire geometry.

  This assumes the non-adiabatic limit $L \to \infty$

$$\boldsymbol{G}^{3j} = \frac{e}{2\pi i} \mathrm{trace}\left(P_{\mathrm{A}} \frac{\partial S}{\partial x_j} S^{\dagger}\right) \qquad \text{[BPT]}$$

- We have analyzed pumping on networks using Green function expressions.
- The diagonal approximation agrees with classical stochastic modeling.
- Our expression for dQ depends on both g_0 and g_T.
- The dispersion ("UCF") of G decreases with Γ.

$$\boldsymbol{G}^{3j} = \left[\frac{g_T}{1-g_T}\right] \left[\frac{1-g_0}{g_0}\right] \times \frac{e}{\pi} k_{\mathrm{F}}$$