From classical pumps of water to quantum pumping of electrons in closed devices

Doron Cohen, Ben-Gurion University

Collaborations:

Tsampikos Kottos (Gottingen)
Holger Schanz (Gottingen)
Itamar Sela (BGU)

Discussions:

Yshai Avishai (BGU)
Markus Buttiker (Geneva)
Thomas Dittrich (Colombia)
Misha Moskalets (Kharkov)

http://www.bgu.ac.il/~dcohen
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$GIF, $ISF

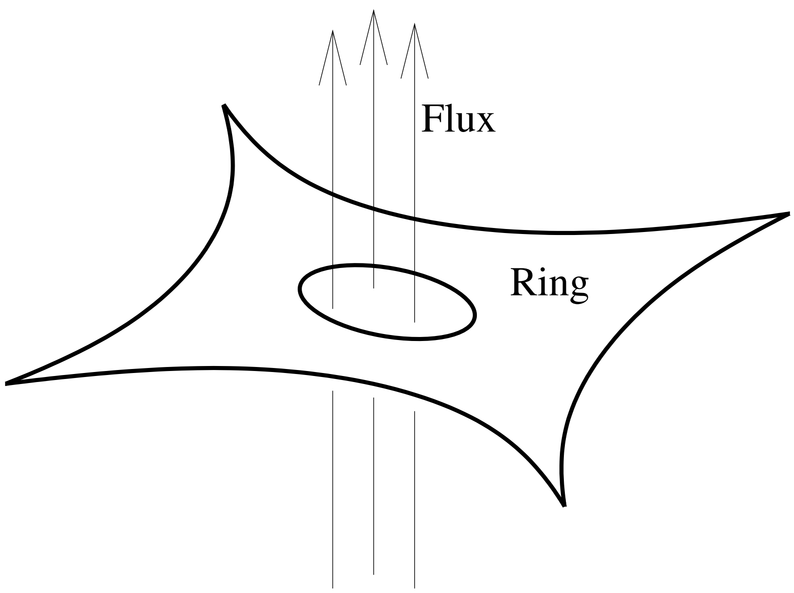
Driven Systems

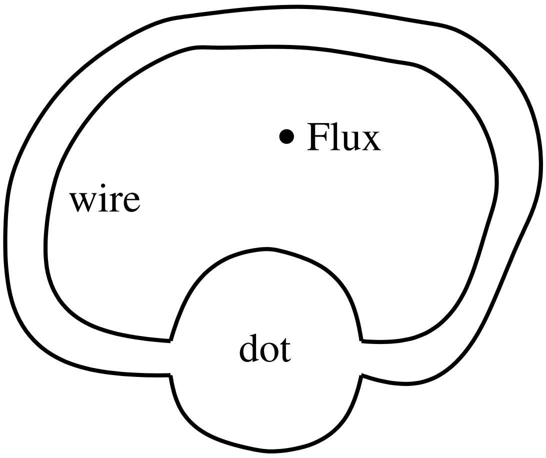
Non interacting "spinless" electrons.

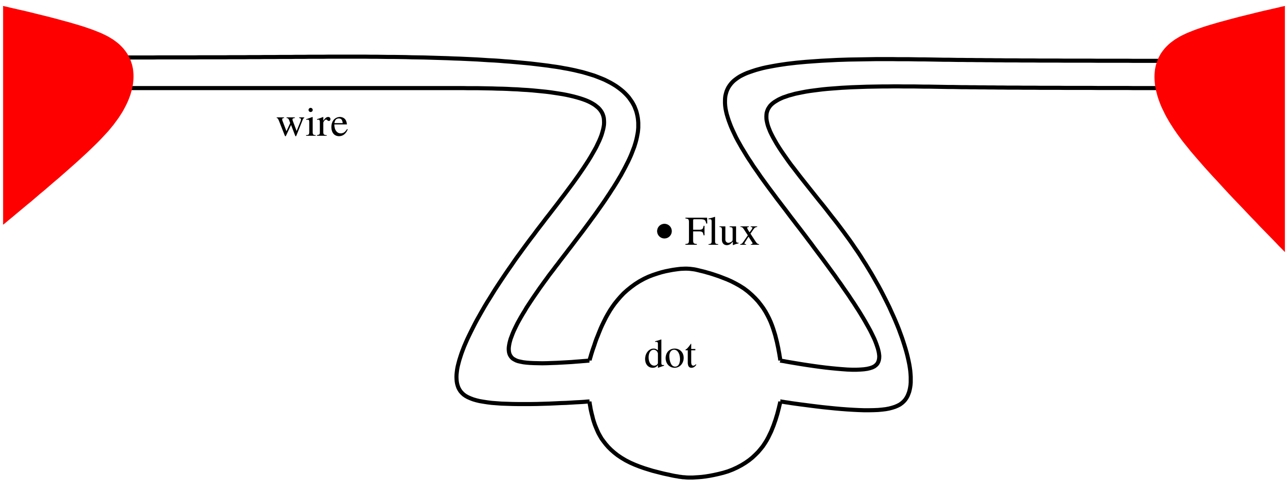
Held by a potential (e.g. AB ring geometry).

$x_1, \; x_2 \;\; = \;\;$ shape parameters

$x_3 \; = \; \Phi \; = \; (\hbar/e)\phi \; = \;$ magnetic flux







“Ohm law”

For one parameter driving by EMF

$$I = \boldsymbol{G}^{33} \times (-\dot{x}_3)$$
$$dQ = -\boldsymbol{G}^{33}\ dx_3$$

For driving by changing another parameter

$$I = -\boldsymbol{G}^{31}\ \dot{x}_1$$
$$dQ = -\boldsymbol{G}^{31}\ dx_1$$

For two parameter driving

$$I = -\boldsymbol{G}^{31}\dot{x}_1 - \boldsymbol{G}^{32}\dot{x}_2$$
$$dQ = -\boldsymbol{G}^{31}\ dx_1 - \boldsymbol{G}^{32}\ dx_2$$
$$Q = -\oint \boldsymbol{G} \cdot dx$$

and in general

$$\langle F^k \rangle = -\sum_j \boldsymbol{G}^{kj}\ \dot{x}_j$$

Linear response theory

$$\mathcal{H} \quad = \quad \mathcal{H}(\boldsymbol{r}, \boldsymbol{p};\ x_1(t), x_2(t), x_3(t))$$

$$F^k \quad = \quad -\frac{\partial \mathcal{H}}{\partial x_k}$$

$$\langle \boldsymbol{F} \rangle_t \ = \int \boldsymbol{\alpha}(t - t')\ \delta \boldsymbol{x}(t')\ dt'$$

$$\alpha^{kj}(t - t')$$

$$\downarrow$$

$$\chi^{kj}(\omega)$$

$$\swarrow \qquad \searrow$$

$$\mathrm{Re}[\chi^{kj}(\omega)] \qquad\qquad (1/\omega) \times \ \mathrm{Im}[\chi^{kj}(\omega)]$$

$$\downarrow$$

$$\boldsymbol{G}^{kj}$$

$$\swarrow \qquad \searrow$$

$$\boldsymbol{\eta}^{kj} \qquad\qquad \boldsymbol{B}^{kj}$$

(dissipative) (non-dissipative)

$$\langle F^k \rangle \quad = \quad -\sum_j \boldsymbol{G}^{kj}\ \dot{x}_j$$

What is the problem?

From Kubo formula we get
a formal expression for $\boldsymbol{G}^{kj}$.

Can we trust this expression? Conditions?
Quantum chaos!

How to use this expression?
The bare Kubo formula gives no dissipation!

To define an energy scale Γ
Beyond first order perturbation theory!

Γ in case of isolated system is due to
non-adiabaticity.

Γ affects both the dissipative and the
non-dissipative (geometric) part of the response.

Some references

Adiabatic transport

Thouless (PRL 1983) - Periodic arrays

Avron, Sadun, Raveh, Zur (1988) - Networks with fluxes

Berry, Robbins (JPA 1993) - Geometric magnetism

Linear response theory and Mesoscopics

Imry, Shiren (PRB 1986) - Using Kubo for a closed ring

Wilkinson, Austin (JPA 1995) - Challenging the validity

DC (PRL 1999) - The "quantum chaos" identification of regimes

DC and Kottos (PRL 2000) - The (A, Ω) regimes diagram

DC (PRB+Rapid 2003) - the Kubo approach to pumping

DC, Kottos, Schanz (cond-mat 2004) - pumping on networks

Sela, DC (in preperation) - pumping on a ring

Open systems, S matrix formalism

The Landauer / Landauer-Buttiker formula (1970,1986)

Fisher, Lee, Baranger, Stone (1981,1989)

The Buttiker Pretre Thomas [BPT] formula (1994)

Brouwer (1998)

Avron, Elgart, Graf, Sadun

Buttiker, Texier, Moskalets

Marcus - experiments

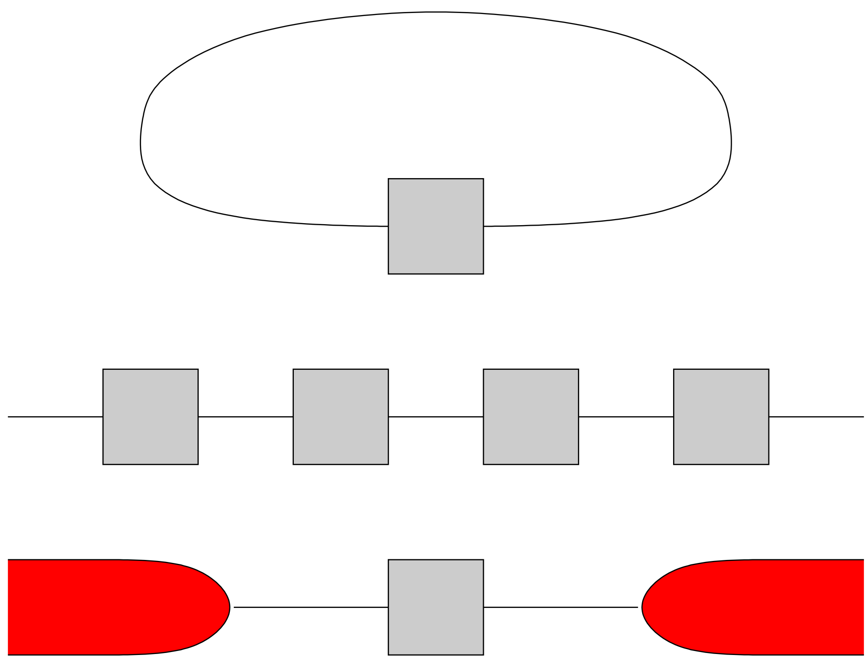
Shutenko, Aleiner, Altshuler (PRB 2000) - quantization?

Entin-Wohlman, Aharony, Levinson (2002) - two delta functions

Driven Systems - classification

$$\mathcal{H} \;=\; \mathcal{H}(\boldsymbol{r},\boldsymbol{p};\; x_1(t), x_2(t), x_3(t))$$

- closed isolated systems
- periodic arrays
- open systems (with reservoirs)
- one of the above interacting with a bath



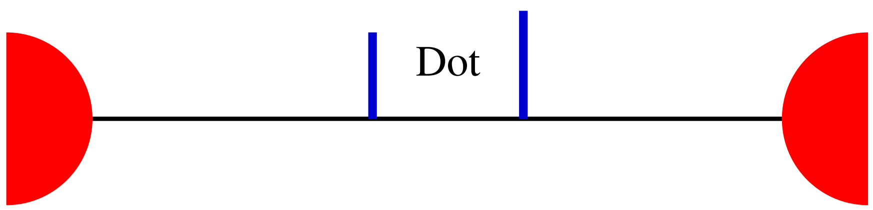
Questions: **Transport? Dissipation?**

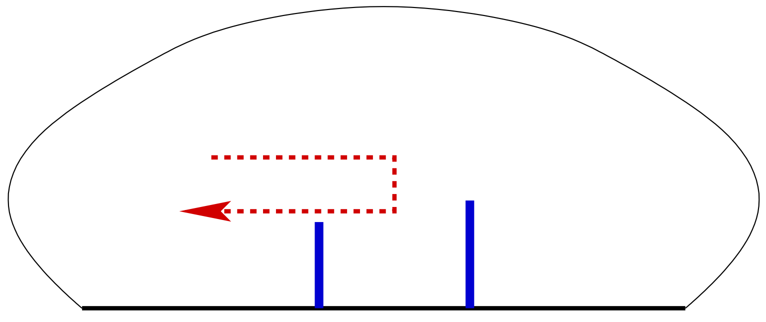
Simple pumping devices

How can we drive current?

- by changing the height of the potential

- by translating the potential

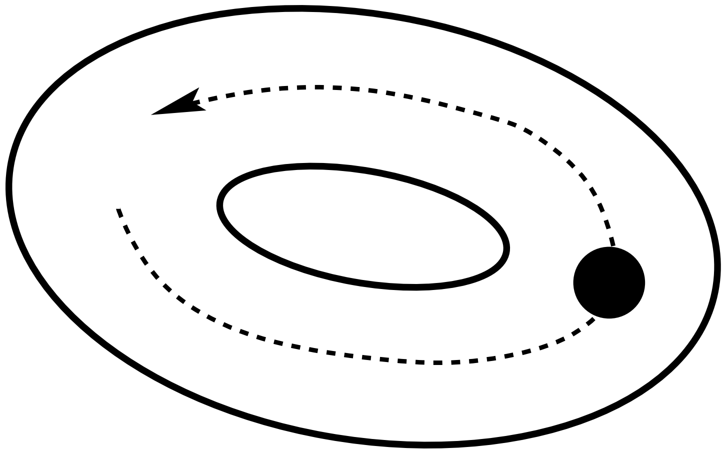




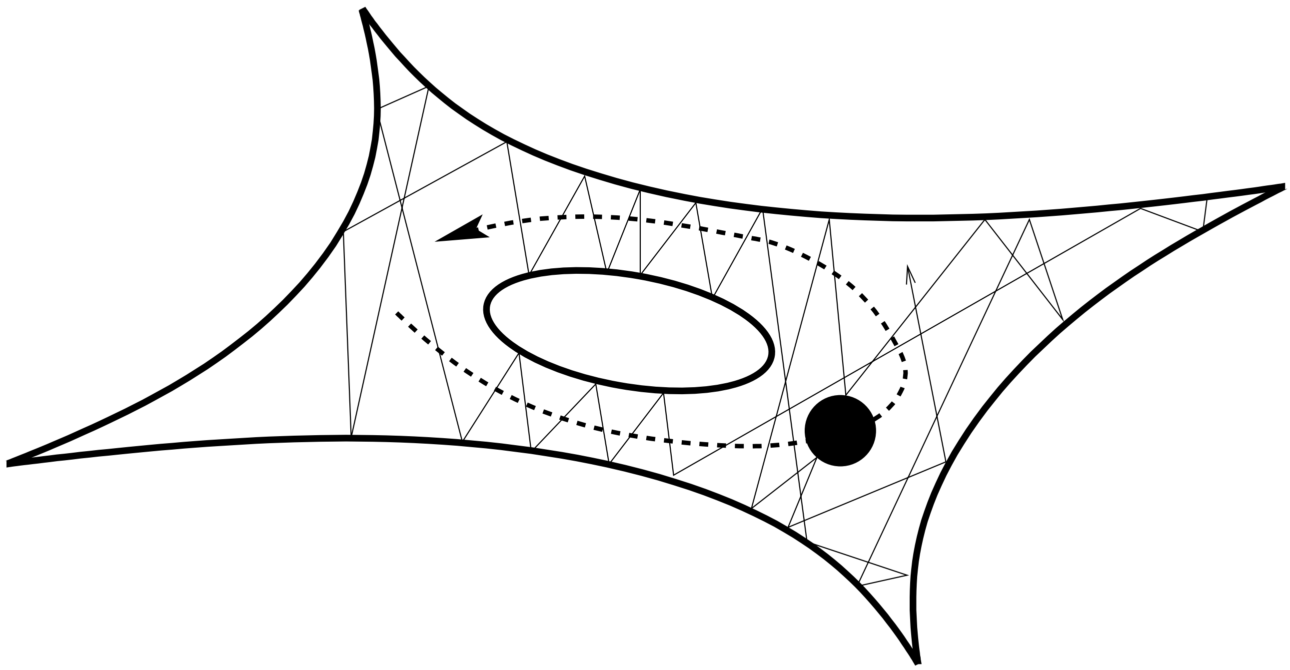
Specific question:

What is the current which is created by translating a scatterer?

The moving scatterer model

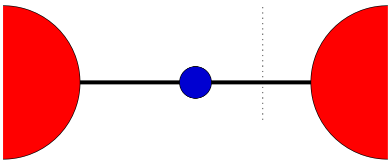


- There is a stationary solution.
- There is no dissipation.
- Pumping: $dQ \propto 1 \times dX$

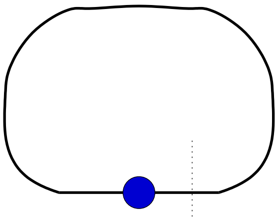


- There is no stationary solution.
- There is dissipation.
- Pumping: $dQ \propto (1 - g_0) \times dX$

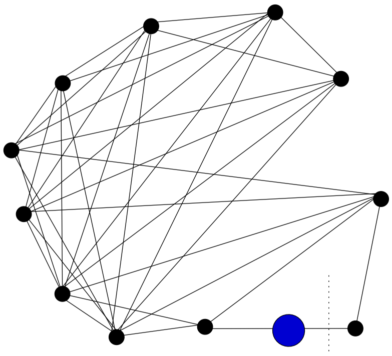
Simple model systems - networks



$$dQ = (1-g_0) \times \frac{e}{\pi} k_{\rm F} \times dX$$



$$dQ = 1 \times \frac{e}{\pi} k_{\rm F} \times dX$$



$$dQ = \left[\frac{g_T}{1-g_T}\right] \left[\frac{1-g_0}{g_0}\right] \times \frac{e}{\pi} k_{\rm F} \times dX$$

Adiabatic versus non-adiabatic result

DC, Kottos, Schanz, cond-mat 2004

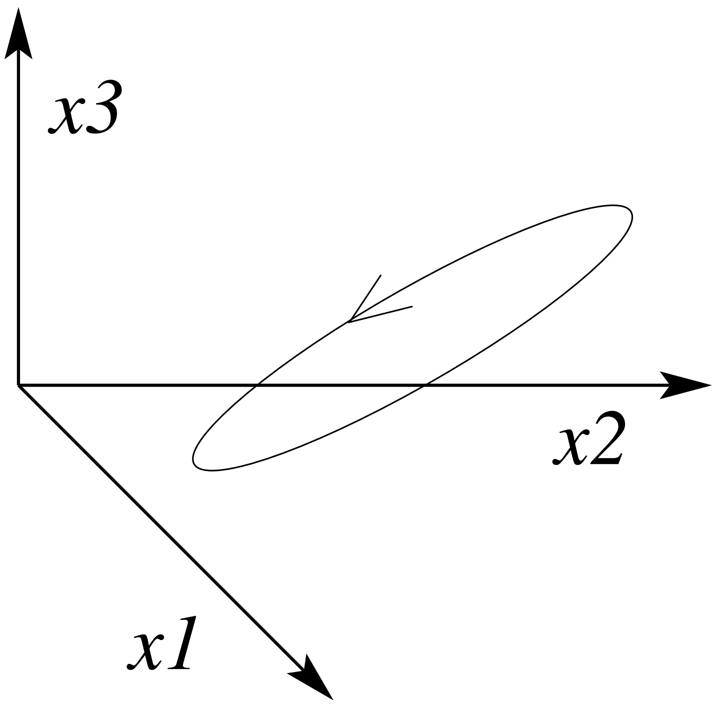
Driven Systems - pumping

Assume periodic ("AC") driving.

Does the current have a "DC" component?

Define charge transported per cycle:

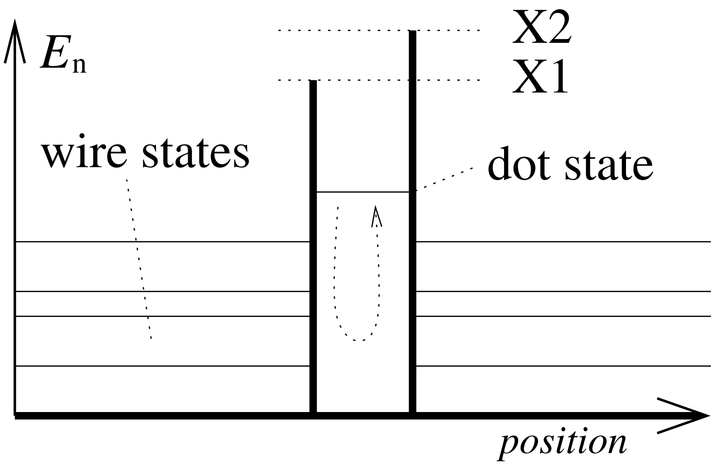
$$Q = \oint I dt = ???$$

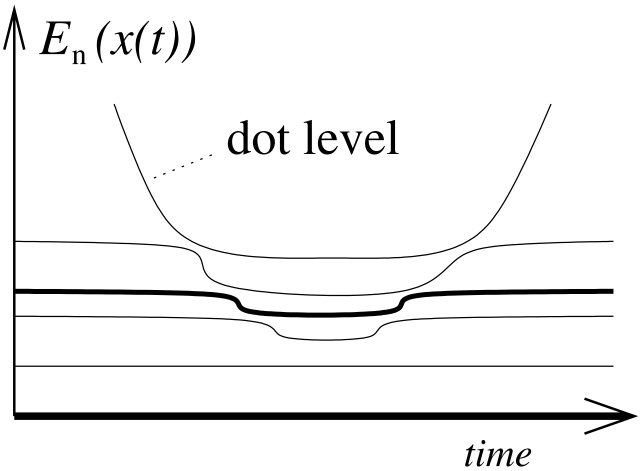


Linear response assumption $\Longrightarrow$
for one parameter driving $Q = 0$.

Ratchests are non-linear devices that use mixed or damped dynamics in order to pump with only one parameter.

The two barrier model - Speculations...





For the open system, using BPT:

$$Q \approx 1 - g$$

- Is it due to non-adiabaticity?
- Is it a dissipative contribution?

What about strict adiabatic cycle in a closed system?

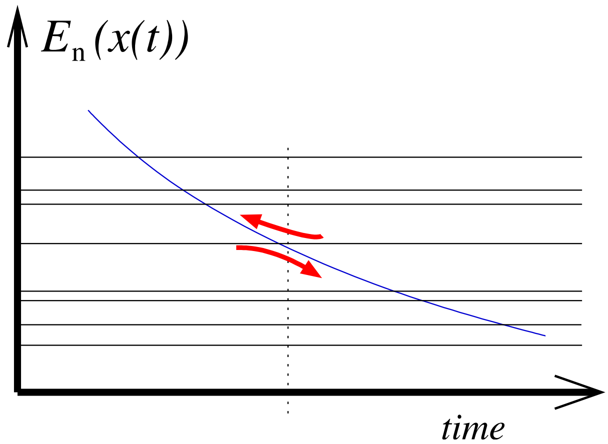
Shutenko, Aleiner, Altshuler (2000):

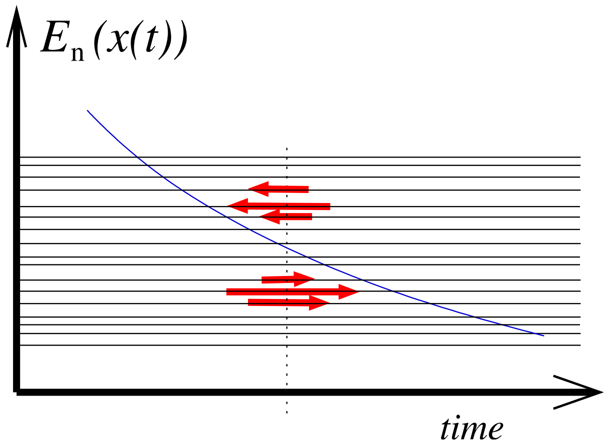
"If the system were closed [and strictly adiabatic], the charge distribution after each period of the perturbation would return to the original distribution, and therefore, the pumped charge would be *exactly quantized*."

Not correct!

Questions

- Take the "two barrier model" as an example.
- Assume Fermi occupation.
- Adiabatic limit: Find the current of each level.
- Is there one level that carries most of the current?
- What is the effect of non-adiabaticity?
- What is the role of dissipation?
- Can we get in a closed system $Q > 1$ or even $Q \gg 1$
- Why in an open system always $Q = 1 - g$

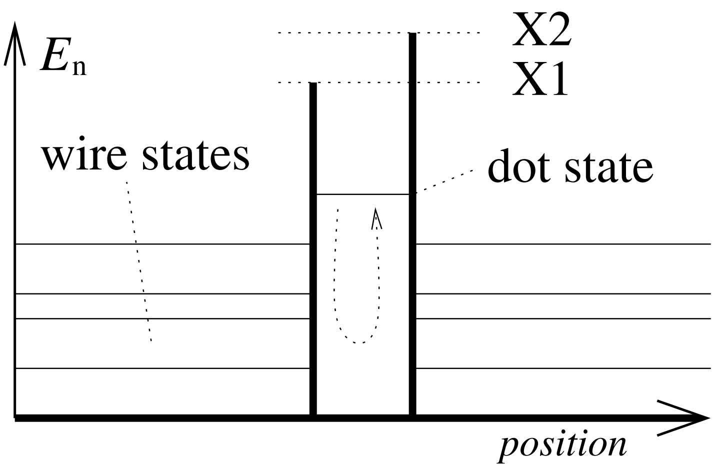


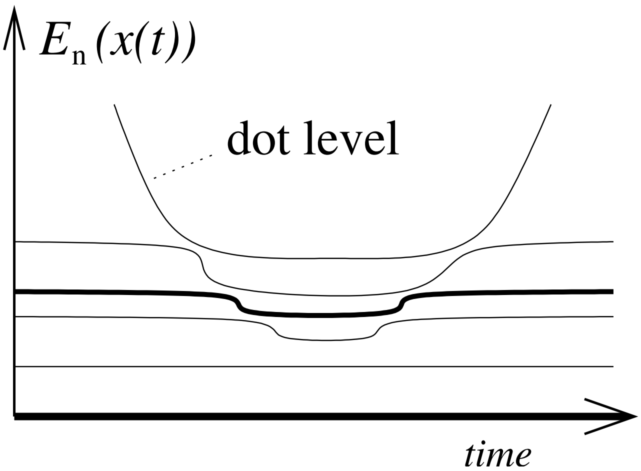


Formalism:

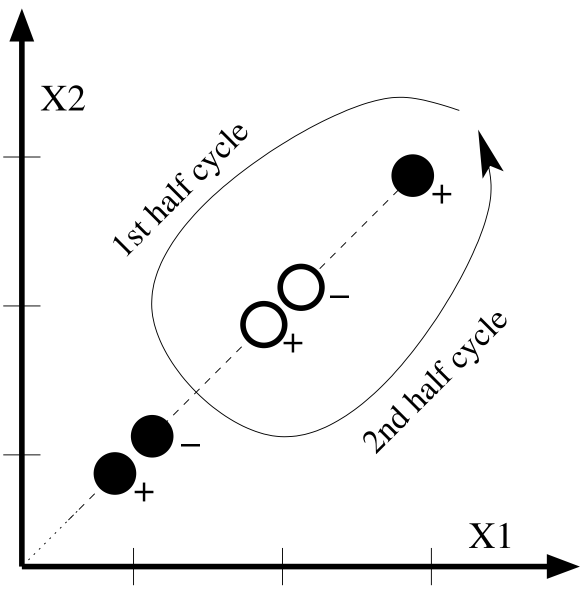
- Start with Kubo formula.
- Derive from it the adiabatic limit.
- Explain the implications of non-adiabaticity.
- Explain the emergence/role of dissipation.
- Take the limit of large L.
- Express the results using the S matrix.
- Does the result coincides with BPT?

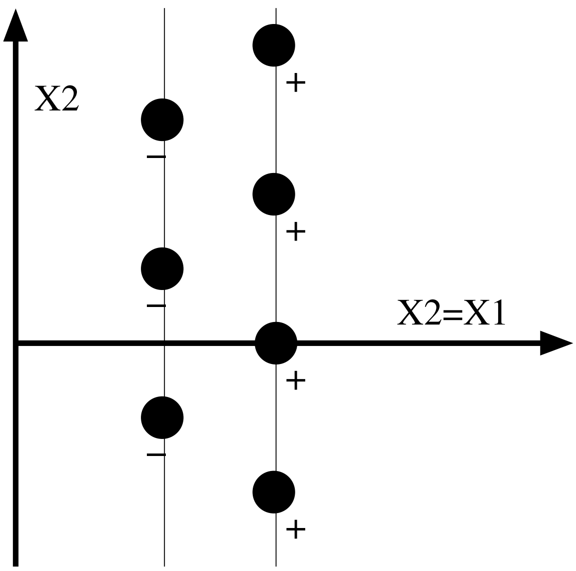
The two barrier model





$X_1 + X_2 \;\sim$ dot potential floor





The dot-wire ring system (I)

$$\mathcal{H} = \mathcal{H}(\boldsymbol{r}, \boldsymbol{p};\ x_1, x_2, x_3)$$

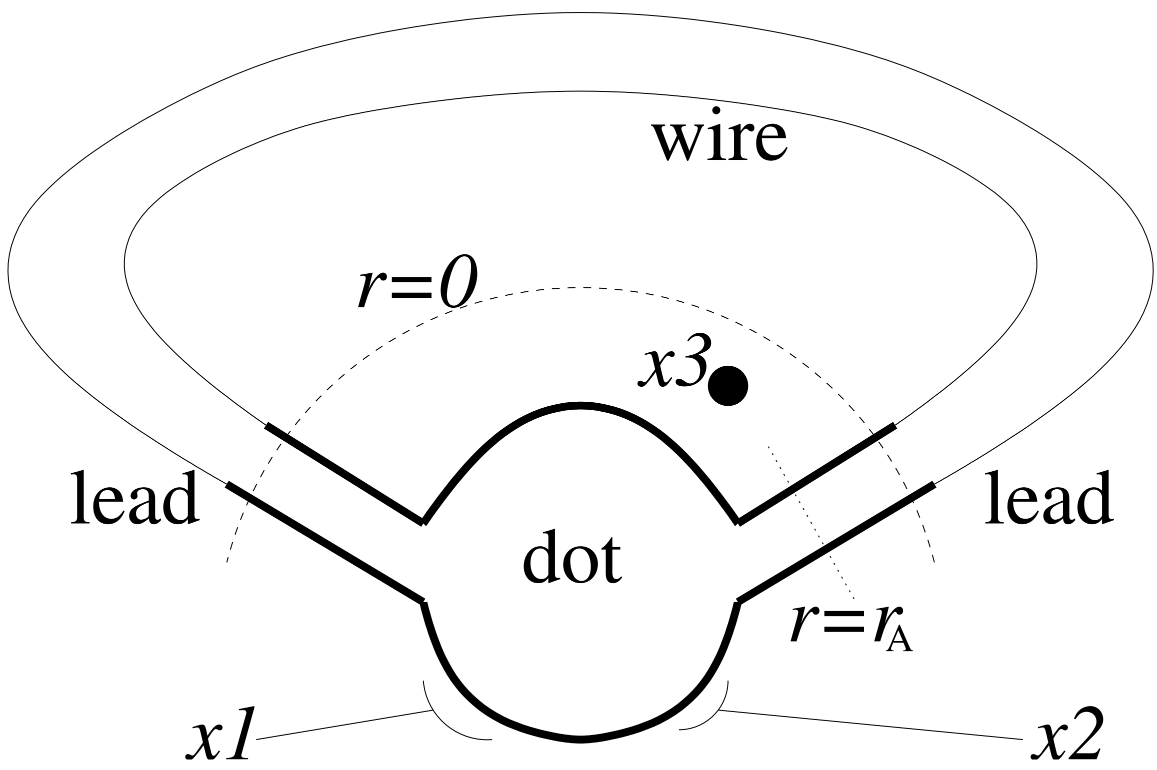
$$x_3 = \Phi = (\hbar/e)\phi = \text{magnetic flux}$$

$$-\dot{x}_3 = \text{electro motive force [Volt]}$$

$$\oint \vec{\mathcal{A}} \cdot \vec{dr} = \Phi$$

There is more than one way to put Φ into $\mathcal{H}$...

Conjugate current operator? Continuity?



The dot-wire ring system (II)

$$\mathcal{H} \;=\; \mathcal{H}(\boldsymbol{r}, \boldsymbol{p};\; x_1, x_2, x_3)$$

$x_1,\; x_2 \;=\;$ shape parameters

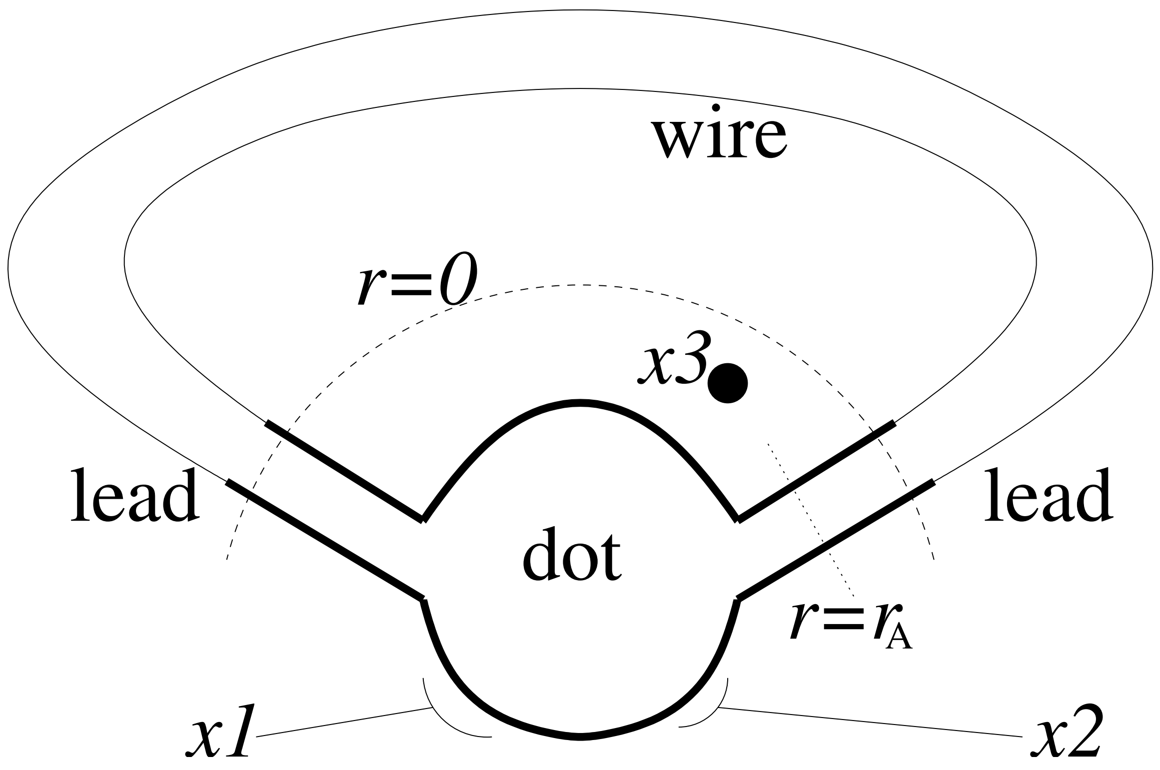
Possibilities:

$x =$ dot potential floor

$x =$ position of a wall element

$x =$ position of a scatterer inside the dot

$x =$ height of a barrier



Generalized forces / currents (I)

$$\mathcal{H} = \mathcal{H}(\boldsymbol{r}, \boldsymbol{p}; x)$$
$$F = -\frac{\partial \mathcal{H}}{\partial x}$$

Example 1:

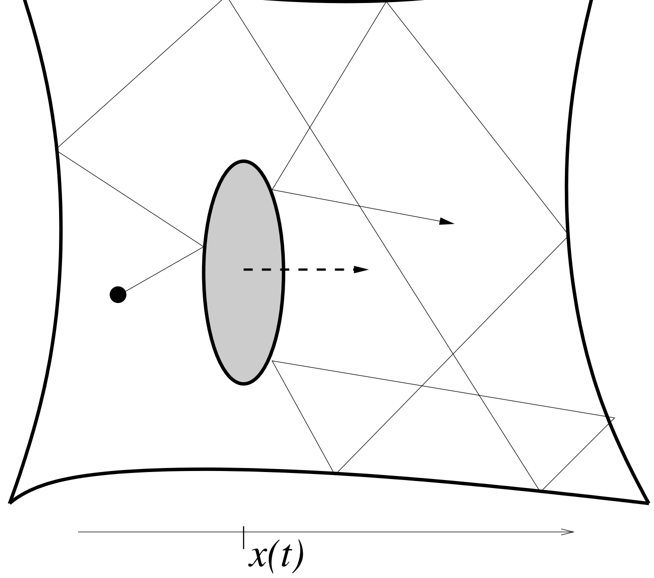
$x =$ position of a wall element (or scatterer)

$\dot{x} =$ wall (or scatterer) velocity

$F =$ Newtonian force

$\langle F \rangle = -\eta \dot{x}$ [friction law]

$\dot{\mathcal{W}} = \eta \dot{x}^2$ [rate of heating]



chaos!

Generalized forces / currents (II)

$$\mathcal{H} = \mathcal{H}(\boldsymbol{r}, \boldsymbol{p}; x)$$
$$F = -\frac{\partial \mathcal{H}}{\partial x}$$

Example 2:

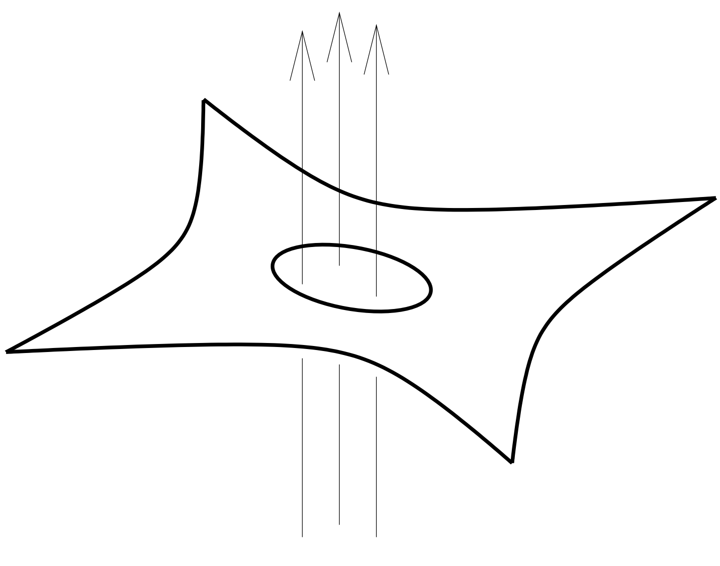
x = magnetic flux through the ring

$\dot{x} = -\mathrm{EMF}$

F = electrical current

$\langle F \rangle = -G\dot{x}$ [Ohm law]

$\dot{\mathcal{W}} = G\dot{x}^2$ [Joule law]



chaos!

Linear response theory

$$\langle \boldsymbol{F} \rangle_t \;=\; \int \boldsymbol{\alpha}(t-t')\;\delta\boldsymbol{x}(t')\;dt'$$

$$\alpha^{kj}(t-t')$$

$$\downarrow$$

$$\chi^{kj}(\omega)$$

$$\swarrow \qquad \searrow$$

$$\mathrm{Re}[\chi^{kj}(\omega)] \qquad\qquad (1/\omega)\times\;\mathrm{Im}[\chi^{kj}(\omega)]$$

$$\downarrow$$

$$\boldsymbol{G}^{kj}$$

$$\swarrow \qquad \searrow$$

$$\boldsymbol{\eta}^{kj} \qquad\qquad \boldsymbol{B}^{kj}$$

(dissipative) (non-dissipative)

$$\langle F^k \rangle \;=\; -\sum_j \boldsymbol{G}^{kj}\;\dot{x}_j$$

$$\langle F \rangle \;=\; -\boldsymbol{G}\cdot\dot{x} \;=\; -\boldsymbol{\eta}\cdot\dot{x} \;-\; \boldsymbol{B}\wedge\dot{x}$$

$$\dot{\mathcal{W}} \;=\; -\langle F \rangle\cdot\dot{x} \;=\; \sum_{kj}\boldsymbol{\eta}^{kj}\;\dot{x}_k\;\dot{x}_j$$

The Kubo Formula

$$\alpha^{kj}(\tau) \;=\; \Theta(\tau) \times \; \frac{i}{\hbar} \langle [F^k(\tau), F^j(0)] \rangle$$

$$\chi^{kj}(\omega) \;=\; \sum_{n,m} f(E_n) \left(\frac{-F^k_{nm} F^j_{mn}}{\hbar\omega - (E_m - E_n) + i0} + \frac{F^j_{nm} F^k_{mn}}{\hbar\omega + (E_m - E_n) + i0} \right)$$

$$\boldsymbol{G}^{kj} \;=\; \frac{1}{\omega} \times \; \mathrm{Im}[\chi^{kj}(\omega)] \; \Big|_{\omega \sim 0}$$

$$\boldsymbol{\eta}^{kj} = \pi\hbar \sum_{n,m} F^k_{nm} F^j_{mn} \; \delta(E_n - E_F) \; \overline{\delta(E_m - E_n)}$$

$$\boldsymbol{B}^{kj} \;=\; 2\hbar \sum_n f(E_n) \sum_{m(\neq n)} \frac{\mathrm{Im}\left[F^k_{nm} F^j_{mn}\right]}{(E_m - E_n)^2 + (\Gamma/2)^2}$$

The Kubo Formula and "quantum chaos"

$\tau_{\rm cl}$ = classical correlation time

$\Delta \propto \hbar^d/L$ = mean level spacing

$\Delta_b \sim \hbar/\tau_{\rm cl}$ = bandwidth

Effective width of the energy levels:

$$\Gamma = \left(\frac{\hbar\sigma}{\Delta^2}|\dot{x}|\right)^{2/3} \times \Delta \sim \left(L\,|\dot{x}|\right)^{2/3}\frac{1}{L}$$

$\Gamma \ll \Delta$ adiabatic regime

$\Delta < \Gamma < \Delta_b$ non-adiabatic regime

$\Delta_b < \Gamma$ non-perturbative regime

$L \to \infty$ is not the semiclassical limit!

The generalized FD relation

$$K^{kj}(\tau) = (i/\hbar)\langle[F^k(\tau), F^j(0)]\rangle$$
$$C^{kj}(\tau) = \frac{1}{2}(\langle F^k(\tau)F^j(0)\rangle + \mathrm{cc})$$

$$\alpha^{kj}(\tau) = \Theta(\tau)\ K^{kj}(\tau) \qquad \text{[“Kubo formula”]}$$

$$\boldsymbol{G}^{kj} = \lim_{\omega\to 0}\frac{\mathrm{Im}[\chi^{kj}(\omega)]}{\omega} = \int_0^\infty K^{kj}(\tau)\tau d\tau$$

$$\boldsymbol{G}^{kj} = \frac{1}{\Delta}\int_0^\infty C^{kj}(\tau)d\tau \qquad \text{[“FD relation”]}$$

$$\boldsymbol{B}^{kj} = \frac{-i}{\Delta}\int_{-\infty}^{\infty}\left[\frac{\tilde{C}^{kj}(\omega)}{\omega}\right]\frac{d\omega}{2\pi}$$

$$\boldsymbol{\eta}^{kj} = \frac{1}{2\Delta}\tilde{C}^{kj}(\omega\sim 0)$$

Kubo formula - Green functions - BPT formula

$$\eta^{3j} = \frac{\hbar}{\pi} \text{trace}\left[F^3 \ \text{Im}[\mathsf{G}^+] \ F^j \ \text{Im}[\mathsf{G}^+]\right]$$

$$= \frac{\hbar}{4\pi}\text{trace}\left[\frac{\partial S^\dagger}{\partial x_3}\frac{\partial S}{\partial x_j}\right]$$

$$\boldsymbol{B}^{3j} = \frac{\hbar}{4\pi}\text{trace}\left[\mathcal{F}^3 \ \mathsf{G}^+ \ \mathcal{F}^j \ \mathsf{G}^- - \mathcal{F}^3 \ \mathsf{G}^- \ \mathcal{F}^j \ \mathsf{G}^+\right]$$

$$= \frac{e}{4\pi i}\text{trace}\left[P_{\text{A}}\left(\frac{\partial S}{\partial x_j}S^\dagger - \frac{\partial S^\dagger}{\partial x_j}S\right)\right] + \text{intrf}$$

$$\boldsymbol{G}^{3j} = \frac{e}{2\pi i}\text{trace}\left(P_{\text{A}}\frac{\partial S}{\partial x_j}S^\dagger\right) \qquad \text{[BPT]}$$

DC, PRB(R) 2003

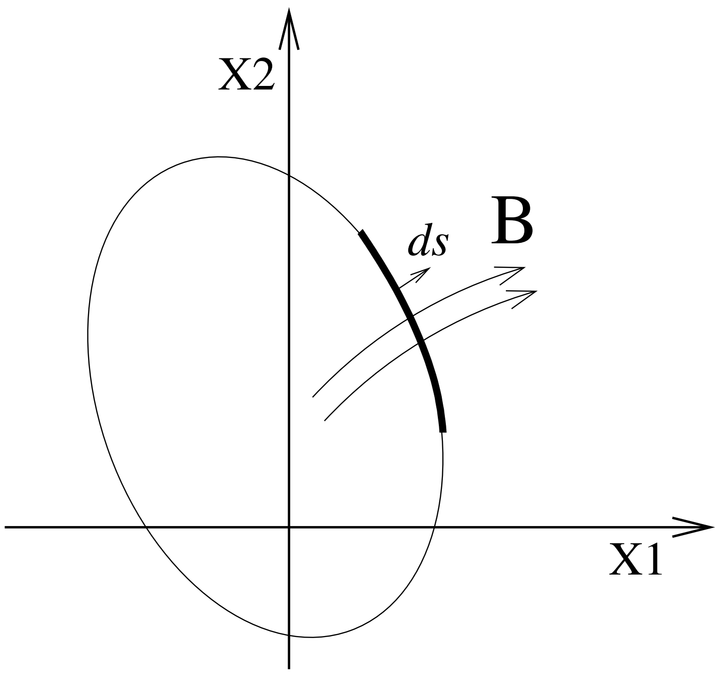
Pumping within the Kubo formulation

$$Q = \oint \langle F^3 \rangle dt$$

$$\langle F \rangle = -\boldsymbol{\eta} \cdot \dot{x} \quad - \boldsymbol{B} \wedge \dot{x}$$

$$Q = \left[-\oint \boldsymbol{\eta} \cdot dx \quad - \oint \boldsymbol{B} \wedge dx \right]_{k=3}$$

Consider a planar (x_1, x_2) pumping cycle.



$$Q = -\oint \boldsymbol{B} \cdot \, ds$$

No magnetic field. Onsager $\Longrightarrow$

$\boldsymbol{\eta}^{31} = \boldsymbol{\eta}^{32} = 0$ (no dissipative contribution)

$\boldsymbol{B}^{12} = 0$ (no vertical component)

The B field

$$Q \;=\; \left[-\oint \boldsymbol{B} \wedge dx\right]_{k=3}$$

$$\boldsymbol{B}^{ij} \;=\; \sum_n f(E_n) \boldsymbol{B}_n^{ij} \qquad \text{[geometric magnetism]}$$

$$\boldsymbol{B}_n^{kj} \;=\; 2\hbar \sum_{m(\neq n)} \frac{\mathrm{Im}\left[F_{nm}^k F_{mn}^j\right]}{(E_m - E_n)^2 + (\Gamma/2)^2}$$

This field is divergenceless (for $\Gamma = 0$)

A chain of degeneracies:

$$\left(x_1^{(0)},\;\; x_2^{(0)},\;\; \Phi^{(0)} + 2\pi\frac{e}{\hbar} \times \text{ integer}\right)$$

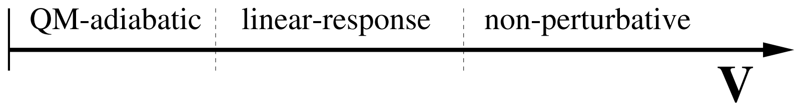
The degeneracies are like Dirac monopoles

- The issue of bandwidth

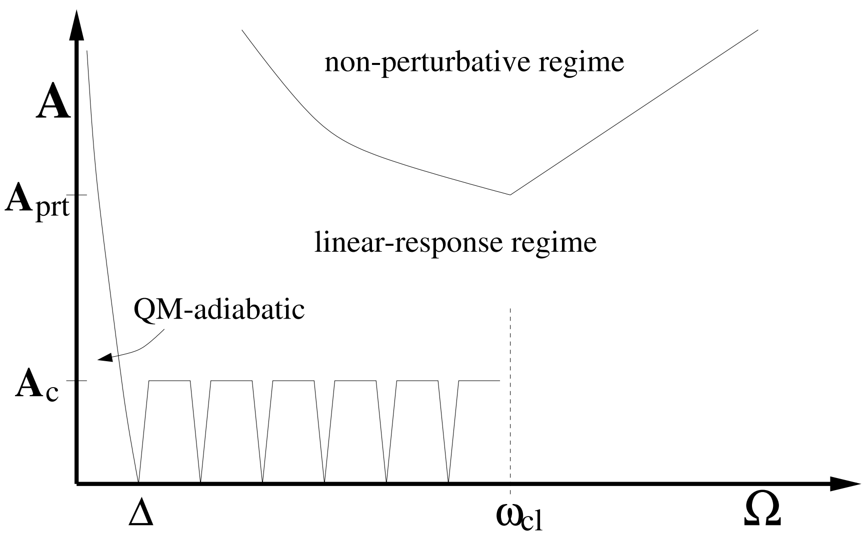
- The effect of screening

Summary

- LRT gives a unified framework for the theory of pumping.
- Derivation of S matrix expressions for $\boldsymbol{\eta}$ and $\boldsymbol{B}$.
- Distinction between adiabatic, non-adiabatic and non-prt regimes.
- "Quantum chaos" considerations are essential (Γ).
- The emergence / relevance of dissipation.
- The $L \to \infty$ limit versus the $\hbar \to 0$ limit.
- Near-field versus far field pumping cycles around "Dirac chains".
- The analysis of deviations from "quantized" pumping.



$$x(t) = Vt$$



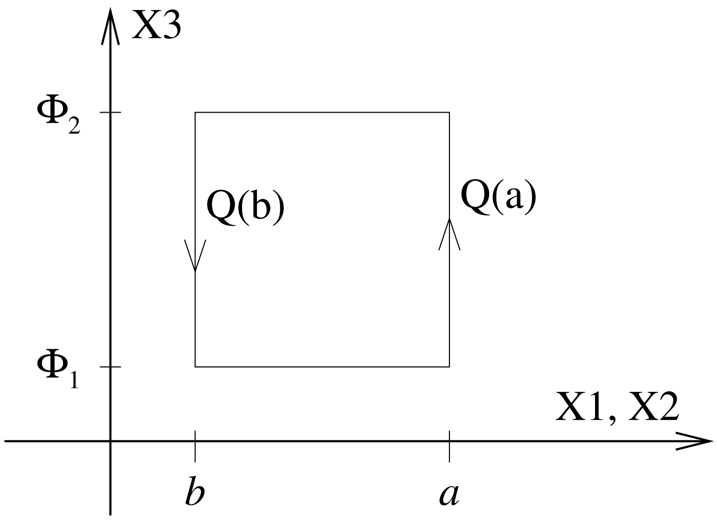
$$x(t) = A\sin(\Omega t)$$

Digression - The simplest pump

Assume that the current is given by Ohm law:

$$I = -G \frac{d}{dt}\Phi = -G \frac{dx_3}{dt}$$

$$Q = \oint I dt = -\oint G dx_3$$



$$Q(a) = -G(a) \times (\Phi_2 - \Phi_1)$$

$$Q(b) = -G(b) \times (\Phi_1 - \Phi_2)$$

The net pumped charge:

$$Q = (G(b) - G(a)) \times (\Phi_2 - \Phi_1).$$

Digression - The BPT formula

$$S = \begin{pmatrix} \boldsymbol{r}_{\mathrm{B}} & \boldsymbol{t}_{\mathrm{AB}} \mathrm{e}^{-i\phi} \\ \boldsymbol{t}_{\mathrm{BA}} \mathrm{e}^{i\phi} & \boldsymbol{r}_{\mathrm{A}} \end{pmatrix}$$

$$P_{\mathrm{A}} = \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$

$$\boldsymbol{G}^{3j} = \frac{1}{2\pi i} \mathrm{trace} \left(P_{\mathrm{A}} \frac{\partial S}{\partial x_j} S^{\dagger} \right)$$

$$\langle F^3 \rangle \quad = \quad -\sum_j \boldsymbol{G}^{3j} \; \dot{x}_j$$

$$S = \mathrm{e}^{i\gamma} \begin{pmatrix} i\sqrt{1-g}\,\mathrm{e}^{i\alpha} & \sqrt{g}\,\mathrm{e}^{-i\phi} \\ \sqrt{g}\,\mathrm{e}^{i\phi} & i\sqrt{1-g}\,\mathrm{e}^{-i\alpha} \end{pmatrix}$$

$$Q \quad = \quad \frac{1}{2\pi} \oint (1-g) \frac{d\alpha}{dx} \cdot \vec{dx} \quad \approx \quad 1 \;-\; g$$

Digression - The S matrix

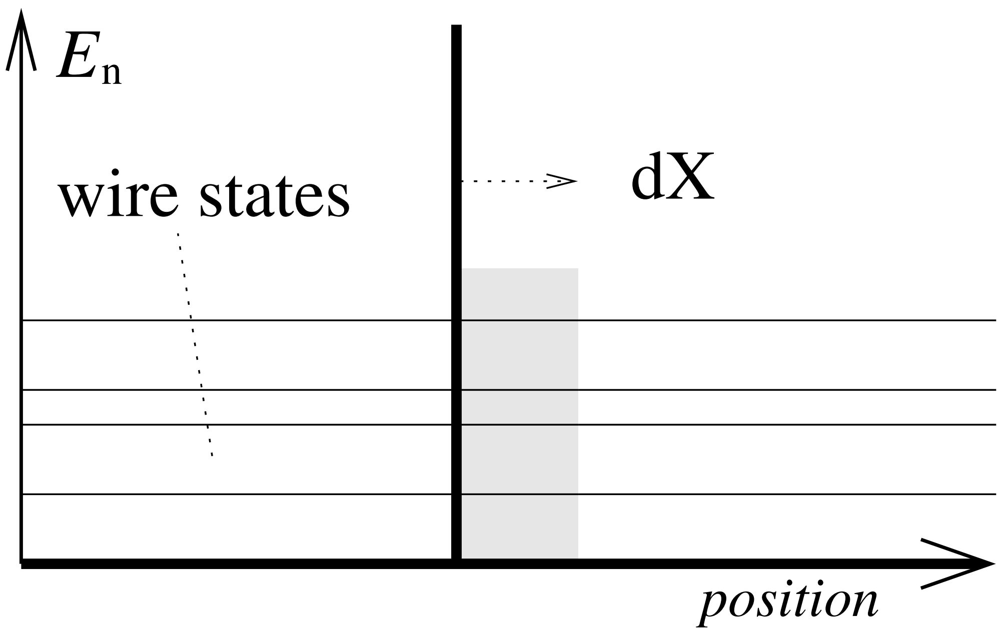
$$S = e^{i\gamma} \begin{pmatrix} i\sqrt{1-g}e^{i\alpha} & \sqrt{g}e^{-i\phi} \\ \sqrt{g}e^{i\phi} & i\sqrt{1-g}e^{-i\alpha} \end{pmatrix}$$

g = transmission

γ = global phase shift

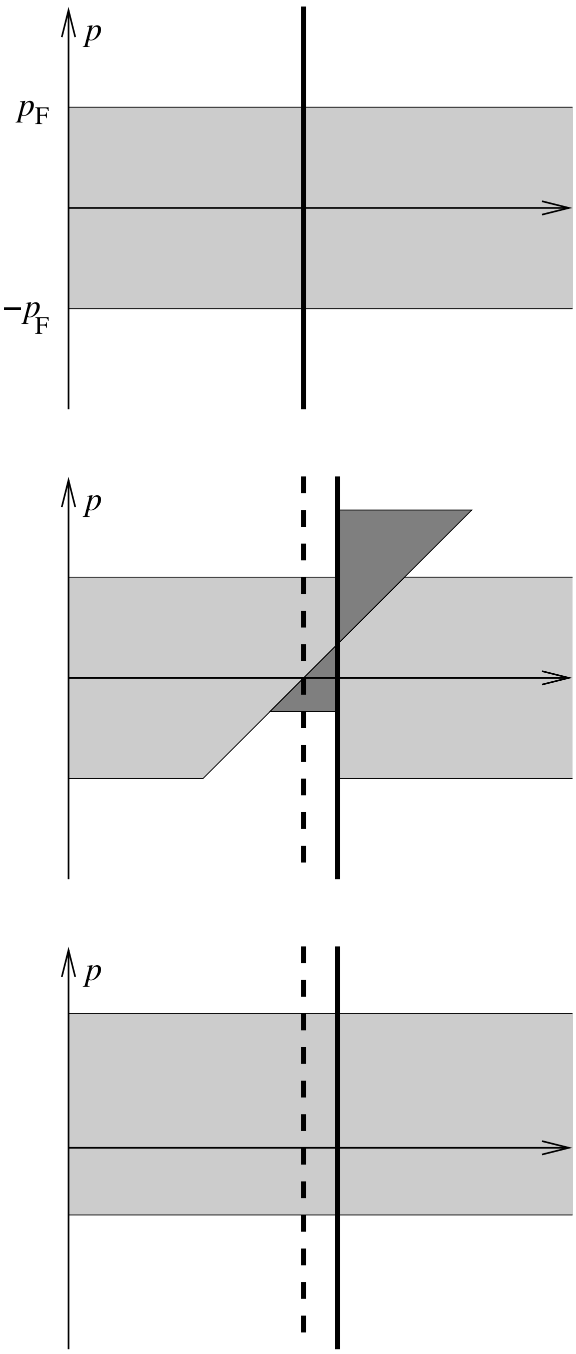
ϕ = magnetic flux phase

α = displacement phase



$$d\alpha = 2kdX$$

Digression - The 1D moving wall model



Strictly adiabatic:

$$dQ_n \;=\; \frac{1}{L} dX$$

$$dQ \;=\; \frac{p_F dX}{\pi\hbar}$$

$$N = \frac{2p_F L}{2\pi\hbar}$$

Non adiabatic:

$$dQ = (1-g) \times \frac{p_F dX}{\pi\hbar}$$