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Coherent transport through a quantum shuttle

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A shuttle site that crosses through a band of network levels induces currents. If the process is re-cycled this can be regarded as "pumping" or "stirring". We show that the analysis reduces to that of calculating time dependent probabilities, as in the stochastic formulation, but with splitting (branching) ratios that are not bounded within $[0, 1]$. Our approach allows to address the adiabatic regime, as well as the Slow and Fast non-adiabatic regimes, on equal footing. We emphasize aspects that go beyond the familiar picture of sequential Landau-Zener crossings, taking into account Wigner-type multilevel mixing due to the crossing shuttle.

Transport in quantum networks is a theme that emerges in diverse contexts, including quantum Hall effect [1], Josephson arrays [2], quantum computation models [3], quantum internet [4], and even in connection with photosynthesis [5]. For some specific models there are calculations of the induced currents in the adiabatic regime [6–10] for both open and closed systems, so called "quantum pumping" [11–18] and "quantum stirring" [19–23] respectively. In the latter context most publications focus on 2-level [24, 25] and 3-level dynamics, while the larger perspective is rather abstract, notably the "Dirac monopoles picture" [9, 19, 21, 22]. This should be contrasted with the analysis of stochastic stirring where the theory is quite mature [26–29].

Considering (e.g.) the unidirectional rotation of a molecular rotor [27], it is possibly allowed to be satisfied with a stochastic picture [26] that relates the currents, via a "decomposition formula", to rates of change of occupation probabilities. Once we turn (e.g.) to the analysis of pericyclic reactions [30] this is no longer possible. In the latter case the method of calculating electronic quantum fluxes had assumed that the latter can be deduced from the continuity equation. Such procedure is obviously not applicable for (say) a ring-shaped molecule: due to the multiple path geometry there is no obvious relation between currents and time variation of probabilities.

In this work we would like to analyze the following prototype problem. Consider a network that consists of N interconnected sites, with on-site energies $\mathcal{E}_i$, and couplings C_{ij}. Additionally there is a shuttle ($i = 0$), where the on-site energy $\mathcal{E}_0 = u(t)$ is varied monotonically from $u = -\infty$ to $u = \infty$. Accordingly the Hamiltoinan is

$$\mathcal{H} = \sum_{i=0}^{N} |i\rangle \mathcal{E}_i \langle i| + \sum_{i \neq j} |i\rangle C_{ij} \langle j|, \quad \mathcal{E}_0{=}u(t), \quad C_{i0}{=}C_i \quad (1)$$

Our interest is in the induced current $I(t)$ that flows through a tagged connecting bond C_a. In the adiabatic limit this current is determined by the so-called geometric conductance G as follows:

$$I = G\dot{u}, \qquad G = 2\mathrm{Im}\left[\left\langle \frac{\partial}{\partial\phi}\Psi \middle| \frac{\partial}{\partial u}\Psi \right\rangle\right]_{\phi=0} \qquad (2)$$

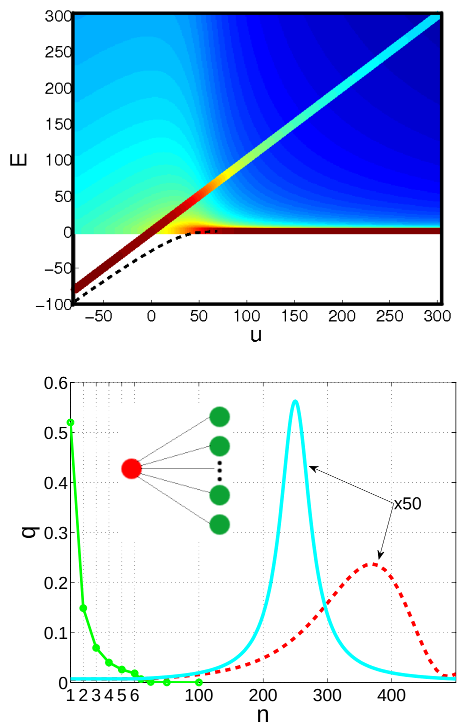

FIG. 1: *Upper panel:* an initially loaded shuttle is crossing a band that contains $N = 500$ levels. The occupation probabilities of the shuttle (p) and of the levels (q_n) are imaged as a function of time. The vertical axis is the energy, and the horizontal axis is $u(t)$. We assume star geometry with level spacing $\Delta{=}1$, and identical couplings $c_n{=}3$. The dashed line illustrates the energy of the lowest adiabatic level. *Lower panel:* plot of q_n vs n in several cases. Green line with markers - the adiabatic scenario of the upper panel at $u = 60$. Cyan solid line - after decay from a standing shuttle ($\dot{u}{=}0$). Red dashed line - after decay from a fast shuttle ($\dot{u}{=}5000$).

Here ϕ is a test flux through the bond of interest, namely $C_a \mapsto C_a e^{i\phi}$, and Ψ is the wave-function of the adiabatic eigenstate. Specific scenarios to be considered are: (i) The shuttle is initially filled with a particle that later is transferred to the network; (ii) One of the levels of the network is initially filled, and later a current is induced via the crossing shuttle. The occupation dynamics in the first scenario is illustrated in Fig.1, which will be further discussed later. For a non-interacting many-body occupation, results can be obtained by simple summation.

We shall consider first a Star geometry (inset in Fig.1), and later a general network geometry. As an appetizer we refer to a 3 site model (inset in Fig.2). We shall see that the integrated current through the C_a bond (indicated in the figure) is $Q = |C_a|^2/(|C_a|^2 + |C_b|^2)$ for $C_0 = 0$, while $Q = C_a/(C_a - C_b)$ for any $C_0 \neq 0$. It is possible to get $|Q| > 1$ in the latter case, indicating that a circulation current is induced in the system. This result has been derived in the past as an *approximation*, using either the "Dirac monopoles picture", or a two-level calculation. In the analysis below the C_a are allowed to be large, such that numerous network levels are mixed during the shuttle crossing. We also address the non-adiabatic scenario.

Outline.– We find explicit expression for the current I that is induced either in adiabatic or non-adiabatic shuttling process. We first consider "star geometry" (inset in Fig.1) and later a general network. For the analysis we introduce the *splitting ratio* phenomenology, and demonstrate it for a dot-wire "ring geometry" (inset in Fig.3). We find it essential to distinguish between two types of processes: *adiabatic crossing* and *adiabatic metamorphosis*. This distinction is important for the understanding of non-adiabatic effects.

Star geometry.– Let us consider the special case of having sites with energies $\mathcal{E}_n = \epsilon_n$ and connections $C_{n0} = c_n$, while all the other couplings are zero. An adiabatic eigenstate of the system is represented by [a]:

$$|\Psi\rangle \quad \mapsto \quad \sqrt{p} \times \left(1, \ \left\{\frac{c_n}{E - \epsilon_n}\right\}_{n=1,\dots,N}\right) \tag{3}$$

Using the notation

$$g(E; c_1, ...c_N) \quad = \quad \sum \frac{|c_n|^2}{E - \epsilon_n} \tag{4}$$

the adiabatic energy is the solution of the equation $g(E) = E - u$. As u is swept from $-\infty$ to $+\infty$, the energy increases monotonically from ϵ_n to ϵ_{n+1}, where n is the starting level. The normalization factor $p = [1 - g'(E)]^{-1}$ is the probability to find the particle in the shuttle. For the following derivation note that $1/p$ is a quadratic form in c_n. Using Eq.(2) we get after differentiation by parts that the current through c_n is

$$\begin{aligned} G \quad &= \quad \frac{|c_n|^2}{(E - \epsilon_n)^2}\left(\frac{\partial p}{\partial u}\right) - 2p\frac{|c_n|^2}{(E - \epsilon_n)^3}\left(\frac{\partial E}{\partial u}\right) \\ &= \quad \frac{\partial}{\partial u}\left[\left(\frac{1}{2}\frac{\partial(1/p)}{\partial c_n}c_n\right) \ p\right] \quad = \quad \frac{\partial}{\partial u}\left[q_n\right] \end{aligned} \tag{5}$$

where $q_n = |\Psi_n|^2$ are identifies as the probabilities to find the particle in $n = 1...N$.

Multiple path geometry.– Needless to say that we do not really need Eq.(2) in order to get the expression for G in the case of a star graph. We could simply deduce Eq.(5) from conservation of probability, i.e. from the continuity equation $I = \dot{q}_n$. This is no longer the case if we have a multiple path geometry: probability conservation alone cannot tell us how the current is split between the different paths. Furthermore, we would like to go beyond the adiabatic transport formalism, and obtain a formula that applies also in non-adibatic circumstances.

Splitting ratio approach.– Let us first see what is the expression for G in the case of a general network. It is natural to switch from the $\mathcal{E}_i$ basis to an ϵ_n basis that diagonalize the network Hamiltonian in the absence of the shuttle. Consequently getting a star geometry with $c_n = \sum_b \langle n|b\rangle C_b$. Our interest is in the current through a tagged bond C_a. We define the "splitting ratio" of the current that flows in the nth levels as

$$\lambda_n[\text{splitting}] \quad = \quad \frac{\langle n|a\rangle C_a}{c_n} \quad = \quad \frac{\langle n|a\rangle C_a}{\sum_b \langle n|b\rangle C_b} \tag{6}$$

A straightforward generalization of the derivation that leads to Eq.(5) implies that the current through C_a is

$$I \quad = \quad \frac{\partial}{\partial t}\left[\sum_n \lambda_n q_n\right] \tag{7}$$

The physical simplicity of Eq. (7) suggests that it can be derived without assuming adiabaticity, where $q_n(t) = |\Psi_n(t)|^2$ is obtained from the solution of the *time dependent* Schrodinger equation. Indeed this is the case. We just have to remember that quite generally $I = C_a \text{Im}[\Psi_a(t)^* \Psi_0(t)]$, substitute $\Psi_a = \sum_n \langle a|n\rangle \psi_n$, use the definition Eq.(6) of the splitting ratio, and the identification $\dot{q}_n = c_n \text{Im}[\psi_n^* \psi_0]$.

The integrated current.– As the simplest example for the application of the splitting ratio approach we consider a process in which a particle has been transferred from the shuttle to the wire in the network that is illustrated in the inset of Fig. 3. In this model the splitting ratio of the even-parity levels is $\lambda_n = \lambda_+ = C_a/(C_a + C_b)$, while for the odd-parity levels we have $\lambda_n = \lambda_- = C_a/(C_a - C_b)$. From Eq.(7) it follows that the integrated current is $Q = \text{average}(\lambda_n)$, where the weighted average is determined by the final occupation of wire levels. For an adibatic process, in which the particle ends up at the lower wire level, we get $Q = \lambda_1$. Unlike the case of a stochastic transition this value is not bounded within $[0, 1]$. rather it may have any value, depending on the relative sign of the amplitudes C_a and C_b. However, if the process is not adiabatic, the probability is distributed over both the odd and the even levels with probabilities that are proportional to $|C_a \pm C_b|^2$ respectively. Then we get from the weighted average a stochastic-like result, namely $Q = |C_a|^2/(|C_a|^2 + |C_b|^2)$.

It is also instructive to consider the process in which a particle that is prepared (say) in an even wire-level, is adiabatically transferred, due to shuttling, into the adjacent odd wire-level. Here the integration over Eq. (7) implies $Q = \lambda_- - \lambda_+$, leading to $Q = 2C_aC_b/(|C_a|^2 - |C_b|^2)$.

The parametric variation of the current.– The results for the integrated current give the impression that the size of the coupling c_n compared with the levels spacing Δ is of no importance. But this is a wrong impression. Once we get deeper into the analysis it becomes clear that the familiar two level approximation for the adiabatic current I, requires the coupling c_n to be very small compared with the level spacing Δ. Our interest below is focused in the case of having a quasi-continuum, meaning that the c_n are larger than Δ, hence many levels are mixed as the shuttle goes through.

Before discussing the quasi-continuum case it is useful to discuss what happens with the $N = 2$ wire system if the c_n are not smaller compared with Δ.

Adiabatic metamorphosis.– The heuristic understanding of adiabatic transport is commonly based on the analysis of "adiabatic crossing". Below we shall see that we have to take into account an additional prototype process that we call "adiabatic metamorphosis". The minimal model for its demonstration requires 3 levels.

Consider the dot-wire geometry with $N = 2$. Assume that $\Delta \ll C_a, C_b$. Clearly during the adiabatic crossing Δ has no effect, so we can neglect it. The dynamics is very simple: there is a dark state $|D\rangle = C_b|1\rangle - C_a|2\rangle$ that does not participate in the dynamics. The crossing is exclusively to $|C\rangle = C_a|1\rangle + C_b|2\rangle$. Using the same procedure as before, one concludes that the splitting ratio is $Q = |C_a|^2/(|C_a|^2+|C_b|^2)$. This looks like a contradiction to our previous analysis that predicts $Q = C_a/(C_a - C_b)$ for any $c_0 \neq 0$. What is wrong?

After some thought one realizes that there is a second distinct stage in the dynamics during which the ground state adiabatically transforms from $|C\rangle$ to $|\epsilon_-\rangle = |1\rangle - |2\rangle$. The reason for this metamorphosis is as follows: the sites $|1\rangle$ and $|2\rangle$ are directly coupled via c_0, but also virtually coupled via C_aC_b/u. This second-order coupling via the shuttle is dominating during the adiabatic crossing, but much later, when u becomes larger than $\gamma \equiv |C_aC_b/\Delta|$, the first-order coupling takes over.

At first glance one may argue that the metamorphosis at such later stage is of no interest: all the probability transfer happens during the adiabatic crossing; after that the particle is confined to the wire; and can merely "rearrange" itself there. However, this reasoning is misleading: we have here *multiple path* geometry, and therefore we can have flow through the shuttle without any transfer of probability. In Fig.2 we demonstrate this observation. If the metamorphosis stage is taken into account, there is no longer contradiction with the general formula.

Adiabatic mixing.– We turn to treat the dot-wire system for $N \gg 1$. For simplicity we focus on some energy range where the levels form a comb-like spectrum

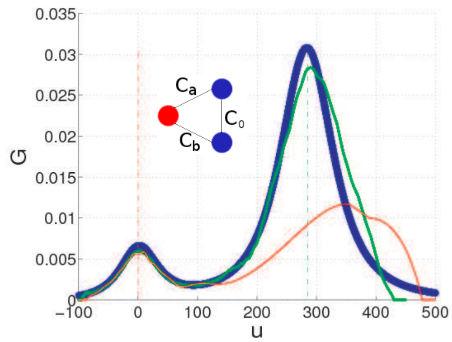

FIG. 2: Flow of the current from the shuttle to an $N = 2$ wire through the C_a bond. The parameters are $C_a = 19$, and $C_b = 15$, and $C_0 = 1$. The thickest line is the exact adiabatic result for $G(u)$. The thinner and the thinnest lines are $I/\dot{u}$ for $\dot{u} = 2$ and for $\dot{u} = 50$. The left and right vertical lines indicate the shuttle-wire crossing point, and the metamorphosis point, with separation $\gamma = 285$. During the adiabatic metamorphosis a current is flowing through the distant shuttle.

with level spacing Δ, hence from the definition Eq.(4)

$$g(E) \;=\; \left(\frac{\pi}{2\Delta}\right)\left[c_-^2 \cot\left(\pi\frac{E}{2\Delta}\right) - c_+^2 \tan\left(\pi\frac{E}{2\Delta}\right)\right] \quad (8)$$

where $c_\pm$ = prefactor $\times$ $(C_a \pm C_b)$ are the couplings to the even and odd levels respectively. For simplicity of presentation we absorb the prefactor into the definition of C_a and C_b [a]. Of interest is the case of having a quasi continuum, meaning that the couplings c_n are larger compared with Δ, hence a two level approximation is out of the question. We shall see that the role of γ in the $N = 2$ analysis is taken by $\gamma = \sin\theta\, \Gamma$, where

$$\Gamma \equiv \pi\frac{c_+^2 + c_-^2}{\Delta}, \qquad \sin(\theta) \equiv \frac{c_+^2 - c_-^2}{c_+^2 + c_-^2}, \quad (9)$$

(here and below we assume starting with even-parity level). With Eq. (8) the secular equation $g(E) = E - u$ becomes a quadratic equation for tan(), and can be solved explicitly. Then it is possible to get an expression for the shuttle occupation probability:

$$p(u) \;=\; [1 - g'(E)]^{-1} \;=\; \Delta \cdot \mathrm{L}\,[u - E;\ \Gamma, \theta] \quad (10)$$

where the distorted Lorentzian $\mathrm{L}\,[x; \Gamma; \theta]$ is

$$\frac{1}{\pi}\left[1 + \frac{\sin\theta\ x}{\sqrt{x^2 + \cos^2\theta\ (\Gamma/2)^2}}\right]^{-1} \frac{\cos^2\theta\ (\Gamma/2)}{x^2 + \cos^2\theta\ (\Gamma/2)^2}$$

In the expression above E is the energy in which the particle has been prepared, and it can be regarded as a constant. Some further straightforward algebra leads to

$$G(u) \;=\; C_a\frac{\partial}{\partial u}\left[p\sum_n \frac{c_n^*\langle n|a\rangle}{(E - \varepsilon_n)^2}\right] \quad (11)$$

$$\;=\; \frac{\partial}{\partial u}C_a\left[\frac{c_+ \sin^2(\varphi(u)) + c_- \cos^2(\varphi(u))}{c_+^2 \sin^2(\varphi(u)) + c_-^2 \cos^2(\varphi(u))}\right] \quad (12)$$

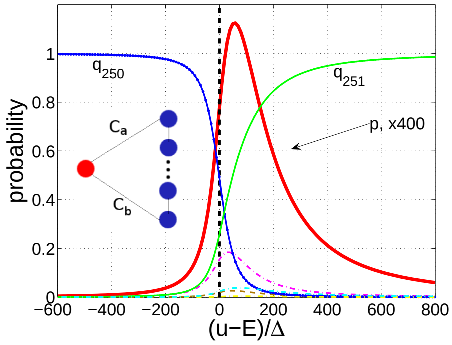

FIG. 3: The variation of the occupation probabilities as a function of u for an adiabatic shuttling process. The particle initially has been placed at $n = 250$. The couplings are $C_a = 6$ and $C_b = 4$. The red thick line is p. The other solid lines are q_{250}, and q_{251}. The dashed lines from up to down are q_{249} and q_{253} and q_{247} and q_{252}.

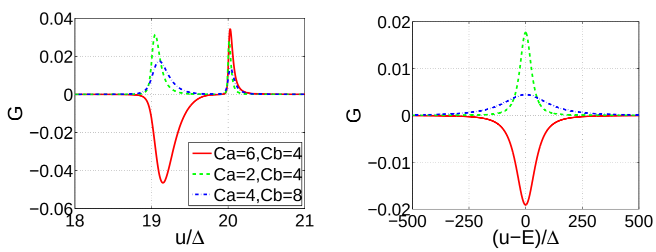

FIG. 4: Flow of the current from the shuttle to the wire through the C_a bond, for the same scenario as in Fig.3. The parameters are indicted in the legend. In the left panel $\Delta = 200$, hence a two level approximation is satisfactory. In the right panel $\Delta = 1$, and consequently the line shape reflects a multi-level crossing.

where $\varphi(u) = (\pi/2)(E/\Delta)$ has a $\pi/2$ variation as u is raised from $-\infty$ to $+\infty$, hence Eq.(12) is a step-like function that varies from λ_+ to λ_- as had been anticipated by inspection of Eq.(7).

The functions $p(u)$ and $G(u)$ are plotted in Fig.3 and in Fig.4. In the latter we contrast with the $c_n \ll \Delta$ case, for which the dynamics can be regarded as a sequence of two-level crossings.

Regimes.– The results for the integrated current give another wrong impression: it looks as if we are dealing with two regimes: either the process is adiabatic or non-adiabatic. A more careful inspection reveals that depending on $\dot{u}$ we have 3 regimes: Adiabatic, Slow and Fast. For star geometry with comb-like quasi continuum of levels, the Slow regime is defined by the condition

$$c^2 \ < \ \dot{u} \ < \ \Gamma^2, \qquad \Gamma \equiv 2\pi \frac{c^2}{\Delta} \tag{13}$$

For simplicity we assume here comb-like quasi continuum with identical couplings $c_n = c$. The left inequality in Eq.(13) means that the adiabatic condition is violated, while the right inequality implies that a first-order perturbative approximation is violated as well. The identification of this intermediate Slow regime parallels the notion of Wigner or FGR or Kubo regime in past studies of time dependent dynamics [19].

Some illustrations for energy spreading are presented in Fig.1. If $c < \Delta$ the transport of probability from the shuttle to the wire would be described using a two level approximation. But the illustration in the upper panel assumes $c > \Delta$, hence many levels are mixed within a parametric range Γ. The time during which this mixing takes place is $\Gamma/\dot{u}$. In the opposite limit of Fast shuttling, which we further discuss below, the decay time of the probability to the quasi-continuum is $1/\Gamma$.

Non adiabatic spreading.– The calculation of I in the non-adiabatic regime requires knowledge of $q_n(t)$. For star geometry this calculation is a variant of the Wigner decay problem, and hence can be solved analytically: instead of a *fixed* level that decays into a quasi-continuum we have a *moving* shuttle. The usual textbook procedure is followed [31] leading to the equations $\partial_t \Psi_0 = [-iu(t) - (\Gamma/2)]\Psi_0$, and $\partial_t \Psi_n = -i\epsilon_n \Psi_n - ic_n \Psi_0$. With $u(t) = \dot{u}t$ one obtains the solution

$$q_n(t) \ = \ \left| c_n \int_0^t d\tau \ \exp\left(i\epsilon_n \tau - i\frac{\dot{u}}{2}\tau^2 - \frac{\Gamma}{2}\tau \right) \right|^2 \tag{14}$$

By inspection one observes that going from the Slow to the Fast regime, the spreading line shape changes from Lorentzian-type to Fresnel-type, as illustrated in the lower panel of Fig.1.

Discussion.– We have found, using elementary considerations, without the need to rely on a complicated transport formalism, that it is possible to replace Eq.(2) by the general expression Eq.(7), that holds both in adiabatic and non-adiabatic circumstances. Hence the problem of calculating currents is reduced to that of calculating time dependent probabilities $q_n(t)$ as in the stochastic formulation [26]. It is important to realize that the "splitting ratio" Eq.(6) unlike the stochastic "partitioning ratio" is not bounded within $[0, 1]$. This observation has implications on the calculation of "counting statistics" and "shot noise" [32, 32].

In the analysis we have emphasized aspects that go beyond the familiar two-level approximation phenomenology. Whenever we have quasi-continuum the network levels are scrambled during the shuttling process: this is what we called "metamorphosis" or "mixing". This scrambling is reflected in the time dependence of the induced currents.

Finally, we realize that the non-adiabatic dynamics is characterized by a stochastic-like result for the integrated current Q. But the detailed temporal variation of the current I has different features depending on whether the shuttling process is Slow or Fast.

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[a] See supplementary material at [URL]

Supplementary material

Eigenstates for star geometry.– For a star geometry we have

$$\mathcal{H} = \begin{pmatrix} u & c_1^* & c_2^* & \dots & c_N^* \\ c_1 & \epsilon_1 & 0 & \dots & 0 \\ c_2 & 0 & \epsilon_2 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ c_N & 0 & \dots & 0 & \epsilon_N \end{pmatrix} \tag{15}$$

The eigenstates satisfy the following set of equations:

$$u\Psi_0 + \sum_{n=1}^{N} c_n^* \Psi_n \;=\; E\Psi_0 \tag{16}$$

$$c_n \Psi_0 + \epsilon_n \Psi_n \;=\; E\Psi_n, \qquad n = 1, 2, ..., N \tag{17}$$

From Eq.(17) we get

$$\Psi_n \;=\; \frac{c_n}{E - \epsilon_n}\sqrt{p}, \qquad p \equiv |\Psi_0|^2 \tag{18}$$

Substitution to Eq. (16) gives the secular equation $g(E) = E - u$, as stated after Eq. (4). From the normalization condition we get the expression for p, namely

$$p \;=\; \left[1 + \sum_{n=1}^{N} \frac{|c_n|^2}{(E - \epsilon_n)^2}\right]^{-1} \;=\; [1 - g'(E)]^{-1} \tag{19}$$

Calculations of p for a dot-wire geometry.– The energy levels of the wire that has length $L = N + 1$, are $\epsilon_n = -2c_0 \cos(k_n)$, where $k_n = (\pi/L)n$. The respective couplings to the shuttle are

$$c_n \;=\; \left[\left(\frac{2}{L}\right)^{1/2} \sin(k_n)\right] \, (C_a \pm C_b) \tag{20}$$

where the $\pm$ reflects the parity of the level. We focus on levels with energy $\epsilon_n \sim E$, such that their spacing Δ can be regarded as constant. Then it is convenient to absorb the factor $[(2/N)^{1/2} \sin(k)]$ into the definition of C_a and C_b. The couplings $c_n = c_\pm$ to the energy levels in the sum Eq.(4) are distinguished by their odd/even parity, hence after summation we get two terms:

$$g(E) \;=\; \left(\frac{\pi}{2\Delta}\right)\left[c_-^2 \cot\left(\pi\frac{E}{2\Delta}\right) - c_+^2 \tan\left(\pi\frac{E}{2\Delta}\right)\right] \tag{21}$$

The secular equation $g(E) = E - u$ becomes a quadratic equation for tan(), and can be solved explicitly.

$$\cot\left(\pi\frac{E}{2\Delta}\right) \;=\; \frac{\Delta}{\pi c_-^2}\left[(E-u) \pm \sqrt{(E-u)^2 + \left(\frac{\pi c_+ c_-}{\Delta}\right)^2}\right] \tag{22}$$

where the $\pm$ refers to the parity that is alternating for subsequent levels. Then it is possible to get an explicit expression for the shuttle occupation probability

$$p = [1-g'(E)]^{-1} = \left[1+\left(\frac{\pi}{2\Delta}\right)^2\left[\frac{c_-^2}{\sin^2\left(\pi\frac{E}{2\Delta}\right)}+\frac{c_+^2}{\cos^2\left(\pi\frac{E}{2\Delta}\right)}\right]\right]^{-1} \tag{23}$$

$$= \left[1+\left(\frac{\pi}{2\Delta}\right)^2\left[c_-^2\left(1+\cot^2\left(\pi\frac{E}{2\Delta}\right)\right)+c_+^2\left(1+\tan^2\left(\pi\frac{E}{2\Delta}\right)\right)\right]\right]^{-1} \tag{24}$$

$$= \left[1\pm\left(\frac{c_+^2-c_-^2}{c_+^2+c_-^2}\right)\frac{(E-u)\left[(E-u)^2+\left(\frac{\pi c_+c_-}{\Delta}\right)^2\right]^{1/2}}{(E-u)^2+\left(\frac{\pi c_+c_-}{\Delta}\right)^2+\left(\frac{2c_+^2c_-^2}{c_+^2+c_-^2}\right)}\right]^{-1}\frac{\left(\frac{2c_+^2c_-^2}{c_+^2+c_-^2}\right)}{(E-u)^2+\left(\frac{\pi c_+c_-}{\Delta}\right)^2+\left(\frac{2c_+^2c_-^2}{c_+^2+c_-^2}\right)} \tag{25}$$

$$\approx \left[1\pm\left(\frac{c_+^2-c_-^2}{c_+^2+c_-^2}\right)\frac{(E-u)}{\left[(E-u)^2+\left(\frac{\pi c_+c_-}{\Delta}\right)^2\right]^{1/2}}\right]^{-1}\frac{\left(\frac{2c_+^2c_-^2}{c_+^2+c_-^2}\right)}{(E-u)^2+\left(\frac{\pi c_+c_-}{\Delta}\right)^2} \tag{26}$$

$$\equiv \Delta\cdot \mathrm{L}\left[E-u;\ \pi\frac{c_+^2+c_-^2}{\Delta},\ \arcsin\left(\pm\frac{c_+^2-c_-^2}{c_+^2+c_-^2}\right)\right] \tag{27}$$

where the distorted Lorentzian is defined as follows:

$$\mathrm{L}\left[x;\Gamma;\theta\right] = \left[1+\frac{\sin\theta\ x}{\left[x^2+\cos^2\theta\ (\Gamma/2)^2\right]^{1/2}}\right]^{-1}\left(\frac{1}{\pi}\right)\frac{\cos^2\theta\ (\Gamma/2)}{x^2+\cos^2\theta\ (\Gamma/2)^2} \tag{28}$$

Note that for $\theta=0$ ($c_+=c_-$) and for $\theta=\pi/2$ ($c_-=0$) it becomes the standard Wigner Lorentzian.

Calculations of G for a dot-wire geometry.– The calculations of G involves similar sums as in the p calculation.

$$G = \frac{\partial}{\partial u}\left[\sum_n q_n\lambda_n\right] = C_a\frac{\partial}{\partial u}\left[p\sum_n\frac{c_n^*\langle n|a\rangle}{(E-\varepsilon_n)^2}\right] \tag{29}$$

$$= C_a\frac{\partial}{\partial u}\left[\frac{\frac{c_-}{\sin^2\left(\pi\frac{E}{2\Delta}\right)}+\frac{c_+}{\cos^2\left(\pi\frac{E}{2\Delta}\right)}}{\left(\frac{2\Delta}{\pi}\right)^2+\frac{c_-^2}{\sin^2\left(\pi\frac{E}{2\Delta}\right)}+\frac{c_+^2}{\cos^2\left(\pi\frac{E}{2\Delta}\right)}}\right] \tag{30}$$

leading to Eq.(12).

Calculations of G for a 3 site geometry.– Here we show how to to re-derive the exact result for $G(u)$ in the case of a 3-site system [19, 23] using the splitting ratio approach. For such system the 2 wire levels $\epsilon_\pm=\pm c_0$ have coupling $c_\pm=(C_a\pm C_b)/\sqrt{2}$ to the shuttle, and accordingly $\lambda_\pm=C_a/(C_a\pm C_b)$. Using Eq.(7) we get

$$G = \frac{\partial}{\partial u}\left[\sum_\pm\lambda_\pm q_\pm\right] = \frac{\partial}{\partial u}\left[\sum_\pm\lambda_\pm\frac{|c_\pm|^2}{(E-\epsilon_\pm)^2}p\right] \tag{31}$$

$$= \frac{\partial}{\partial u}\left[\left(1+\sum_\pm\frac{|c_\pm|^2}{(E-\epsilon_\pm)^2}\right)^{-1}\sum_\pm\frac{C_a(C_a\pm C_b)}{2(E-\epsilon_\pm)^2}\right] \tag{32}$$

$$= \frac{\partial}{\partial u}\left[\frac{C_a^2E^2+2c_0C_aC_bE+c_0^2C_a^2}{E^4+(C_a^2+C_b^2-2c_0^2)E^2+2c_0C_aC_bE+c_0^2(c_0^2+C_a^2+C_b^2)}\right] \tag{33}$$