



## Purely Electric Spin Pumping in One Dimension

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(Received 22 May 2009; revised manuscript received 24 July 2009; published 10 May 2010)

We show theoretically that a simple one-dimensional system (such as metallic wire) can display quantum spin pumping possibly without pushing any charge. It is achieved by applying two slowly varying orthogonal gate electric fields on different sections of the wire, thereby generating local spin-orbit (Rashba) terms such that unitary transformations at different places do not commute. This construction is a unique manifestation of a spin-orbit observable effect in purely one-dimensional systems with potentials respecting time-reversal symmetry.

DOI: 10.1103/PhysRevLett.104.196601

PACS numbers: 72.10.-d, 72.25.Ba

*Motivation.*—A standard way of achieving charge transfer across a conducting system is to apply two gate voltages and change them adiabatically and periodically: Under certain conditions, a charge is transferred across the system during each period. This is referred to as quantum (charge) pumping [1–5]. In recent years, pumping of spin polarization has become a focus of attention. One option to get a polarized current is to introduce a Zeeman splitting term [6], or employing ferromagnetic leads [7]. In some cases it costs a great deal of dissipated energy and besides, time-reversal invariance is broken. That motivates the quest for achieving spin pumping without the application of magnetic fields [8–11] (see also Ref. [12] where spin filtering is discussed). It is naturally expected that pertinent experiments are rather difficult to carry out, and hence, an obvious desirable property required from a model describing spin pumping is that it should be simple and experimentally feasible.

In the present work we show that spin pumping can be achieved in a simple one-dimensional device (wire), by exploiting the spin-orbit (SO) interaction of the electron with electric fields applied on two different sections of the wire (referred below as *Rashba barriers*). The model is characterized by the following attractive properties: (1) It demonstrates that spin pumping is one of the few manifestations of observable SO effects in (strictly) one-dimensional systems; (2) it enables pure spin (without charge) pumping; (3) the expressions obtained are simple, given in analytic form; (4) it serves as a pedagogical manifestation of the basic concepts of generalized forces and generalized charges.

*Outline.*—The order of presentation is as follows: First we derive an expression for the scattering matrix of a single Rashba barrier, and then recall a composition rule for computing the  $S$  matrix for scattering off two successive barriers. Once the  $S$  matrix of the whole device is obtained, the formalism of Refs. [2,3] (see also [13]) is employed in order to analyze the pumping process. An expression for

the pumped spin polarization ( $\vec{P}$ ) is derived, based on the concept of generalized forces and charges. Together with Refs. [9,14] it can be regarded as an  $SU(2)$  extension of the Brouwer formula for the pumped charge ( $Q$ ). Experimental aspects and quantitative estimates of the spin pumping current are then presented, followed by a short summary.

*One-dimensional model.*—The arena of our discussion is that of noninteracting electrons confined in a straight one-dimensional wire (along  $x$ ) possibly experiencing a scattering potential  $V(x)$ , and subject to a *perpendicular* electric field  $\mathbf{E}(x, t)$ . The time dependence will often be omitted. The Pauli Hamiltonian is

$$\mathcal{H} = \frac{1}{2m} p^2 - \frac{e\hbar}{8m^2 c^2} (\mathbf{E} \times \mathbf{p} - \mathbf{p} \times \mathbf{E}) \cdot \boldsymbol{\sigma} + V(x) \quad (1)$$

where  $m$  and  $e$  are the effective mass and the charge of the electron. Concretely, we have in mind a simple and experimentally feasible example where the wire passes through a couple of barriers  $i = 1, 2$  composed of plate capacitors  $C_i$  with different orientations (see Fig. 1). The fields  $\mathbf{E}_1(x) = (0, E_1, 0)$  and  $\mathbf{E}_2(x) = (0, 0, E_2)$  are non overlapping and concentrated within the intervals  $-L < x < 0$ , and  $0 < x < L$ . Barriers 1,2 are governed by Hamiltonians

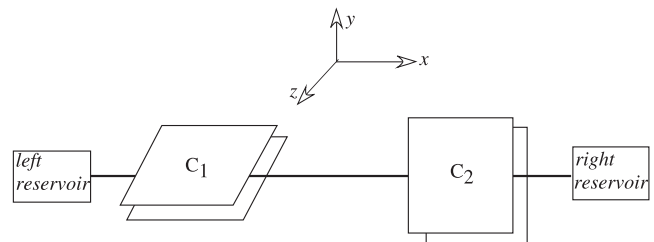


FIG. 1. The pumping device (schematic). Electrons move on a one-dimensional wire along the  $x$  direction between two reservoirs. Two capacitors  $C_1$  and  $C_2$  apply perpendicular electric fields  $\mathbf{E}_1 = (0, E_1, 0)$  and  $\mathbf{E}_2 = (0, 0, E_2)$  whose strength is controlled and varied periodically by an external circuit.

$$\mathcal{H}_1 = \frac{\hbar^2[k - \alpha_1(x)\sigma_z]^2}{2m} + v_1(x), \quad (2)$$

$$\mathcal{H}_2 = \frac{\hbar^2[k + \alpha_2(x)\sigma_y]^2}{2m} + v_2(x), \quad (3)$$

where  $k = -i\partial_x$  and  $\alpha_i(x) = eE_i(x)/4mc^2$ . The scalar potentials  $v_i(x) = V_i(x) - \hbar^2\alpha_i(x)^2/2m$  are nonvanishing within the respective intervals. For realistic circumstances, the second term in  $v_i(x)$  is small on any energy scale, and may be neglected [15]. Otherwise, there is charge pumping due to the barrier modulation, which can be calculated using the same basic formalism developed below. The dimensionless parameters that characterize the field interaction are,

$$\theta_i = 2 \int_{-\infty}^{\infty} \alpha_i(x') dx' \quad i = 1, 2. \quad (4)$$

The time dependence of  $\theta_i$  is assumed to be periodic and very smooth, justifying the use of the adiabatic approximation. Practically then, the time is used as a parameter that will be employed at a later stage when the spin pumping is discussed (hence it will not be specified before that). Our first goal is to find the  $S$  matrix for scattering through the system (see Fig. 1). The strategy would be to write down the Pauli equation and solve the scattering problem separately for each barrier thereby obtaining the corresponding  $S$  matrices  $S^{(1)}$  and  $S^{(2)}$  and then combine them to obtain  $S = S^{(1)} * S^{(2)}$ , the total  $S$  matrix.

*Scattering from a single Rashba barrier.*—The electric field  $E_1(x)$  in the left barrier is constant deep inside the capacitor and decays as a third power (in distance) outside it. For definiteness let us assume that the capacitor  $C_1$  is centered at  $x = -L/2$  and that  $L$  is sufficiently large so that  $E_1(x)$  is non-negligible only within  $-L < x < 0$ . We can then solve the stationary Schrödinger equation for scattering at energy  $\varepsilon$  for each barrier separately. For the first barrier, the equation is  $\mathcal{H}_1\psi_1(x) = \varepsilon\psi_1(x)$  where  $H_1$  is given in Eq. (2). Direct solution yields the  $4 \times 4$   $S$  matrix  $S_{ab}^{(1)}$  of the first barrier where the channel index is  $a = 1 \uparrow, 1 \downarrow, 2 \uparrow, 2 \downarrow$ . Within 2 block structure in Lead  $\otimes$  Spin space, it reads,

$$S^{(1)} = \begin{pmatrix} R_1 & T_1' \\ T_1 & R_1 \end{pmatrix} = \begin{pmatrix} r\mathbf{1} & tU_1^{-1} \\ tU_1 & r\mathbf{1} \end{pmatrix}. \quad (5)$$

Here  $\mathbf{1}$  is the  $2 \times 2$  identity matrix and  $U_1 = e^{i\theta_1\sigma_z}$  is an  $SU(2)$  spin rotation matrix [see also Eq. (6)]. The reflection and transmission amplitudes  $r$  and  $t$  are determined by the potential  $v_1(x)$ . Similar consideration applies to the second barrier as well. Assuming (just for convenience) that the second barrier has the same reflection and transmission amplitudes ( $r$  and  $t$ ), its  $S$  matrix has an identical structure as  $S^{(1)}$  albeit with different spin rotation matrix  $U_2 \neq U_1$ , since the spin-orbit term in the second barrier is  $\alpha_2(x)\sigma_y$  [compare Eqs. (2) and (3)]. In brief, the corresponding spin rotation matrices for barriers 1 and 2 are,

$$U_1 = e^{+i\theta_1\sigma_z/2}, \quad U_2 = e^{-i\theta_2\sigma_y/2}. \quad (6)$$

*Scattering from two Rashba barriers.*—The  $S$  matrix of the whole device is constructed by composing the two (nonoverlapping) barriers in a series, employing the following prescription [16] for calculating the transmission and reflection amplitudes:

$$\begin{aligned} T &= T_2(1 - R_1' R_2)^{-1} T_1, & T' &= T_1'(1 - R_2 R_1')^{-1} T_2', \\ R &= R_1 + T_1'(1 - R_2 R_1')^{-1} R_2 T_1, \\ R' &= R_2' + T_2(1 - R_1' R_2)^{-1} R_1' T_2'. \end{aligned} \quad (7)$$

In the *absence* of SO the transmission and reflection amplitudes due to the total potential  $v_1(x) + v_2(x)$  are

$$\tau = \frac{t^2}{1 - r^2}, \quad \rho = r \left( 1 + \frac{t^2}{1 - r^2} \right). \quad (8)$$

In the *presence* of SO, Eqs. (7) imply,

$$S = \begin{pmatrix} R & T' \\ T & R' \end{pmatrix} = \begin{pmatrix} \rho\mathbf{1} & \tau U_1^{-1} U_2^{-1} \\ \tau U_2 U_1 & \rho\mathbf{1} \end{pmatrix}, \quad (9)$$

where  $U_1$  and  $U_2$  are defined in Eq. (6). The noncommutativity of the  $SU(2)$  spin rotations,  $[U_1, U_2] \neq 0$ , is crucial for the operation of the pumping device, (discussed below) because only then the two pumping parameters  $\theta_1$  and  $\theta_2$  are generically independent.

*Gauge considerations in spin pumping.*—Our formalism employs the approximate  $SU(2)$  invariance of the Pauli equation [15]. For a one-dimensional system, it might be tempting to eliminate the  $SU(2)$  vector potential  $\mathcal{A}_x(x, t) = \frac{\hbar}{4mc} [\mathbf{E}(x, t) \times \boldsymbol{\sigma}]_x$  from the kinetic energy term  $\frac{1}{2m} [p_x + \frac{e}{c} \mathcal{A}_x(x, t)]^2$  by a suitable gauge transformation  $\psi(x, t) \rightarrow g(x, t)\psi(x, t)$ , such that

$$\mathcal{A}_x \rightarrow \mathcal{A}_x + g \mathcal{A}_x g^{-1} + g \partial_x g^{-1} = 0, \quad (10)$$

where  $g(x, t)$  is an  $SU(2)$  valued function. This is possible in principle, but there is a price, since pumping is essentially a time dependent problem (albeit treated adiabatically). The gauge transformation will generate a time component of the  $SU(2)$  vector potential such that  $i\hbar\partial_t \rightarrow i\hbar\partial_t - g(x, t)\partial_t[g(x, t)]^{-1}$ . The upshot is that in *static* problems, the  $SU(2)$  vector potential can be completely eliminated by an  $SU(2)$  gauge transformation, but in a time dependent problem, such as pumping, it cannot.

*Operation of the pumping device.*—We now consider the situation displayed in Fig. 1 where the SO dimensionless parameters  $\theta_1(t)$  and  $\theta_2(t)$  of Eq. (4) are controlled by slowly varying the fields inside the capacitors with a common period  $2\pi/\Omega$ . The adiabatic picture implies that the driving frequency  $\Omega$  is very small. By that we mean the usual condition for so-called adiabatic pumping, namely, if we have parametric driving with rate  $\dot{\theta}$  and frequency  $\Omega$  then the condition is  $\hbar\Omega, \hbar\dot{\theta} \ll \Delta E$ , where  $\Delta E$  is the energy scale over which the transmission fluctuates. For a device with a simple barrier (no resonance)  $\Delta E$  is simply  $\hbar v_F/L$  where  $L$  is the length of the device. Our goal is to

study the pumped charge and the pumped spin polarization during a single period. The generalized conductance  $G^{a,i}(\theta_i)$  is defined as in [13] via the relation:

$$dQ_a = - \sum_{i=1,2} G^{a,i}(\theta_i) d\theta_i, \quad (11)$$

where  $dQ_a$  is the charge pushed into channel  $a$ . If only one control parameter is manipulated (call it  $\theta$ ), and only one lead (say the left) is inspected, then one can use the simpler notations  $dQ_\uparrow = -G^\uparrow d\theta$  and  $dQ_\downarrow = -G^\downarrow d\theta$ . Accordingly, the net charge which is pushed into the specified lead is  $dQ = -(G^\uparrow + G^\downarrow) d\theta$  while the net spin polarization is  $dP_z = -(G^\uparrow - G^\downarrow) d\theta$ . Below it is shown that  $dP_x$  and  $dP_y$  can be calculated as well, and that our pump generates net spin polarization current while the net charge current vanishes at any moment.

*Pumping of spin polarization.*—The generalized conductance can be calculated using the Buttiker-Thomas-Pretre formula [2,3]. With our notations it reads:

$$G^{a,i}(\theta) = \frac{1}{2\pi i} \left[ \frac{\partial S}{\partial \theta_i} \mathbf{S}^\dagger \right]_{aa} \equiv -\frac{1}{2\pi} \left[ \mathbf{H}^{(i)} \right]_{aa}. \quad (12)$$

If one regards the  $\mathbf{S}(\theta)$  matrices as a group of unitary transformations, then the  $\mathbf{H}^{(i)}$  are interpreted as their generators. For the problem under consideration:

$$\begin{aligned} \mathbf{H}^{(1)} &= \frac{1}{2} \begin{pmatrix} |\tau|^2 \sigma_z & \tau \rho^* \sigma_z U_1^\dagger U_2^\dagger \\ -\tau \rho^* U_2 U_1 \sigma_z & -|\tau|^2 U_2 \sigma_z U_2^\dagger \end{pmatrix} \\ \mathbf{H}^{(2)} &= \frac{1}{2} \begin{pmatrix} -|\tau|^2 U_1^\dagger \sigma_y U_1 & -\tau \rho^* U_1^\dagger U_2^\dagger \sigma_y \\ \tau \rho^* \sigma_y U_2 U_1 & |\tau|^2 \sigma_y \end{pmatrix}. \end{aligned} \quad (13)$$

Simple manipulations show that the diagonal  $2 \otimes 2$  blocks in the above expressions are proportional to Pauli matrices which are traceless. Hence, the net charge which is pushed into any of the two leads vanishes,  $G^\uparrow + G^\downarrow = 0$ . On the other hand, the spin polarization current is determined by  $G^\uparrow - G^\downarrow$ , which generically does not vanish. It is estimated below to be of the order of  $|\tau|^2 \theta_0^2$  where  $|\tau|^2$  is the transmission coefficient [Eq. (8)], and  $\theta_0$  is the amplitude of the pumping parameters  $\theta_i$  defined in Eq. (4).

In order to get physical understanding of the spin pumping one should note that if the channel basis is changed, then  $\mathbf{H}$  undergoes a similarity transformation  $\mathbf{H} \mapsto \mathcal{T}^{-1} \mathbf{H} \mathcal{T}$  where  $\mathcal{T}$  is the transformation matrix from the old to the new basis. In particular, one is interested in block diagonal  $\mathcal{T}$ 's, such that each of the two  $2 \times 2$  blocks represents an  $SU(2)$  rotation of the axes that are attached to the respective lead. By an appropriate choice of axes, a given lead-related  $2 \times 2$  block of a given  $\mathbf{H}$  matrix can be transformed into the canonical form  $\mathbf{H}_{\text{lead}} \mapsto \frac{1}{2} |\tau|^2 \sigma_Z$  where  $Z$  is the new  $z$  axis. This means that the net spin polarization which is pushed into a lead is

$$dP_Z = |\tau|^2 \frac{d\theta}{2\pi}, \quad (14)$$

where  $\theta$  is either  $\theta_1$  or  $\theta_2$ . It should be appreciated that the *direction* ( $Z$ ) of the spin polarization current depends on whether  $\theta_1$  or  $\theta_2$  is being changed, and it is not the same for the left and for the right lead. Specifically, if  $\theta_1$  is being varied, then the spin polarization of the current in the left lead is in the  $z$  direction, while in the right lead it is in the  $xy$  plane, with an angle  $\theta_1$  relative to the  $y$  direction.

*SU(2) extension of the Brouwer formula.*—For a general pumping cycle the net pumping is given by a line integral over the conductance. Following Brouwer [3], one can replace this line integral by an area integral using Stokes theorem. Namely,

$$Q_a = \oint \mathbf{G} \wedge d\boldsymbol{\theta} = \iint C_{aa} d\theta_1 d\theta_2 \quad (15)$$

where  $\boldsymbol{\theta} = (\theta_1, \theta_2)$ , and  $\mathbf{G} = (G^{a,1}, G^{a,2})$ . In the above expression we have introduced the ‘‘rotor’’  $C_{aa}$  of  $\mathbf{G}$ , which can be regarded as a diagonal element of the matrix  $\mathbf{C} = -(1/2\pi)[\partial_1 \mathbf{H}_2 - \partial_2 \mathbf{H}_1]$ ; hence,

$$\mathbf{C} = \frac{1}{\pi} \Im \left[ \left( \frac{\partial \mathbf{S}}{\partial \theta_2} \right) \left( \frac{\partial \mathbf{S}^\dagger}{\partial \theta_1} \right) \right] = \frac{1}{2\pi i} [\mathbf{H}_2, \mathbf{H}_1]. \quad (16)$$

Note that if one changes the channel basis, then  $\mathbf{C}$  undergoes a similarity transformation. Calculating  $\mathbf{C}$  for our model system, one observes that the derivatives bring down  $\sigma_y \sigma_z = i\sigma_x$  and each  $\sigma_x$  is rotated by the corresponding  $SU(2)$  rotation matrix, by angles  $-\theta_1$  around  $z$  for  $U_1$  and  $\theta_2$  around  $y$  for  $U_2$ , leading to

$$\begin{aligned} \mathbf{C} &= \frac{i}{4\pi} |\tau|^2 \begin{pmatrix} U_1^\dagger \sigma_y \sigma_z U_1 & 0 \\ 0 & U_2 \sigma_y \sigma_z U_2^\dagger \end{pmatrix} \\ &= -\frac{1}{4\pi} |\tau|^2 \begin{pmatrix} \cos \theta_1 \sigma_x + \sin \theta_1 \sigma_y & 0 \\ 0 & \cos \theta_2 \sigma_x - \sin \theta_2 \sigma_z \end{pmatrix}. \end{aligned} \quad (17)$$

The  $U(1)$  Brouwer expression for the pumped charge and its  $SU(2)$  extension for the pumped spin polarization are,

$$Q_{\text{lead}} = \iint \text{Tr}(\mathbf{C} \mathbf{1}_{\text{lead}}) d\theta_1 d\theta_2 \quad (18)$$

$$\vec{P}_{\text{lead}} = \iint \text{Tr}(\mathbf{C} \vec{\sigma}_{\text{lead}}) d\theta_1 d\theta_2. \quad (19)$$

The matrix  $\mathbf{1}_{\text{lead}}$  is a projector on (say) the left lead, which means in practical terms that one can keep only the upper right  $2 \times 2$  block of  $\mathbf{C}$ , and sum only over the channels of the left lead. Equation (17) automatically entails  $\text{Tr}(\mathbf{C}) = 0$  and hence by Eq. (18)  $Q_{\text{lead}} = 0$ . This is to be expected when the effect of spin-orbit interaction appears as a pure gauge: it affects the wave function merely through an  $SU(2)$  phase factor. On the other hand, spin pumping is not zero since  $\mathbf{C}$  is multiplied by spin matrices before being traced, and  $\vec{P} \neq 0$ . Estimates below indicate that it is not necessarily small.

*Experimental aspects.*—In Ref. [17] it was stressed that the high purity of 2DEGs grown by molecular beam epi-

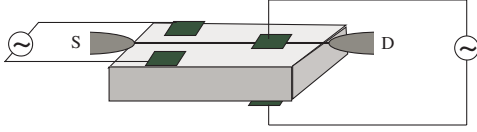


FIG. 2 (color online). Suggested experimental device for realizing spin pumping (schematic). The wire is stretched on a metallic substrate between source and drain and subject to horizontal and vertical ac fields at different regions so that they do not overlap.

taxy enables an almost collisionless motion of an electron through an experimental quasi-1D electronic system which has an arbitrary number of occupied transverse modes. Experiments on quantum-wire based devices that exploit both the charge and spin of an electron then become feasible (see Fig. 2). For a pumping cycle of frequency  $\Omega$  the pumped (vector) spin current  $\mathbf{I}^s$  is obtained by an integral over the area of the pumping cycle,

$$\mathbf{I}^s = \frac{e\Omega}{\pi} \int \text{Tr}[\mathbf{C}\boldsymbol{\sigma}] d\theta_1 d\theta_2.$$

For a very clean wire perfect transmission is assumed,  $|\tau|^2 = 1$ . Employing Eq. (17), it is easy to check that

$$\text{Tr}[\mathbf{C}\boldsymbol{\sigma}] = -\frac{1}{4\pi} [\cos\theta_1 + \cos\theta_2, \sin\theta_1, -\sin\theta_2].$$

The size of spin pumping is then determined solely by the dimensionless parameters  $\theta_{1,2}$  defined in Eq. (4). The fields  $E_{1,2}(x, t)$  inside the capacitors 1,2  $-L \leq x \leq 0$  and  $0 \leq x \leq L$  are chosen such that  $\alpha_{1,2}(x, t)$  and hence  $\theta_{1,2}$  can be approximated by  $\alpha_{1,2}(x, t) = \alpha_0 \cos\Omega t$  ( $\alpha_0 \sin\Omega t$ ) and  $\theta_{1,2}(x, t) = \theta_0 \cos\Omega t$  ( $\theta_0 \sin\Omega t$ ) where  $\theta_0 = \alpha_0 L$ . Assuming a pumping cycle in the form of a square  $0 \leq \theta_i \leq \theta_0$  the pumped spin currents read,

$$\begin{aligned} I_x^s &= \frac{e\Omega}{4\pi^2} \int_0^{\theta_0} \int_0^{\theta_0} (\cos\theta_1 + \cos\theta_2) d\theta_1 d\theta_2 \\ &= -\frac{2e\Omega\theta_0 \sin\theta_0}{4\pi^2}, \\ I_y^s &= -I_z^s = \frac{2e\Omega\theta_0(1 - \cos\theta_0)}{4\pi^2}. \end{aligned}$$

It remains to estimate the parameter  $\theta_0$ . In Ref. [18], the quantity  $\eta \equiv \frac{\hbar^2}{2m} \alpha_0$  is experimentally found for the  $\text{In}_{0.53}\text{Ga}_{0.47}\text{As}/\text{In}_{0.52}\text{Al}_{0.48}\text{As}$  heterostructure. For gate voltage  $V_g = -1.5$  V,  $\eta \approx 10^{-11}$  eV m (see Fig. 5 of Ref. [18]). For  $m = 0.05m_0$  [18], this gives  $\alpha \approx 2\mu^{-1}$ , and hence  $\theta_0 = \alpha_0 L = 0.8$ . This generous estimate is based on a 2D experimental setup, but as pointed out in Ref. [17], the value of  $\alpha$  should not be much affected in 1D systems as well.

*Discussion.*—On the practical level it has been demonstrated in this work that in a strictly 1D device it is possible to push polarization into the leads without pushing charge, and that this can be carried out using two gates and without magnetic fields. This should be contrasted with more complicated arrangements that were suggested for this purpose,

e.g., in Ref. [12]. The scheme considered in our analysis is based on a pumping (time dependent) setup, instead of the more conventional transmission filter setup. On the theoretical side, a very simple result for the pumped spin polarization has been obtained, namely, Eq. (14). As demonstrated, it can also be formulated as an  $SU(2)$  extension of the Brouwer formula for charge pumping, noting that the geometric (Kubo) conductance  $G$  is formally a 2-form (curvature), while  $C$  is a 3-form (scalar).

We have illuminated the gauge consideration in the theory: while in the time-independent setting it is possible to transform away the SO interaction, in spite of the non-commutativity of the  $SU(2)$  gauge transformations, this is no longer true for the time dependent Hamiltonian that describes the pumping scenario.

We would like to thank P. Sharma, C. Chamon, J. Nitta, A. Yacoby, and A. Hamilton for fruitful discussions. This research is partially supported by: BSF, and DIP grants (D. C.); ISF grant (Y. A.); Grant-in-Aids No. 19048015, No. 21244053, NAREGI Nanoscience Project from the Ministry of Education, Culture, Sports, Science and Technology, Japan, and Strategic Japanese-German Joint Research Program, JST, Japan (N. N.).

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