Quantum stirring of particles

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Outline:

Quantum Stirring of BEC in a 3 site system;

The linear response / Kubo analysis;

Counting Statistics in closed geometries;

Analysis of a double path adiabatic passage;

Analysis of a double path Bloch transition.

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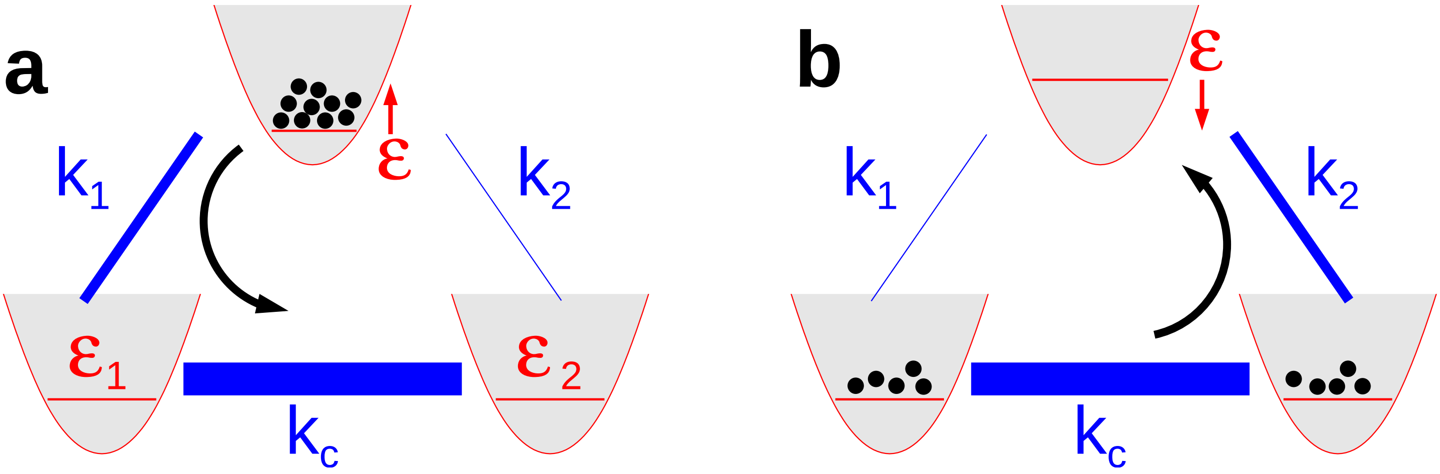
Quantum Stirring in a 3 site system



$$
\begin{aligned}
\hat{\mathcal{H}} &= \sum_{i=0}^{2} \varepsilon_i n_i + \frac{U}{2} \sum_{i=0}^{2} \hat{n}_i(\hat{n}_i - 1) - k_c(\hat{b}_1^\dagger \hat{b}_2 + \hat{b}_2^\dagger \hat{b}_1) \\
&\quad -k_1(\hat{b}_0^\dagger \hat{b}_1 + \hat{b}_1^\dagger \hat{b}_0) - k_2(\hat{b}_0^\dagger \hat{b}_2 + \hat{b}_2^\dagger \hat{b}_0)
\end{aligned}
$$

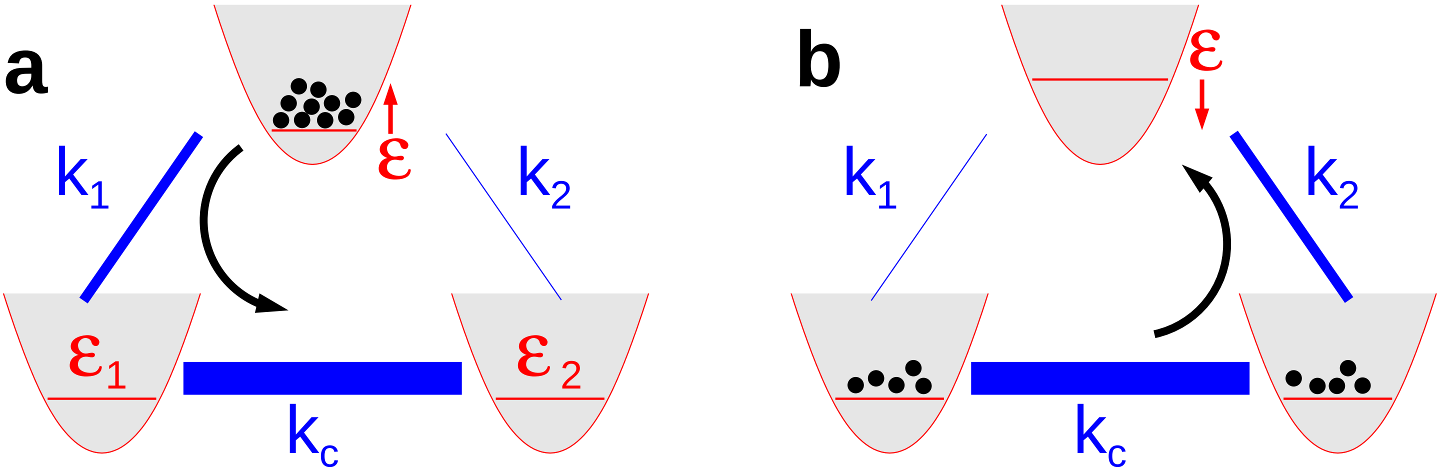
Trimer system [Hiller, Kottos, Geisel, Bodyfelt, Stickney, Anderson, Zozulya]

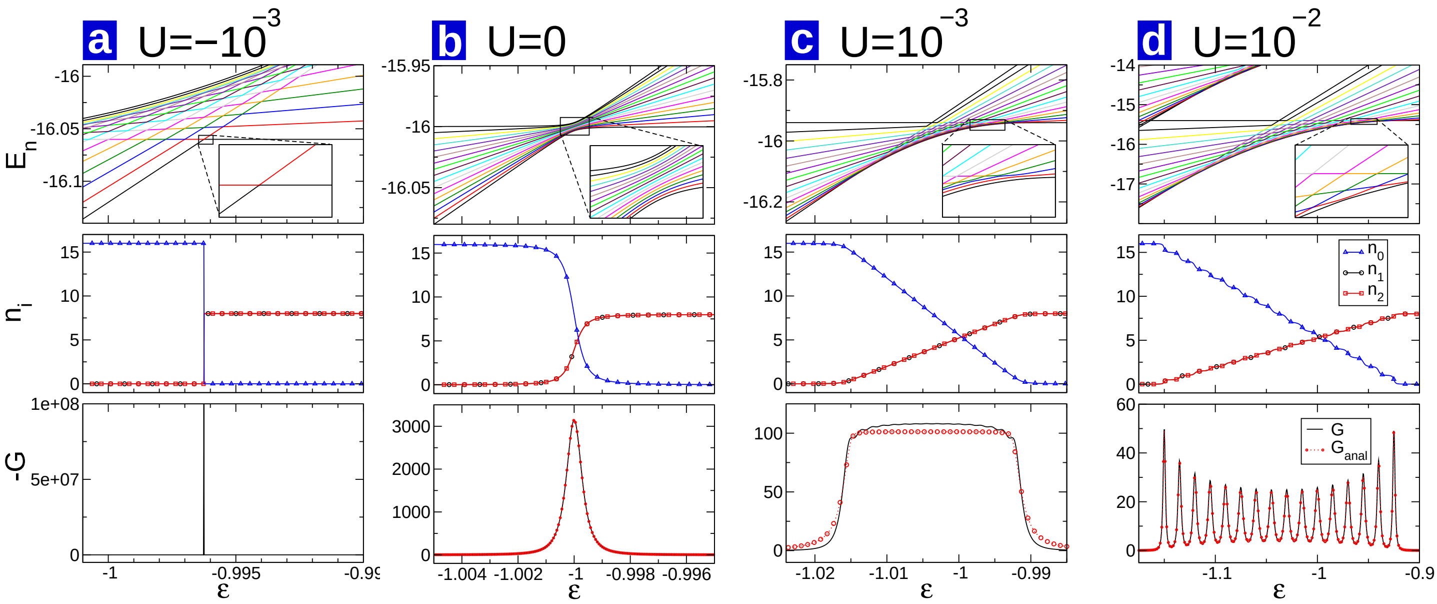
$$U = \text{the inter-atomic interaction}$$

Control parameters:

$$
\begin{aligned}
X_1 &= \left(\frac{1}{k_2} - \frac{1}{k_1}\right) \\
X_2 &= \varepsilon
\end{aligned}
$$

Dynamical scenarios





$$
\begin{aligned}
I &= -G\ \dot{X} \\
dQ &= -G\ d\varepsilon
\end{aligned}
$$

strong attractive interaction: classical ball dynamics

negligible interaction ($|U| \ll \kappa$): mega-crossing

weak repulsive interaction: gradual crossing

strong repulsive interaction ($U \gg N\kappa$): sequential crossing

Results for the geometric conductance

For $U = 0$, mega crossing

$$G \;=\; -N \frac{(k_1^2 - k_2^2)/2}{[(\varepsilon - \varepsilon_-)^2 + 2(k_1 + k_2)^2]^{3/2}}$$

For $\kappa \ll U \ll N\kappa$, gradual crossing

$$G \;\approx\; -\left[\frac{k_1 - k_2}{k_1 + k_2}\right] \frac{1}{3U}$$

For $U \gg N\kappa$, sequential crossing

$$G \;=\; -\left(\frac{k_1 - k_2}{k_1 + k_2}\right) \sum_{n=1}^{N} \frac{(\delta\varepsilon_n)^2}{[(\varepsilon - \varepsilon_n)^2 + (2\delta\varepsilon_n)^2]^{3/2}},$$

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Linear response theory

For one parameter driving by EMF

$$
\begin{aligned}
I &= \boldsymbol{G}^{33} \times (-\dot{X}_3) \\
dQ &= -\boldsymbol{G}^{33}\ dX_3
\end{aligned}
$$

For driving by changing another parameter

$$
\begin{aligned}
I &= -\boldsymbol{G}^{31}\ \dot{X}_1 \\
dQ &= -\boldsymbol{G}^{31}\ dX_1
\end{aligned}
$$

For two parameter driving

$$
\begin{aligned}
I &= -\boldsymbol{G}^{31}\dot{X}_1\ -\boldsymbol{G}^{32}\dot{X}_2 \\
dQ &= -\boldsymbol{G}^{31}\ dX_1\ -\boldsymbol{G}^{32}\ dX_2 \\
Q &= -\oint \boldsymbol{G} \cdot dX
\end{aligned}
$$

(*) In general

$$
\langle F^k \rangle = -\sum_j \boldsymbol{G}^{kj}\ \dot{X}_j
$$

The Kubo formula approach

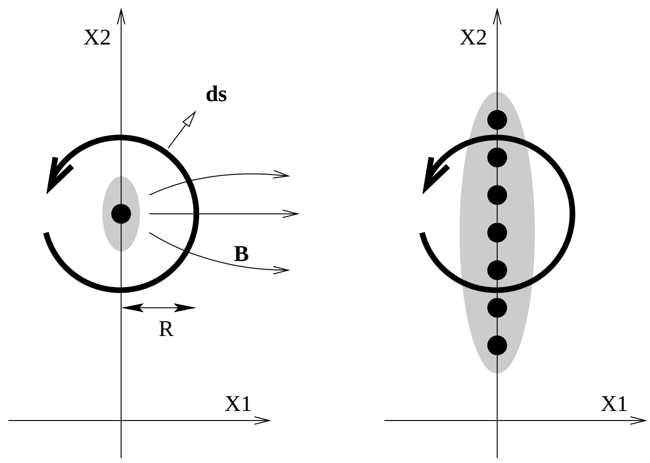
$$Q = -\oint (G^{31} dX_1 + {\color{red} G^{32} dX_2}) = \oint \boldsymbol{B} \cdot \vec{ds}$$

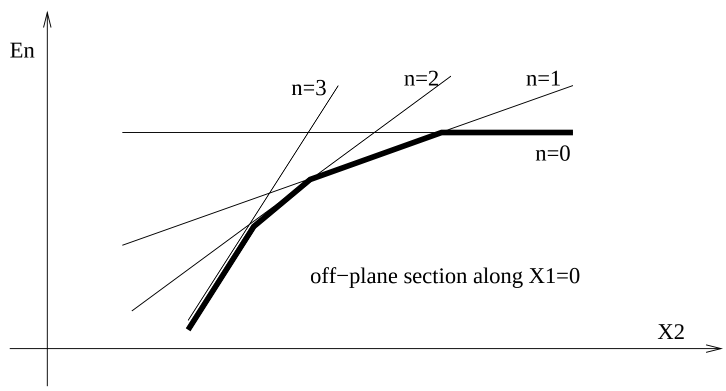
$$\boldsymbol{B} = (-G^{32}, G^{31}) \qquad\qquad \vec{ds} = (dX_2, -dX_1)$$

$$\boldsymbol{B}_j = \sum_{n \neq n_0} \frac{2 \, \mathrm{Im}[\mathcal{I}_{n_0 n}] \; \mathcal{F}^j_{n n_0}}{(E_n - E_{n_0})^2}.$$

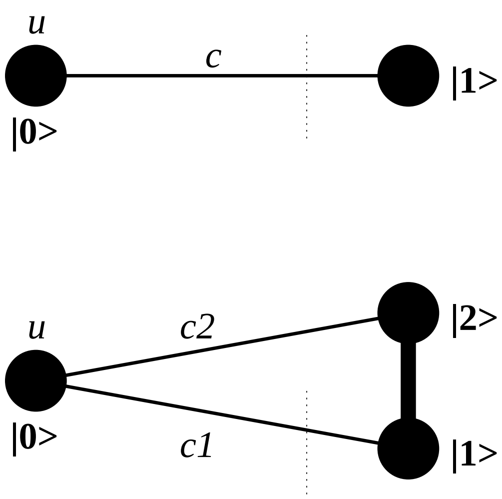
$$X_1 = \left(\frac{1}{k_2} - \frac{1}{k_1}\right)$$

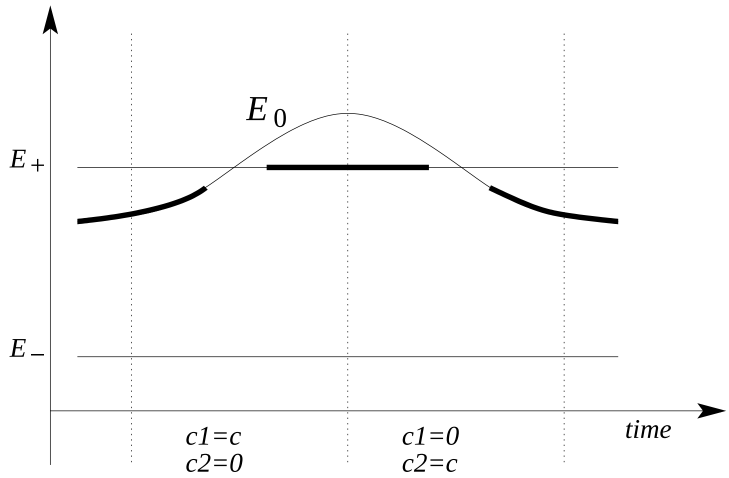
$$X_2 = \varepsilon$$





Counting Statistics





$$\mathcal{Q} \;=\; \int_0^t \mathcal{I}(t')dt'$$

$\langle \mathcal{Q} \rangle =???$ $\qquad \delta Q =???$ $\qquad P(Q) =???$

Simple example: adiabatic passage with $c_1 = c_2$.
We say "$\langle \mathcal{Q} \rangle = \frac{1}{2}$". What do we mean by that?

- maybe $P(1/2)$=100% and hence $\delta Q = 0$
- maybe: $P(0)$=$P(1)$=50% and hence $\delta Q = 1/2$

How Q is measured

Naive answer:

The distribution $\mathrm{P}(Q)$ can be determined using a continuous measurement scheme.

In such setup the current induces (so to say) a "translation" of a Von-Neumann pointer. After time t the position of the pointer is measured.

Proper answer:

One can measure the quasi distribution $\mathrm{P}(Q;x)$

$$\rho_t(q,x) = \int \mathrm{P}(q-q';x)\rho_0(q',x)dq'$$

where

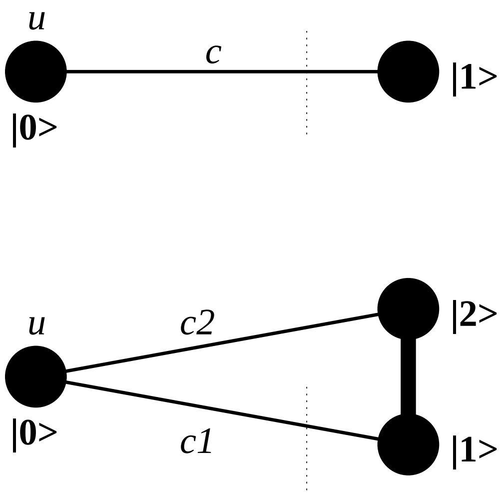
$$\mathrm{P}(Q;x{=}0) = \frac{1}{2\pi}\int \left\langle \left[\mathcal{T}\mathrm{e}^{-i(r/2)\mathcal{Q}}\right]^{\dagger} \left[\mathcal{T}\mathrm{e}^{+i(r/2)\mathcal{Q}}\right] \right\rangle \mathrm{e}^{-iQr}dr$$

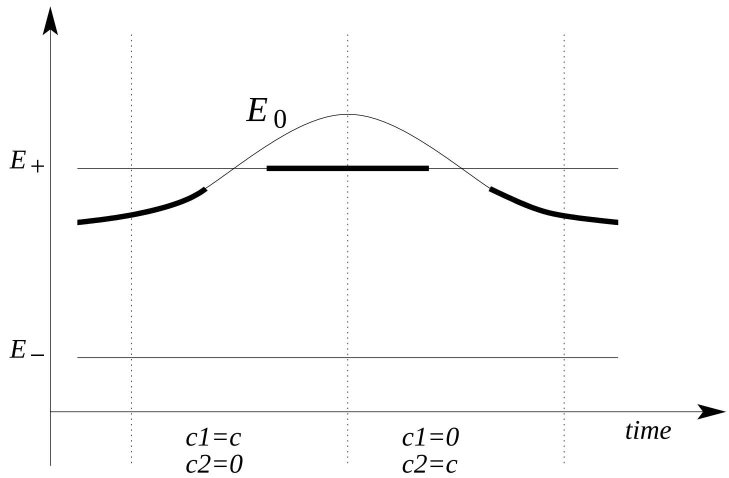
If we ignore time ordering we get:

$$\mathrm{P}(Q) \;=\; \frac{1}{2\pi}\int \left\langle \mathrm{e}^{+ir\mathcal{Q}}\right\rangle \mathrm{e}^{-iQr}dr \;=\; \langle\delta(Q-\mathcal{Q})\rangle$$

H. Everett, Rev. Mod. Phys. 29, 454 (1957).
L.S. Levitov and G.B. Lesovik, JETP Letters (1992).
L.S. Levitov and G.B. Lesovik, JETP Letters (1993).
Y.V. Nazarov and M. Kindermann, EPJ B (2003).

Hamiltonians for 2 and 3 site systems





$$\mathcal{H} = \begin{pmatrix} u & c \\ c & 0 \end{pmatrix}, \qquad \mathcal{I} = \begin{pmatrix} 0 & ic \\ -ic & 0 \end{pmatrix}$$

$$\mathcal{H} = \begin{pmatrix} u & c_1 & c_2 \\ c_1 & 0 & 1 \\ c_2 & 1 & 0 \end{pmatrix}, \qquad \mathcal{I} = \begin{pmatrix} 0 & ic_1 & 0 \\ -ic_1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathcal{H} = \begin{pmatrix} u & \frac{(c_1+c_2)}{\sqrt{2}} \\ \frac{(c_1+c_2)}{\sqrt{2}} & 1 \end{pmatrix}, \qquad \mathcal{I} = \frac{c_1}{\sqrt{2}} \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$$

Double path adiabatic passage

$$\mathcal{H} = \begin{pmatrix} u(t) & c \\ c & 1 \end{pmatrix}, \qquad \mathcal{I} = \lambda \begin{pmatrix} 0 & ic \\ -ic & 0 \end{pmatrix}$$

with effective coupling and splitting ratio

$$c \equiv \frac{(c_1+c_2)}{\sqrt{2}}, \qquad \lambda \equiv \frac{c_1}{c_1+c_2}$$

Accordingly:

$$\begin{aligned} \langle \mathcal{Q} \rangle &= \lambda p \\ \delta Q^2 &= \lambda^2 \underbrace{(1-p)p} \quad \neq \quad (1-\lambda p)\lambda p \end{aligned}$$

If c_1 and c_2 have opposite signs then (say) λ becomes larger than unity, while $(1-\lambda)$ is negative. This reflects that the driving induces a circulating current within the ring, and illuminates the fallacy of the classical peristaltic point of view.

Counting statistics for a coherent transition

Naive expectation:

Given the probability p to make the transition

$$\mathrm{P}(Q) = \begin{cases} 1-p & \text{for } Q = 0 \\ p & \text{for } Q = 1 \end{cases}$$

$$\langle \mathcal{Q}^k \rangle = \mathrm{P}(1)\cdot 1^k + \mathrm{P}(0)\cdot 0^k = p$$

$$\delta Q^2 = (1-p)p$$

Quantum result:

$$\mathrm{P}(Q) = \begin{cases} p_- & \text{for } Q = Q_- \\ p_+ & \text{for } Q = Q_+ \end{cases}$$

where

$$Q_\pm = \pm\sqrt{p}$$

$$p_\pm = \frac{1}{2}(1 \pm \sqrt{p})$$

hence

$$\langle \mathcal{Q}^k \rangle = p_+ Q_+^k + p_- Q_-^k = p^{\left\lfloor \frac{k+1}{2} \right\rfloor}$$

Restricted quantum-classical correspondence

$\mathcal{N}$ = occupation operator (eigenvalues $= 0, 1$)

$\mathcal{I}$ = current operator

Heisenberg equation of motion:

$$\frac{d}{dt}\mathcal{N}(t) \;=\; \mathcal{I}(t)$$

leads to

$$\mathcal{N}(t) - \mathcal{N}(0) = \mathcal{Q}$$

hence

$$\langle \mathcal{Q}^k \rangle \;=\; \langle \mathcal{N}^k \rangle_t \;=\; p \qquad\qquad \text{for } k = 1, 2.$$

Restricted QCC is robust

Detailed QCC is fragile

Restricted QCC survives even in the presence of diffraction:

A. Stotland and D. Cohen, J. Phys. A 39, 10703 (2006).

Digression: Analysis of quantum stirring

Assume $|c_1| > |c_2| > 0$ in the 1st half of the cycle and interchange them in the 2nd half of the cycle.

Naive expectation:

$\langle \mathcal{Q} \rangle < 1$ [per cycle]

Quantum result:

$$\langle \mathcal{Q} \rangle = \frac{c_1 - c_2}{c_1 + c_2} \qquad \text{[per cycle]}$$

An optional way to derive this result is to make a full 3 level calculation using the Kubo formula:

`D. Cohen, Phys. Rev. B 68, 155303 (2003).`

Long time Counting Statistics

Naive expectation:

Probabilistic point of view implies

$$\delta Q \quad \propto \quad \sqrt{t}$$

Quantum result:

The eigenvalues $Q_{\pm}$ of the $\mathcal{Q}$ operator grow linearly with the number of cycles

$$\delta Q \quad \propto \quad t$$

If we have good control over the preparation we can select it to be a Floque state of the quantum evolution operator. For such preparation the linear growth of δQ is avoided, and it oscillates around a residual value.

Conclusions

Quantum mechanics is a "deterministic" rather than a "probabilistic" theory.

Coherent splitting unlike probabilistic splitting of a wavepacket is "exact".

In a double path adiabatic passage one may find that (say) 170% of the particle goes via one path, while -70% goes via the second path.

There is at most restricted quantum-classical correspondence for the first and second moments.

If we have N condensed particles the sign and the strength of the interactions determines the nature of the dynamics: mega crossing or gradual crossing or sequential crossing, or classical ball dynamics.