Control of atomic currents using a quantum stirring device

Moritz Hiller,[1,2,3] Tsampikos Kottos,[1,2] and Doron Cohen[4]
[1]*Department of Physics, Wesleyan University, Middletown, Connecticut 06459, USA*
[2]*MPI for Dynamics and Self-Organization, Bunsenstraße 10, D-37073 Göttingen, Germany*
[3]*Fakultät für Physik, Universität Göttingen, Friedrich-Hund-Platz 1, D-37077 Göttingen, Germany*
[4]*Department of Physics, Ben-Gurion University, Beer-Sheva 84105, Israel*

We propose a BEC stirring device which can be regarded as the incorporation of a quantum pump into a closed circuit: it produces a DC circulating current in response to a cyclic adiabatic change of two control parameters of an optical trap. We demonstrate the feasibility of this concept and point out that such device can be utilized in order to probe the interatomic interactions.

The realization of Bose-Einstein Condensation (BEC) of ultra-cold atoms in optical lattices (OL) [1, 2, 3, 4, 5] and the availability of atom chips [6, 7] is considered to be a major breakthrough with potential applications in the arena of quantum information processing. A major advantage of BEC based devices, as compared to conventional solid-state structures, lies in the extraordinary degree of precision and control that is available over the confining potential and the interatomic interactions. This opens the possibility to design BEC-based high performance or precision devices.

The possibility to induce DC currents by periodic (AC) modulation of the potential is familiar from the context of electronic devices. If an open geometry is concerned, it is known as "quantum pumping" [8, 9, 10, 11, 12, 13], while for closed geometry we use the term "quantum stirring" [14]. We consider below the stirring of condensed ultra-cold atoms [15, 16] due to the periodic variation of the on-site potentials and of the tunneling rates between adjunct confining traps. We show that the nature of the transport process depends crucially on the sign and on the strength of the interatomic interactions. We distinguish between 3 types of dynamical behavior: For strong repulsive interaction the particles are transported one-by-one; As the repulsive interaction becomes weaker the sequential tunneling process is replaced by a gradual or by a coherent mega crossing; For strong attractive interaction the particles are glued together and behave like a huge classical ball that rolls from trap to trap.

The simplest model that captures the physics of quantum stirring is a 3 site Bose-Hubbard Hamiltonian (BHH) [17, 18, 19], which is illustrated in Fig. 1. One site (i=0) is regarded as a "shuttle" (energy ε), while the other two sites (i=1, 2) are regarded as a two level "canal" (energies ε_+ and ε_-). The Hamiltonian for an N body system is:

$$\hat{\mathcal{H}} = \sum_{i=0}^{2} v_i n_i + \frac{U}{2}\sum_{i=0}^{2} \hat{n}_i(\hat{n}_i - 1) - k_c(\hat{b}_1^\dagger \hat{b}_2 + \hat{b}_2^\dagger \hat{b}_1) - k_1(\hat{b}_0^\dagger \hat{b}_1 + \hat{b}_1^\dagger \hat{b}_0) - k_2(\hat{b}_0^\dagger \hat{b}_2 + \hat{b}_2^\dagger \hat{b}_0) \quad (1)$$

Without loss of generality we use mass units such that $\hbar$=1, and time units such that intra canal hopping amplitude is k_c=1. The on-site potential at site i is defined as v_i. Below we assume for simplicity that $v_1 = v_2 = 0$, and hence the single particle canal levels are $\varepsilon_\pm = \pm 1$. We consider $v_0 = \varepsilon$ as one control parameter of the pumping cycle. The annihilation and creation operators $\hat{b}_i$ and $\hat{b}_i^\dagger$ obey the canonical commutation relations $[\hat{b}_i, \hat{b}_j^\dagger] = \delta_{i,j}$ while the operators $\hat{n}_i = \hat{b}_i^\dagger \hat{b}_i$ count the number of bosons at site i. The interaction strength between two atoms in a single site is given by $U = 4\pi\hbar^2 a_s V_{\text{eff}}/\mathsf{m}$ where V_{eff} is the effective volume, m is the atomic mass, and a_s is the s-wave scattering length. Below we assume that $NU \ll 1$

The couplings between the shuttle and the two ends of the canal are k_1 and k_2. We assume that both are much smaller than k_c. Their inverse $1/k_1$ and $1/k_2$ are like barrier heights, and changing them is like switching valves on and off. It is convenient to define the two control parameters of the pumping as follows:

$$X_1 = \left(\frac{1}{k_2} - \frac{1}{k_1}\right), \qquad X_2 = \varepsilon \quad (2)$$

By periodic cycling of the parameters (X_1, X_2) we can imitate a classical peristaltic mechanism and obtain a non-zero amount (Q) of transported atoms per cycle. The pumping cycle is illustrated in Figs. 1,2. Initially all the particles are located in the shuttle which has a sufficiently negative on-site potential energy ($X_2 < 0$). In the first half of the cycle the coupling is biased in favor of the k_1 route ($X_1 > 0$) while X_2 is raised until (say) the shuttle is empty. In the second half of the cycle the coupling is biased in favor of the k_2 route ($X_1 < 0$), while X_2 is lowered until the shuttle is full. Assuming U=0, the shuttle is depopulated via the k_1 route into the lower energy level ε_- during the first half of the cycle, and re-populated via the k_2 route during the second half of the cycle. Accordingly the net effect is to have a non-zero Q. If we had a single particle in the system, the net effect would be to pump roughly one particle per cycle. If we have N non-interacting particles, the result of the same cycle is to pump roughly N particles per cycle. We would like to know what is the actual result using a proper quantum mechanical calculation, and furthermore we would like to investigate what is the effect of the interatomic interaction U on the result.

In analogy with Ohm's law (where X is the magnetic flux, and $-\dot{X}$ is the electro motive force), the current is $I=-G_1\dot{X}_1$ if we change X_1 and $I=-G_2\dot{X}_2$ if we change X_2, where G_1 and G_2 are elements of the geometric conductance matrix. Accordingly

$$Q=\oint_{\text{cycle}} I dt=-\oint (G_1 dX_1+G_2 dX_2). \tag{3}$$

In order to calculate the geometric conductance we use the Kubo formula approach to quantum pumping [20] which is based on the theory of adiabatic processes [21, 22, 23]. It turns out that in the strict adiabatic limit G is related to the vector field $\boldsymbol{B}$ (also know as "two-form") in the theory of Berry phase. Namely, using the notations $\boldsymbol{B}_1=-G_2$ and $\boldsymbol{B}_2=G_1$ we can rewrite Eq.(3) as

$$Q=\oint \boldsymbol{B}\cdot\vec{ds}, \tag{4}$$

where we define the normal vector $\vec{ds}=(dX_2,-dX_1)$ as illustrated in Fig. 1. The advantage of this point of view is in the intuition that it gives for the result: Q is related to the flux of a field $\boldsymbol{B}$ which is created by "magnetic charges" in X space. For U=0 all the magnetic charge is concentrated in one point. As the interaction U becomes larger the "magnetic charge" disintegrates into N elementary "monopoles" (see Fig. 2). In practice the calculation of $\boldsymbol{B}$ is done using the following formula:

$$\boldsymbol{B}_j=\sum_{n\neq n_0}\frac{2\ \mathrm{Im}[\mathcal{I}_{n_0 n}]\ \mathcal{F}^j_{nn_0}}{(E_n-E_{n_0})^2}. \tag{5}$$

Above $\mathcal{I}=i/2\left[(\hat{b}_0^\dagger\hat{b}_1-\hat{b}_1^\dagger\hat{b}_0)+(\hat{b}_2^\dagger\hat{b}_0-\hat{b}_0^\dagger\hat{b}_2)\right]$ is the averaged current along the bonds $0\mapsto 1$, $2\mapsto 0$ and $\mathcal{F}^j=-\partial\mathcal{H}/\partial X_j$ is the generalized force associated with the control parameter X_j. The index n distinguishes the eigenstates of the many-body Hamiltonian. We assume from now on that n_0 is the BEC ground state.

Below we find convenient to define the average coupling as $\kappa=(k_1+k_2)/\sqrt{2}$. In the zero order approximation k_1 and k_2 are neglected, and later we take them into account as a perturbation. For $\kappa=0$ the number of particles in the dot n becomes a good quantum number. Furthermore, we adopt a "two orbital approximation": we assume that there is non-zero occupation only in the shuttle and in the lower canal level, which is valid if $NU\ll k_c$=1. Accordingly the many body energies are

$$E_n=E_{\text{shuttle}}(n)+E_{\text{canal}}(N-n),\qquad n=0,1,2,..,N \tag{6}$$

where

$$\begin{aligned} E_{\text{shuttle}}(n) &= \varepsilon n+(1/2)U(n-1)n \\ E_{\text{canal}}(N-n) &= -(N-n)+(1/4)U(N-n-1)(N-n) \end{aligned} \tag{7}$$

The location $\varepsilon_n=-1+\frac{1}{2}U\times(N-3n+2)$ of the $n\mapsto(n-1)$ crossing is determined from the degeneracy condition $E_n-E_{n-1}=0$, where $n=1,2,...N$ can be associated with the number of particles in the shuttle. The crossings are distributed within

$$-1-(N-1)U\ \ \leq\ \ \varepsilon\ \ \leq\ \ -1+\frac{1}{2}(N-1)U. \tag{8}$$

The rescaled version of the control variable is $\hat{\varepsilon}=(\varepsilon+1)/((N-1)U)$, and its support is $-1<\hat{\varepsilon}<1/2$. Within this range we have N crossings. The distance between the crossings, while varying the dot potential ε, is U. Once we take κ into account we get *avoided* crossings of width $\delta\varepsilon_n=[(N+1-n)n/2]^{1/2}\kappa$. If κ/U is large these avoided crossings merge and eventually we get one mega crossing. For the purpose of further analysis we write the many body Hamiltonian in the two-orbital approximation as a matrix (say for $N=4$):

$$\mathcal{H}=\begin{pmatrix} E_0 & -\kappa_1 & 0 & 0 & 0 \\ -\kappa_1 & E_1 & -\kappa_2 & 0 & 0 \\ 0 & -\kappa_2 & E_2 & -\kappa_3 & 0 \\ 0 & 0 & -\kappa_3 & E_3 & -\kappa_4 \\ 0 & 0 & 0 & -\kappa_4 & E_4 \end{pmatrix} \tag{9}$$

where the couplings are defined as $\kappa_n=\langle n-1|\mathcal{H}|n\rangle$. The calculation of κ_n involves the matrix elements of $b_i^\dagger b_0$, leading to $\kappa_n=[(N+1-n)n]^{1/2}\kappa$. Analogous expression applies to the current operator where κ is replaced by $(k_1-k_2)/\sqrt{2}$. For large U, as ε is varied, we encounter a sequence of distinct Landau-Zener transitions ($|4\rangle\mapsto|3\rangle\mapsto|2\rangle\mapsto|1\rangle\mapsto|0\rangle$). The distance between avoided crossings is of order U while their width is $\delta\varepsilon_n=\kappa_n$. The widest crossings are at the center with $\delta\varepsilon_n\sim N\kappa$. This should be contrasted with the energy scales U and NU that describe the span of the crossings. Accordingly we deduce the following regimes:

$U\ll\kappa$	mega crossing regime
$U\gg N\kappa$	sequential crossing regime

We observe that the regime of behavior depends on the ratio κ/U. If N is not too large one can resolve a sequence of two level crossings. In the following paragraphs we summarize the results in the various regimes. In particular Eq.(12) below is obtained from Eq.(5) with a two level approximation for each crossing.

The predominant contribution to Q comes during the dX_2 variation, therefore we refer from now on to $G_2=G$ only. An overview of the numerical results for the conductance is shown in Fig. 3, where we plot G as a function of X_2 for various interaction strengths U. In Fig. 4 more details are presented: besides G we also plot the X_2-dependence of the energy levels, and of the site population. Four representative values of U are considered including also the $U<0$ case.

Let us try to understand the observed results. For U=0 the analytical calculation is just N times the single

particle result:

$$G = -N\frac{(k_1^2 - k_2^2)/2}{[(\varepsilon - \varepsilon_-)^2 + 2(k_1{+}k_2)^2]^{3/2}}, \qquad (10)$$

which can be expressed as a function of the control parameters (X_1, X_2). Integrating over a full cycle one obtains

$$Q = N\frac{[1 + (\kappa R)^2]^{1/2} - 1}{\kappa R}, \qquad (11)$$

where κ is the average value of the coupling constants, and R is the radius of the pumping cycle (see Fig. 2). For small cycles we get $Q \approx N\kappa R/2$, while for large cycles we get the limiting value $Q \approx N$. For $U{=}0$ and also for small values of U all the particles cross "together" from the shuttle orbital to the ε_- canal orbital. We call this type of dynamics "mega crossing". For very repulsive interaction ($U{>}0$) we get

$$G = -\left(\frac{k_1 - k_2}{k_1 + k_2}\right)\sum_{n=1}^{N}\frac{(\delta\varepsilon_n)^2}{[(\varepsilon - \varepsilon_n)^2 + (2\delta\varepsilon_n)^2]^{3/2}}. \qquad (12)$$

For intermediate values of U (weak repulsive interaction), namely in the range $\kappa \ll U \ll N\kappa$, we find neither the sequential crossing of Eq.(12), nor the mega-crossing of Eq.(10), but rather a gradual crossing. Namely, in this regime, over a range $\Delta X_2 = 3/2(N{-}1)U$ we get a constant geometric conductance:

$$G \approx -\left[\frac{k_1 - k_2}{k_1 + k_2}\right]\frac{1}{3U} \qquad (13)$$

which reflects in a simple way the strength of the interaction. This formula has been deduced by extrapolating Eq.12, and then was validated numerically (see lower panel of Fig. 3c).

As discussed above for large positive U the N-fold "degeneracy" of the $U{=}0$ Landau-Zener crossing is lifted, and we get a sequence of N Landau-Zener crossings (for schematic illustration see the lower panel of Fig. 2, and compare with the numerical results in the upper panels of Fig. 4). Also for $U{<}0$ this N-fold "degeneracy" is lifted, but in a different way: the levels separate in the "vertical" (energy) direction rather than "horizontally" (see upper panels of Fig. 4). In the latter regime all the particles execute a single two-level transition from the shuttle to the canal (see Fig. 4a). In fact, for sufficiently strong attractive interactions all the particles are glued together and behave like a classical ball that rolls from the shuttle to *one* of the canal sites. When the sign of X_1 is reversed the ball rolls from one end of the canal to the other end (not shown). This should be clearly distinguished from the N-fold degenerated transition to the lower canal *level* which is observed in the $U{=}0$ case.

In summary, we have proposed the creation of controlled atomic current on the basis of BEC stirring [15, 16] and optical lattice technology [24]. The actual measurement of induced neutral currents poses a challenge to experimentalists. In fact there is a variety of techniques that have been proposed for this purpose. For example one can exploit the Doppler effect at the perpendicular direction, which is known as the rotational frequency shift [25]. The analysis of the prototype trimer system reveals the crucial importance of interactions. The interactions are not merely a perturbation: rather they determine the nature of the transport process. We expect the induced circulating atomic current to be extremely accurate, which would open the way to various applications, either as a new metrological standard, or as a component of a new type of quantum information or processing device.

[1] B. P. Anderson, and M. A. Kasevich, Science **282**, 1686 (1998).
[2] D. Jaksch, C. Bruder, J. I. Cirac, C. W. Gardiner, P. Zoller, Phys. Rev. Lett. 81, 3108 (1998).
[3] C. Orzel, A. K. Tuchman, M. L. Fenselau, M. Yasuda, M. A. Kasevich, Science **291**, 2386 (2001).
[4] M. Greiner, O. Mandel, T. Esslinger, T. W. Hansch, I. Bloch, Nature **415**, 39 (2002).
[5] I. Bloch, Nature Phys. **1**, 23-30 (2005)
[6] R. Folman, P. Kruger, D. Cassettari, B. Hessmo, T. Maier, J. Schmiedmayer, Phys. Rev. Lett. 84, 4749 (2000).
[7] W. Hansel, P. Hommelhoff, T. W. Hansch, J. Reichel, Nature **413**, 498 (2001).
[8] D. J. Thouless,, Phys. Rev. B **27** 6083 (1983).
[9] Q. Niu and D. J. Thouless J. Phys. A **17** 2453 (1984).
[10] M. Buttiker, H. Thomas and A Pretre, Z. Phys. B-Condens. Mat., **94**, 133-137 (1994).
[11] P. W. Brouwer, Phys. Rev. B **58**, R10135 (1998).
[12] B. L. Altshuler, L. I. Glazman, Science **283**, 1864 (1999).
[13] M. Switkes, C. M. Marcus, K. Campman, A. C. Gossard, Science **283**, 1905 (1999).
[14] G. Rosenberg and D. Cohen, J. Phys. A **39**, 2287 (2006), and further references therein.
[15] K. W. Madison, F. Chevy, W. Wohlleben, J. Dalibard, Phys. Rev. Lett. 84, 806 (2000).
[16] C. Raman et. al., Phys. Rev. Lett. **83**, 2502 (1999).
[17] M. Hiller, T. Kottos, and T. Geisel, Phys. Rev. A **73**, 061604(R) (2006).
[18] J. D. Bodyfelt, M. Hiller, and T. Kottos, Europhys. Lett. **78**, 50003 (2007).
[19] J. A. Stickney, D. Z. Anderson, A. A. Zozulya, Phys. Rev. A **75**, 013608 (2007).
[20] D. Cohen, Phys. Rev. B **68**, 155303 (2003).
[21] M.V. Berry, Proc. R. Soc. Lond. A **392**, 45 (1984).
[22] J.E. Avron, A. Raveh and B. Zur, Rev. Mod. Phys. **60**, 873 (1988).
[23] M.V. Berry and J.M. Robbins, Proc. R. Soc. Lond. A **442**, 659 (1993).
[24] L. Amico, A. Osterloh, F. Cataliotti, Phys. Rev. Lett. **95**, 063201 (2005).
[25] I. Bialynicki-Birula and Z. Bialynicka-Birula, Phys. Rev. Lett. **78**, 2539 (1997).

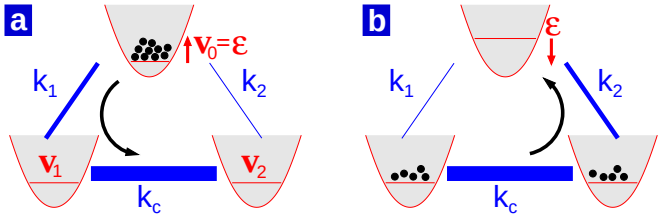

FIG.1: **Illustration of the model system.** In the first half of the cycle **a** the particles are transported from the "shuttle" to the "canal" via the k_1 bond, while in the second half of the cycle **b** the particles are transported back from the "canal" to the "shuttle" via the k_2 bond. See the text for further details.

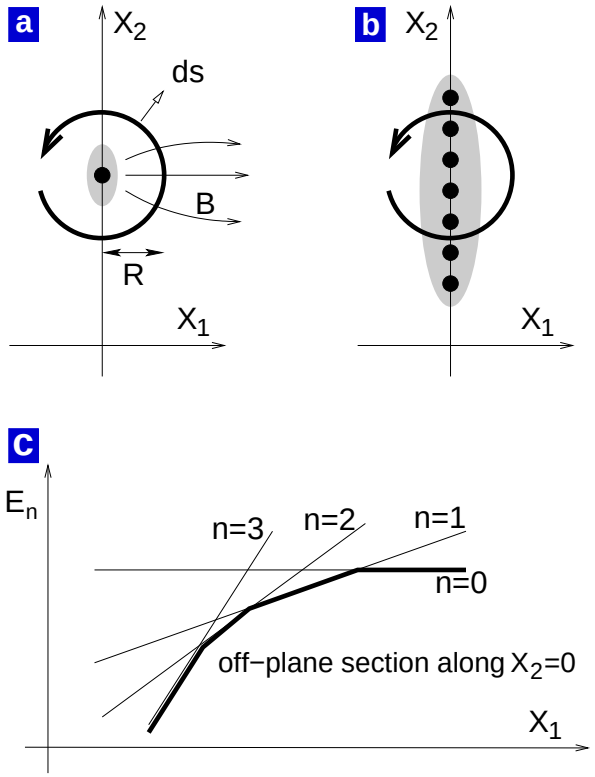

FIG.2: **Illustration of the pumping cycle.** See the text for further details. For a large cycle that encircles the whole shaded region we have $Q \approx N$. Position of the monopoles: **a** no interactions, **b** with interactions. **c**, the energy levels along the $X_1 = 0$ axis are schematically plotted for an $N = 3$ system.

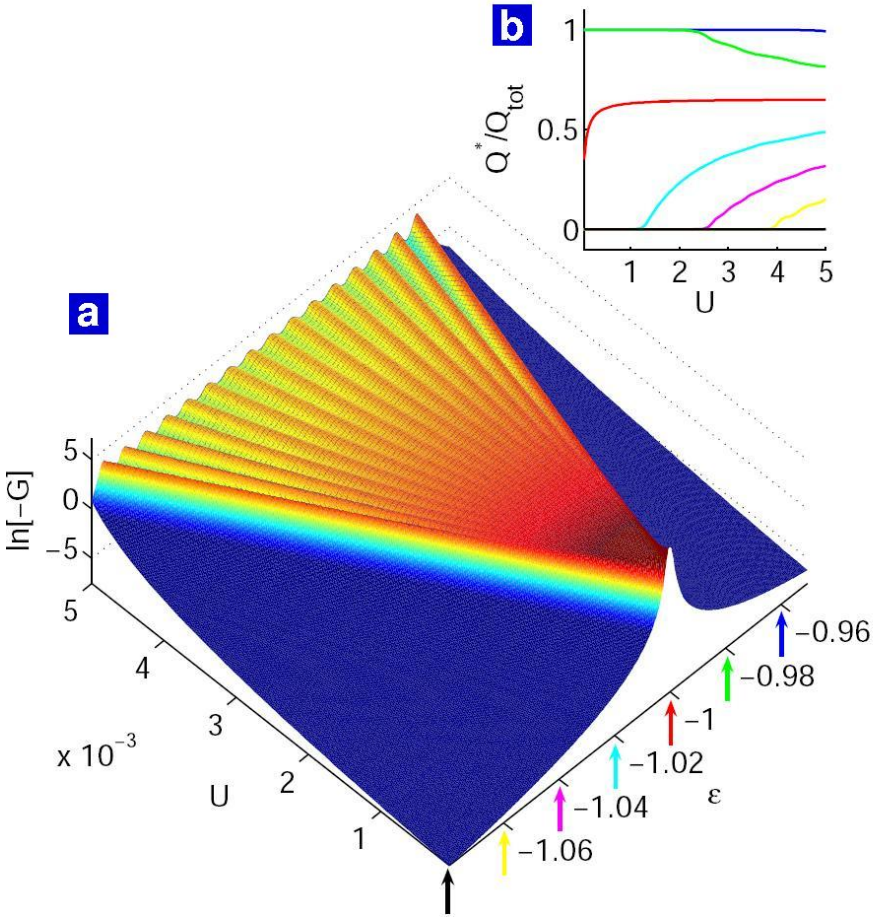

FIG.3: **Conductance during the first half of the pumping cycle.** **a**, the conductance G_2 as a function of the on-site potential ε, for various values of U. The other parameters are $N = 16$ particles, $\kappa = 0.0003/\sqrt{2}$ and $(k_1 - k_2)/\sqrt{2} = 0.0001/\sqrt{2}$. As the interaction U becomes larger one observes the crossover from a single to individual peaks in the conductance. **b**, the U-dependence of the integrated charge Q^*, calculated for wide rectangular cycles for which X_2 is varied within $[-\infty, \varepsilon^*]$. The values of ε^* are indicated by arrows of the same color in the main panel.

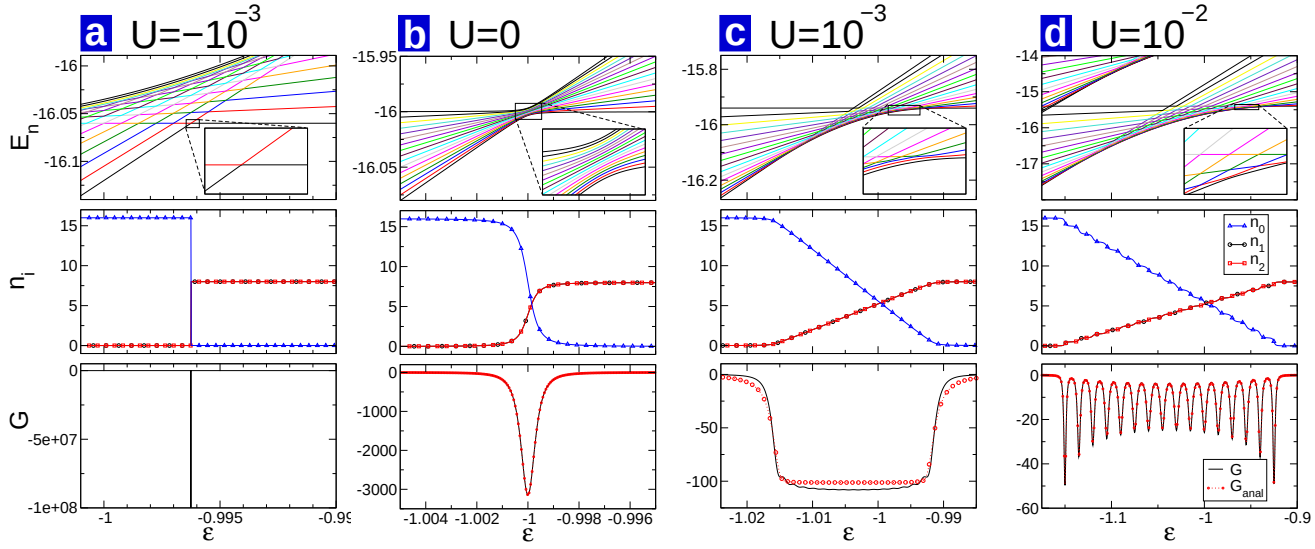

FIG.4: **Evolution of the energy levels, the site occupation and the conductance.** Further details relating to the data of Fig. 3. We refer to four representative values of U, which are indicated on top of each set of panels. *Upper panels*: The lowest $N+1$ energy levels E_n which dominate the conductance G_2 are plotted as a function of $X_2 = \varepsilon$. The insets represent magnifications of the indicated areas. *Middle panels*: the site occupations n_0 (blue $\triangle$), n_1(black $\circ$), n_2 (red $\square$). *Lower panels*: The corresponding conductance G_2 as a function of ε. Numerical results are represented by solid black lines while the dotted red line corresponds to the analytical result (10) in **b** and to (12) in **c**, **d**