

# Overview: Energy Absorption by Driven Mesoscopic Systems

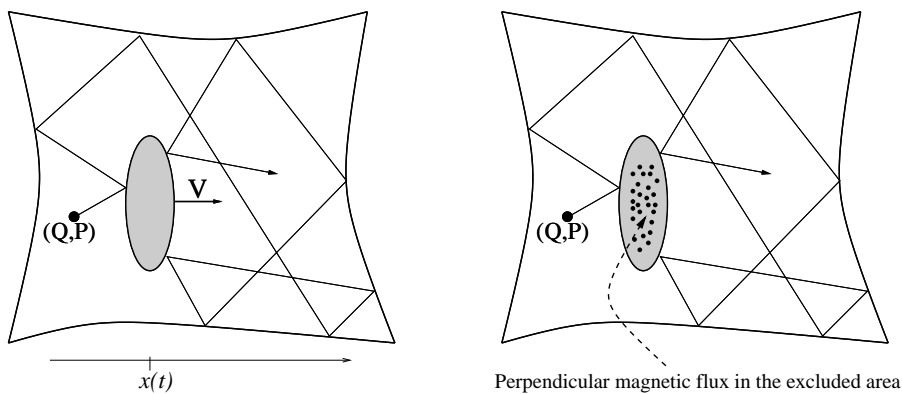
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**Abstract.** There are three regimes in the theory of energy absorption: The adiabatic regime, the linear-response (Kubo) regime, and the non-perturbative regime. The mesoscopic Drude formula for electrical conductance, and the wall formula for friction, can be regarded as special cases of the general formulation of the dissipation problem. The overview is based on a research report for 1998-2000.

The *wall formula* for the calculation of friction in nuclear physics [1], and the *Drude formula* for the calculation of conductance in mesoscopic physics, are just two special results of a much more general formulation of ‘dissipation theory’. The general formulation is as follows: Assume a time-dependent chaotic Hamiltonian  $\mathcal{H}(Q, P; x(t))$  with  $x(t)=Vt$ . Assume also that  $V$  is slow in a classical sense. For  $V = 0$  the energy is constant of the motion. For non-zero  $V$  the energy distribution evolves, and the *average* energy increases with time. This effect is known as *dissipation*.

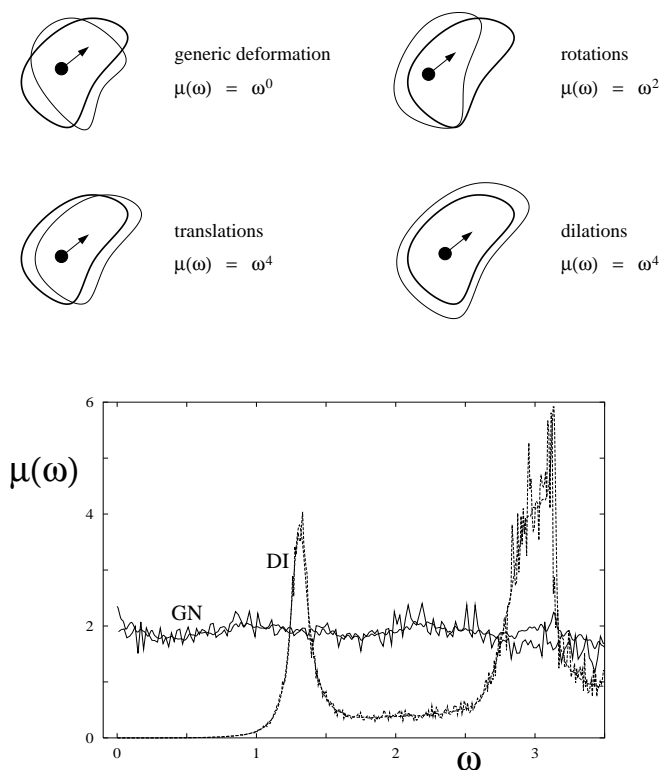
Ohmic dissipation means  $d\langle\mathcal{H}\rangle/dt = \mu V^2$ , where  $\mu$  is defined as the dissipation coefficient. Ohmic dissipation, with an associated (generalized) Fluctuation-Dissipation relation for the calculation of  $\mu$ , can be established within the framework of classical mechanics [2,3] using general classical considerations [4]. See [P2] for detailed presentation, and for discussion of validity conditions.



**Fig.1.** Two examples where the general theory of dissipation can be applied. The left illustration is for the ‘wall formula’, and the right illustration is for the ‘Drude formula’. See text for explanations.

The two leading examples where the above general formulation can be applied are illustrated in Fig.1. In case of the wall formula,  $(Q, P)$  is a particle moving inside a chaotic ‘cavity’, and  $x$  controls the deformation of the boundary. Ohmic dissipation (in the sense defined above) implies a friction force which is proportional to the velocity, where  $\mu$  is the ‘friction coefficient’, and  $\mu V^2$  is the ‘heating’ rate. In case of the mesoscopic Drude formula,  $(Q, P)$  is a charged particle moving inside a chaotic ‘ring’, and  $x$  is the magnetic flux through the hole in the ring. Ohmic dissipation implies Ohm law, where  $V \equiv \dot{x}$  is the electro-motive-force,  $\mu$  is the conductance, and  $\mu V^2$  is the ‘heating’ rate.

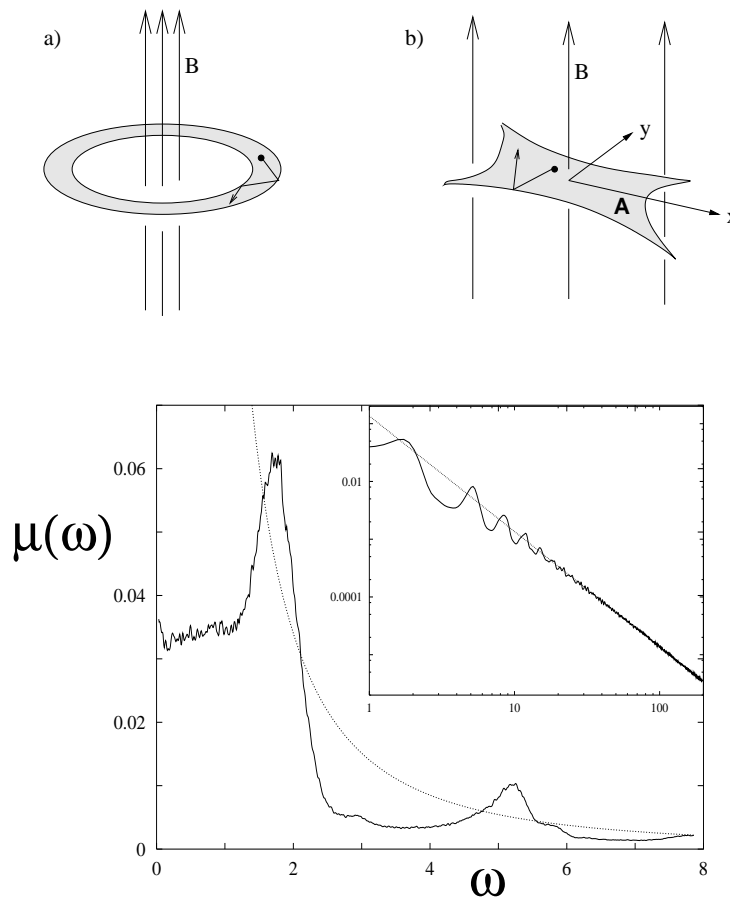
The rate of energy absorption of driven chaotic cavities has been studied [P5,P6] with *A. Barnett* and *E. Heller*. For various reasons a satisfactory theory for the frequency dependent dissipation coefficient  $\mu(\omega)$  has not been introduced in past studies of nuclear friction. It is somewhat surprising that to ‘shake’ a cavity is not an effective way to heat up the ‘gas’ inside it (Fig.2).



**Fig.2.** Consider a system of non-interacting particles confined inside a chaotic cavity. Assume periodic driving  $x(t) = A \sin(\omega t)$ , where  $x$  controls the deformation of the boundary. For special type of deformations  $\mu(\omega)$  vanishes for  $\omega \rightarrow 0$ . A numerical calculation for a stadium shaped cavity is displayed on the right. DI is dilation around the center, while GN is generic deformation. Thick lines are for classical calculation while thin lines are for quantum mechanical (linear response) calculation. Note the remarkable agreement.

A renewed interest in the wall formula is anticipated in the field of mesoscopic physics. Quantum dots can be regarded as small 2D cavities whose shape is controlled by electrical gates. In such case we can use our improved version of the wall formula [P6], and incorporate other corrections [to be published] that take the nature of the dynamics into account.

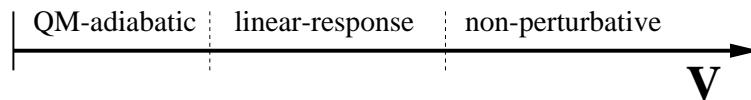
Driving a quantum dot by time-dependent magnetic field (Fig.3) is an obvious option. We already have mentioned the ring geometry (Fig.1, right) where the dissipation coefficient  $\mu$  is just the conductance. However, there is nothing special about ‘rings’. One may consider a simple two dimensional quantum dot driven by a time-dependent *homogeneous* magnetic field. For the latter geometry it is better not to use the term conductance while referring to  $\mu$ .



**Fig.3.** We consider non-interacting electrons driven by electro motive force which is induced by a time-dependent magnetic field. Rather than ring geometry (a) we have a chaotic dot (b). The numerical result can be regarded as a mesoscopic version of Drude formula. The log-log inset on the right demonstrates the large frequency  $1/\omega^2$  behavior.

It is important to realize that the quantum-mechanical (QM) version of linear response theory (LRT) is in remarkable correspondence with the classical result (see eg Fig.2). One wonders whether QM effects are important. This subject has been addressed in [P1-P4]. The main observation is that in the theory of *quantum* dissipation there are *three* distinct regimes (Fig.4):

- The QM-adiabatic regime.
- The linear response (Kubo) regime.
- The non-perturbative regime.

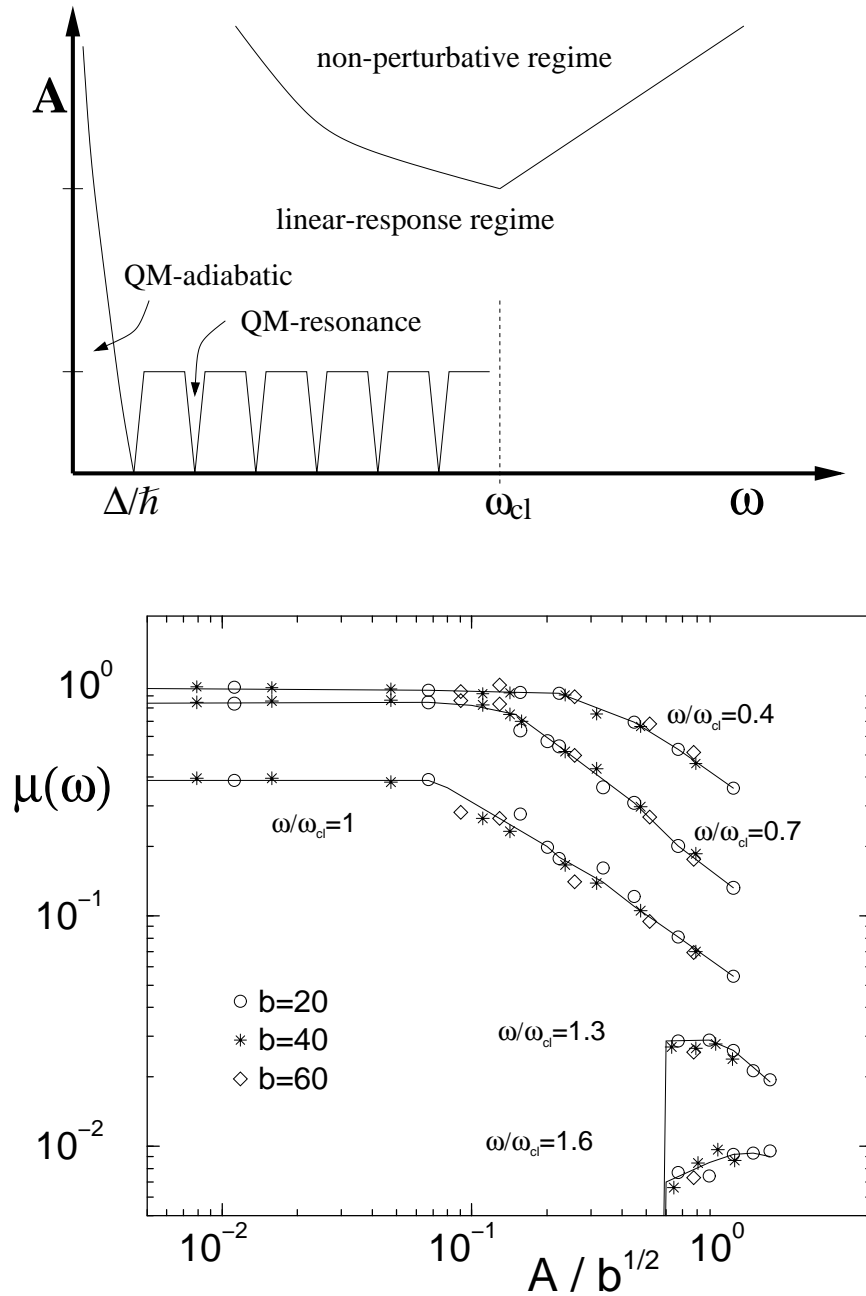


**Fig.4.** This diagram illustrates the various  $V$  regimes in the theory of quantum dissipation for linear driving  $x(t) = Vt$ . The more complicated diagram for the case of periodic driving is presented in Fig.5 and discussed below.

Past studies of quantum dissipation were focused on the QM-adiabatic regime (extremely slow driving), and have dealt with either the Landau-Zener mechanism [5,6] or else with the Debye relaxation absorption mechanism [7] for dissipation. The appearance of the QM-adiabatic regime is related to the existence of a finite mean level spacing  $\Delta$ .

The surprising message of [P1-P4] is that there is a new regime in the theory of quantum dissipation, where QM LRT fails. This failure is not related to having finite mean level spacing  $\Delta$ , but rather to having finite bandwidth  $b \times \Delta$  of the perturbation matrix. A well known semiclassical relation [8] relates the bandwidth to the dropoff frequency of the classical  $\mu(\omega)$ . Namely  $b\Delta = \hbar\omega_{cl}$ . In the context of mesoscopic physics the bandwidth is known as the Thouless energy. Another observation of [P1-P3] is that the semiclassical regime is contained inside the non-perturbative regime.

The various  $(\omega, A)$  regimes for periodic driving  $x(t) = A \sin(\omega t)$  are illustrated in Fig.5. The QM-adiabatic regime (excluding the narrow stripes of resonances) is defined by having *vanishing* first-order probability to go to other levels. In the LRT regime it is assumed that there is strong response for  $\omega < \omega_{cl}$  and vanishingly small response otherwise. Quantal non-perturbative response appears provided the driving amplitude  $A$  is large enough. The theory has been tested [P4] in collaboration with *T. Kottos* for a RMT model.



**Fig.5.** Upper: Diagram illustrating the various  $(\omega, A)$  regimes in the theory of quantum dissipation for periodic driving. Lower: The results of RMT simulations (Wigner model). The dependence of  $\mu(\omega)$  on the driving amplitude  $A$  is displayed.  $\mu = \text{const}$  behavior is implied by LRT. The observed failure of LRT, and the horizontal scaling with respect to the bandwidth  $b$ , are in accordance with the theoretical expectations.

**References:**

- [1] J. Blocki et al. *Ann. Phys.* **113**, 330 (1978).
- [2] M. Wilkinson, *J. Phys. A* **23**, 3603 (1990).
- [3] C. Jarzynski, *Phys. Rev. E* **48**, 4340 (1993).
- [4] E. Ott, *Phys. Rev. Lett.* **42**, 1628 (1979).
- [5] Y. Gefen and D.J. Thouless, *Phys. Rev. Lett.* **59**, 1752 (1987).
- [6] M. Wilkinson, *J. Phys. A* **21**, 4021 (1988).
- [7] R. Landauer and M. Buttiker, *Phys. Rev. Lett.* **54**, 2049 (1985).
- [8] M. Feingold and A. Peres, *Phys. Rev. A* **34**, 591 (1986).

**Publications:**

- [P1] D. Cohen, "Quantum Dissipation due to the interaction with chaotic degrees-of-freedom and the correspondence principle", *Phys. Rev. Lett.* **82**, 4951 (1999).
- [P2] D. Cohen, "Chaos and Energy Spreading for Time-Dependent Hamiltonians, and the various Regimes in the Theory of Quantum Dissipation", *Annals of Physics* **283**, 175 (2000).
- [P3] D. Cohen, "Chaos, Dissipation and Quantal Brownian Motion", Proceedings of the International School of Physics 'Enrico Fermi' Course CXLIII "New Directions in Quantum Chaos", Edited by G. Casati, I. Guarneri and U. Smilansky, IOS Press, Amsterdam (2000).
- [P4] D. Cohen and T. Kottos, "Quantum-mechanical non-perturbative response of Driven Chaotic Mesoscopic Systems, cond-mat/0004022, *Phys. Rev. Lett.* (2000, in press).
- [P5] A. Barnett, D. Cohen and E.J. Heller, "Deformations and dilations of chaotic billiards, rate of dissipations, and quasi-orthogonality of the boundary wavefunctions", *Phys. Rev. Lett.* **85**, 1412 (2000).
- [P6] A. Barnett, D. Cohen and E.J. Heller, "Rate of energy absorption for a driven chaotic cavity", nlin.CD/0006041, *J. Phys. A* (2000, to be published).