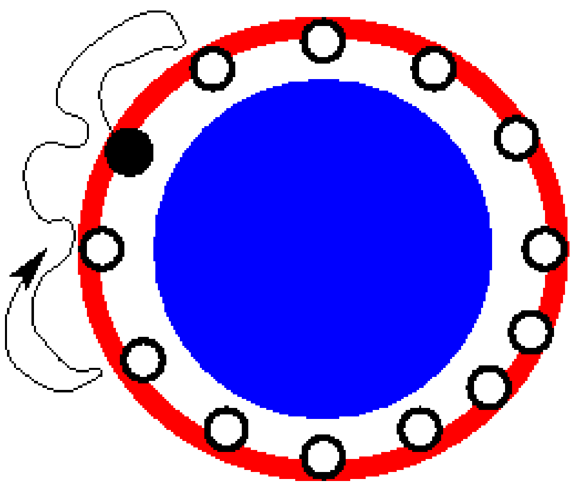
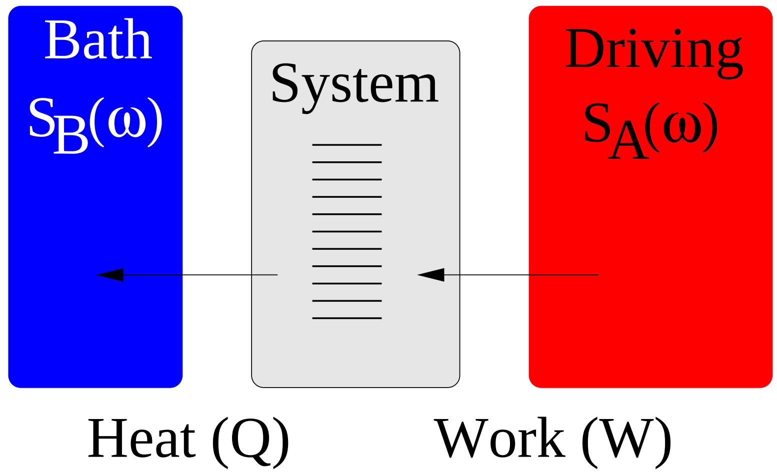
The non-equilibrium steady state of sparse systems with non trivial topology

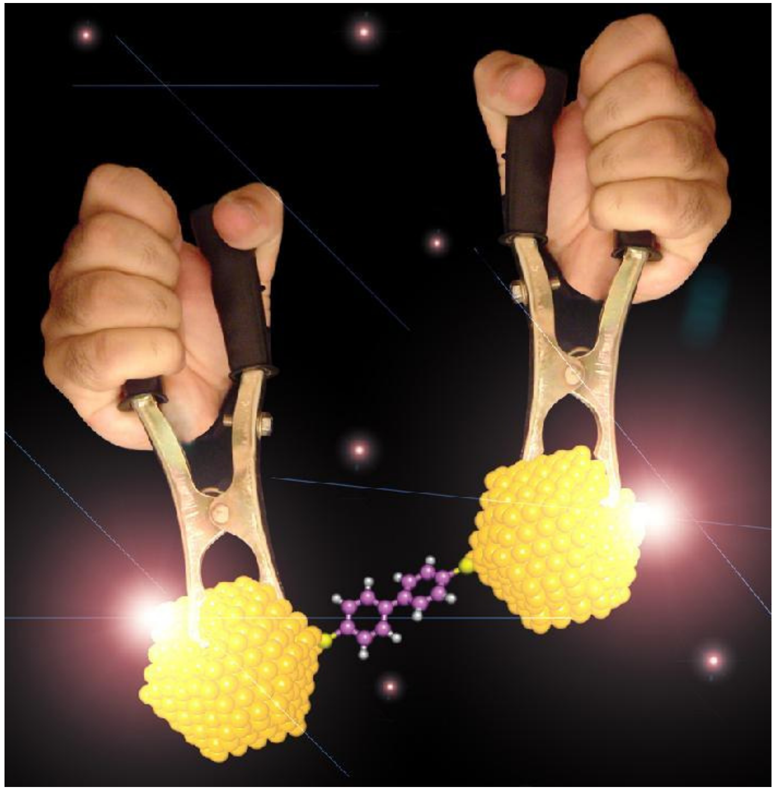
Daniel Hurowitz, Ben-Gurion University

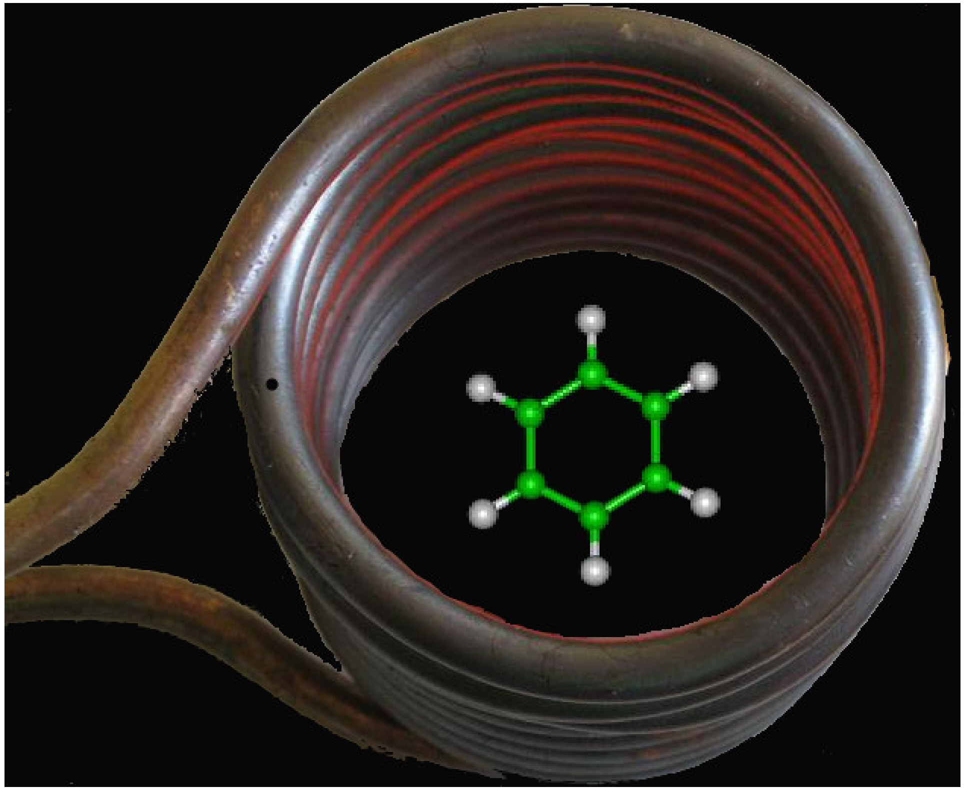




[1] D. Hurowitz and D. Cohen, Europhysics Letters 93, 60002 (2011)
[2] D. Hurowitz, S. Rahav and D. Cohen, Europhysics Letters 98, 20002 (2012)
[3] D. Hurowitz, S. Rahav and D. Cohen, arXiv (2013)

Let us go wireless

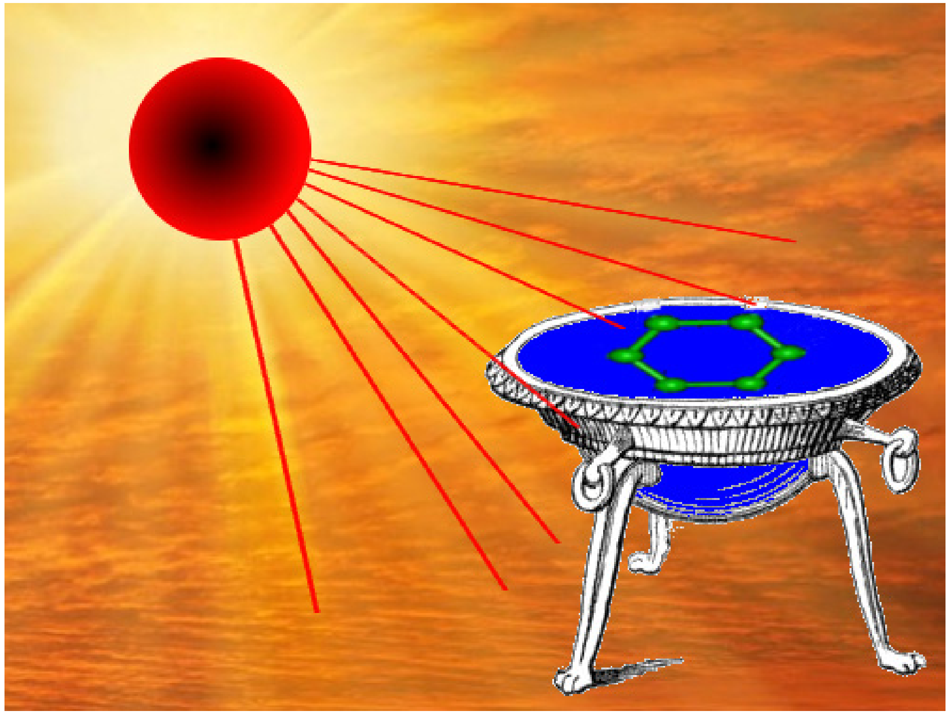




We would like to induce current in a closed device (no leads), even if the the particles have no charge.

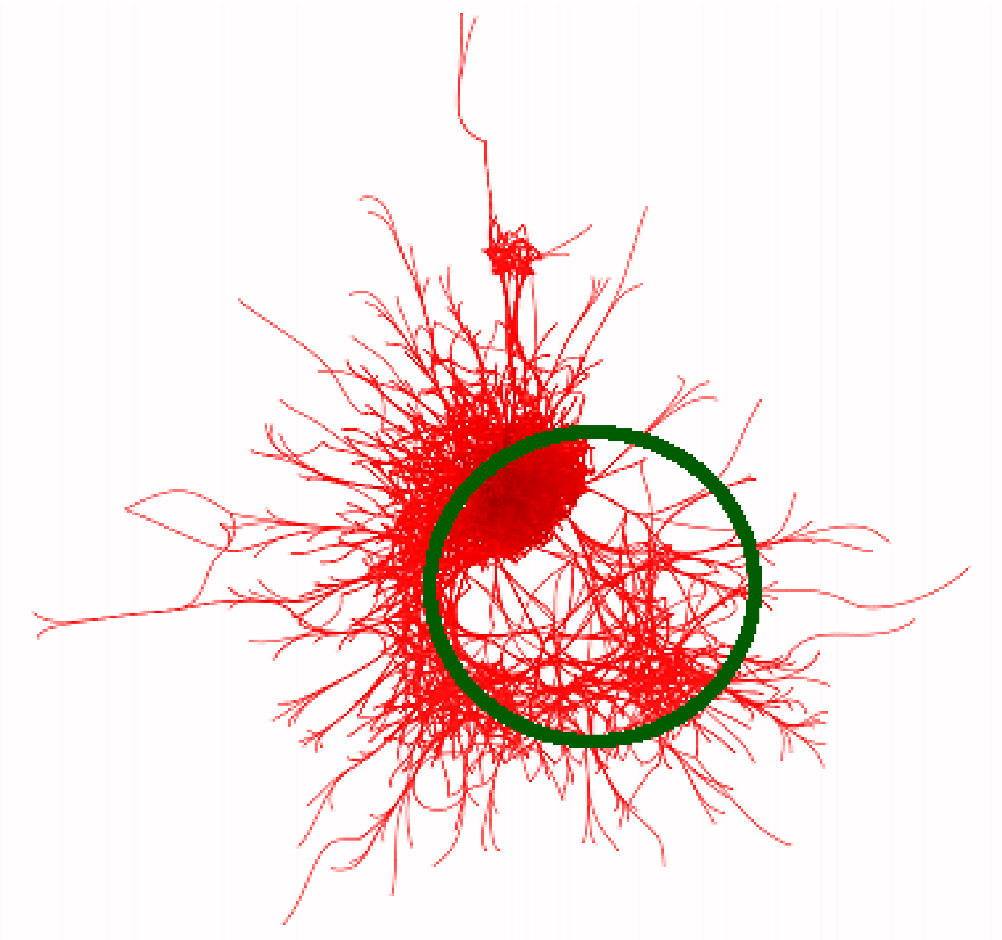
[Left figure is courtesy of Amir Yacobi]

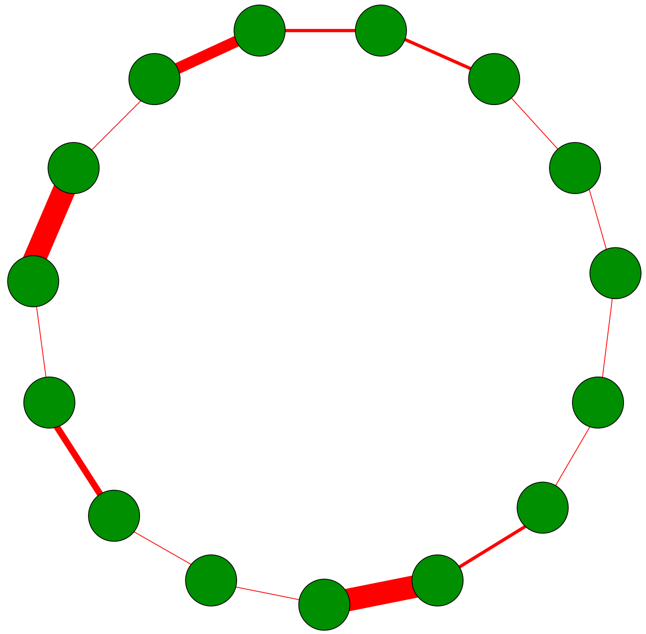
(Non Equilibrium) Statistical Mechanics of Small Systems



We would like to push down the laws of thermodynamics
into the mesoscopic scale,
where fluctuations and quantum mechanics dominate.

Sparse systems



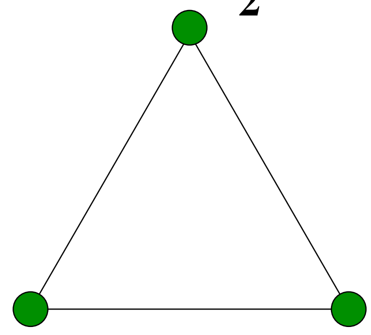


In our study we consider systems that are "sparse" or "glassy", meaning that many time scales are involved. Standard thermodynamics does not apply to such systems.

Non Equilibrium Statistical Mechanics

Master Equation: $\dot{\mathbf{p}} = \mathcal{W}\mathbf{p}$

$$\begin{aligned}
\dot{p}_1 &= -\left(w_{21} + w_{13}\right) p_1 + w_{12} p_2 + w_{13} p_3 \\
\dot{p}_2 &= w_{21} p_1 - \left(w_{12} + w_{32}\right) p_2 + w_{23} p_3 \\
\dot{p}_3 &= w_{31} p_1 + w_{32} p_2 - \left(w_{13} + w_{23}\right) p_3
\end{aligned}$$



Stochastic fields:

$$\begin{aligned}
\mathcal{E}_{12} &= \ln \frac{w_{12}}{w_{21}} \\
\mathcal{E}_{23} &= \ln \frac{w_{23}}{w_{32}} \\
\mathcal{E}_{31} &= \ln \frac{w_{31}}{w_{13}}
\end{aligned}$$

In equilibrium

$$\mathcal{E}_{12} + \mathcal{E}_{23} + \mathcal{E}_{31} = 0$$

$$\begin{aligned}
\mathcal{E}_{12} &= E_2 - E1 \\
\mathcal{E}_{23} &= E_3 - E_2 \\
\mathcal{E}_{31} &= E_1 - E_3
\end{aligned}$$

$$p_i \propto \exp\left(-E_i/T\right)$$

Non equilibrium

$$\mathcal{E}_{12} + \mathcal{E}_{23} + \mathcal{E}_{31} \neq 0$$

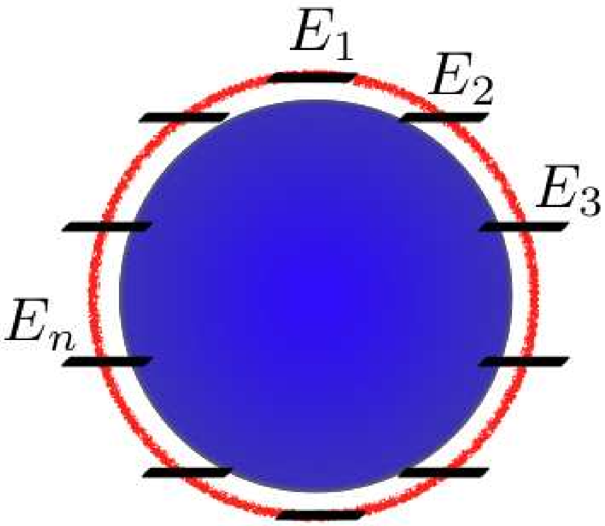
$$\Updownarrow$$

$$w_{12} w_{23} w_{31} \neq w_{13} w_{32} w_{21}$$

If $\oint \mathcal{E}(x)dx = 0$ for all closed loops the steady state will be an equilibrium state.
Otherwise, the system will reach a Non-Equilibrium Steady State (NESS).

The model

System + Bath + Driving

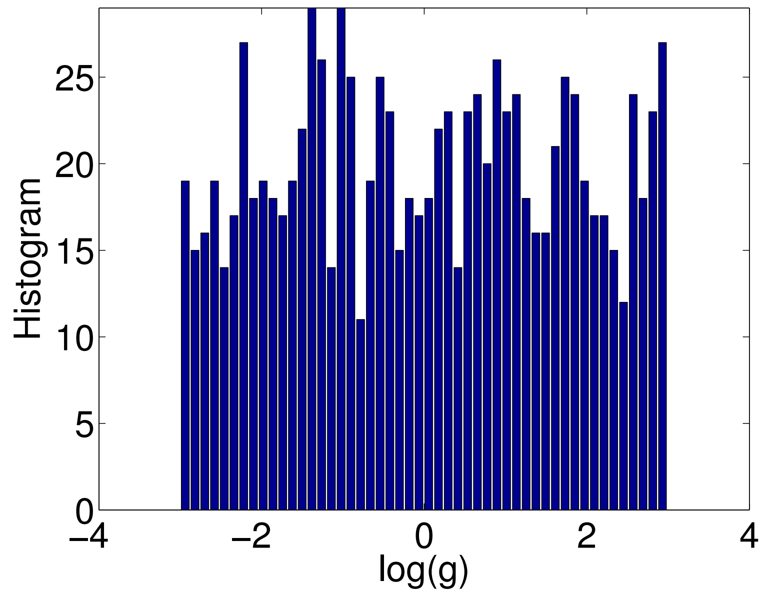


$$w^{total}_{n+1,n} = w^{\beta}_{n+1,n} + \nu g_n$$

w^{β} corresponds to $T_B =$ finite

w^{ν} corresponds to $T_A = \infty$

Histogram of couplings



$\longleftarrow$ few decades $\longrightarrow$

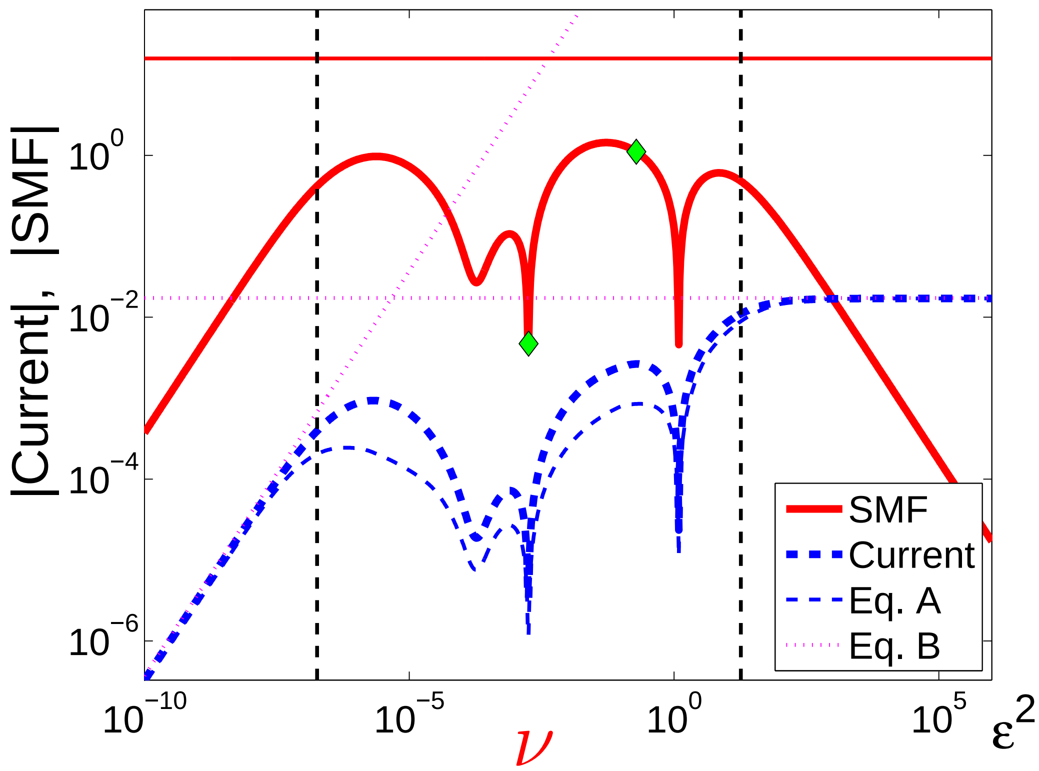
$g_n =$ couplings

"sparsity" $=$ log wide distribution of couplings

Current vs. driving

Driving $\rightsquigarrow$ Stochastic Motive Force $\rightsquigarrow$ Current

Regimes: LRT regime, Sinai regime, Saturation regime



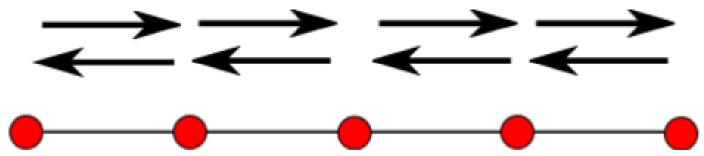
Due to the **sparsity**, we have an intermediate **Sinai regime**.

The width of the Sinai regime is determined by the **log-width of the distribution**.

Sinai Diffusion

Conventional random walk :

Equal & symmetric transition rates

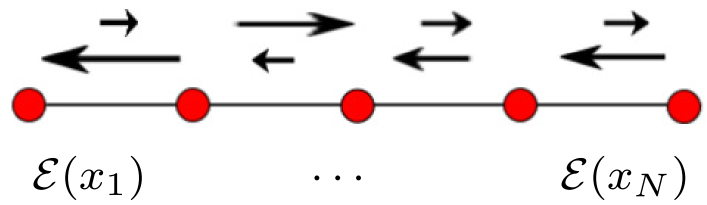


$$I \propto \frac{1}{N}$$

Sinai Random Walk:

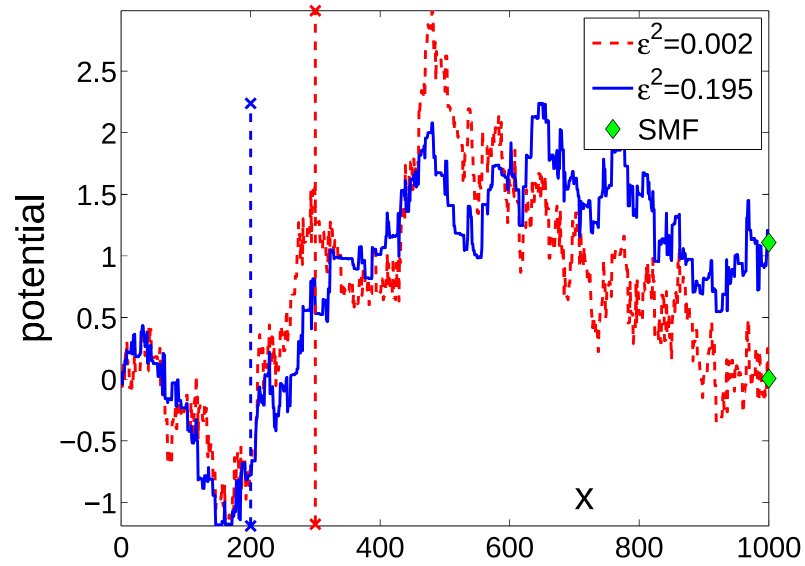
Uncorrelated & non symmetric transition rates

$\rightsquigarrow$ build up of activation barrier



$$I \propto e^{-\sqrt{N}}$$

$$\int_0^x \mathcal{E}(x')dx' \sim \sqrt{N}$$



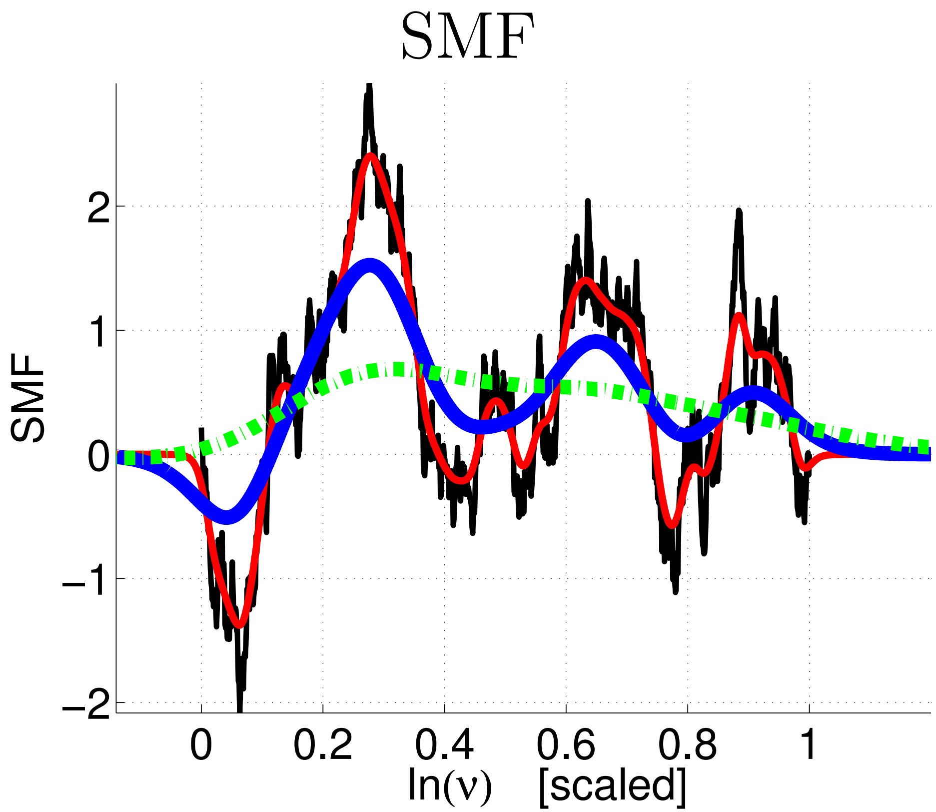
Our model:

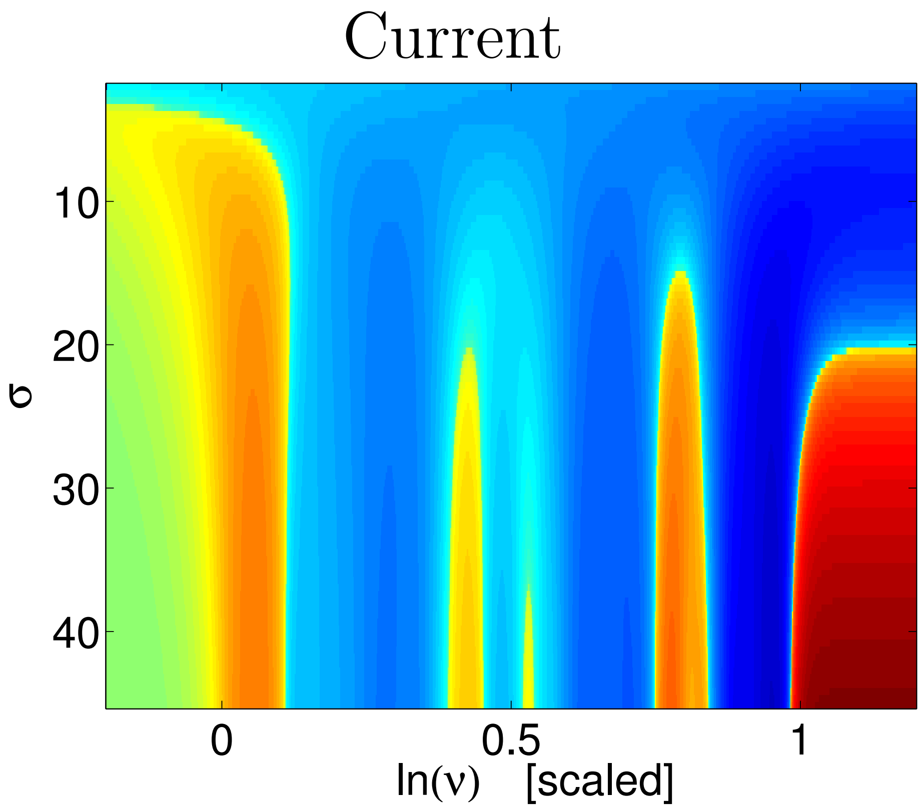
Telescopic correlations: $\mathcal{E}(x_n) \sim \Delta_n \equiv (E_n - E_{n+1})$

Yet... we have sparsely distributed couplings

$$I \sim \frac{1}{N} \overline{w} \exp\left[-\frac{\mathcal{E}_\cap}{2}\right] 2\sinh\left(\frac{\mathcal{E}_\circlearrowleft}{2}\right)$$

Direction of the Current in Sinai Regime





The direction of the current is determined by the SMF

The SMF in the Sinai Regime

Stochastic field : $$\mathcal{E}(x_n) \;\equiv\; \ln\left[\frac{w_{\overrightarrow{n}}}{w_{\overleftarrow{n}}}\right] \;\approx\; -\left[\frac{1}{1+g_n\nu}\right]\frac{E_n - E_{n-1}}{T_B}$$

Stochastic Motive Force: $$\mathcal{E}_{\circlearrowleft} \;\equiv\; \ln\left[\frac{\prod_n w_{\overrightarrow{n}}}{\prod_n w_{\overleftarrow{n}}}\right] \;=\; \oint \mathcal{E}(x)\,dx$$

Coarse grained random walk: $$\mathcal{E}_{\circlearrowleft}(\tau) = -\sum_{n=1}^{N} f_\sigma(\tau-\tau_n)\,\frac{E_n - E_{n-1}}{T_B}$$

$$\sigma \;=\; \ln\frac{g_{\mathsf{max}}}{g_{\mathsf{min}}}, \qquad \text{[log-width of distribution]}$$

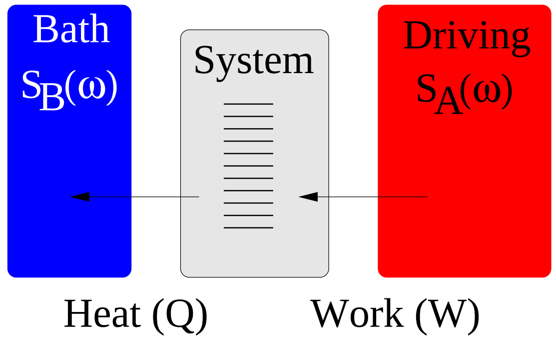
$$\tau \;\equiv\; \frac{1}{\sigma}\ln(g_{\mathsf{max}}\nu), \qquad \tau_n = \frac{1}{\sigma}\ln\left(\frac{g_{\mathsf{max}}}{g_n}\right)$$

$$f_\sigma(t) \;\equiv\; [1+\mathrm{e}^{\sigma t}]^{-1} \qquad \text{[”step” function]}$$

Expected number of sign changes $\approx \sqrt{\pi\sigma}$

FD phenomenology for a "sparse" system

$$w_{nm} \quad = \quad w^{\beta}_{nm} + \nu g_{nm}$$



$$\dot{W} \quad = \quad \text{rate of heating} = \frac{D(\nu)}{T_{\text{system}}}$$

$$\dot{Q} \quad = \quad \text{rate of cooling} = \frac{D_B}{T_B} - \frac{D_B}{T_{\text{system}}}$$

Hence at the NESS:

$$T_{\text{system}} \quad = \quad \left(1 + \frac{D(\nu)}{D_B}\right) T_B$$

$$\dot{Q} = \dot{W} \quad = \quad \frac{1/T_B}{D_B^{-1} + D(\nu)^{-1}}$$

Experimental way to extract response:

$$D(\nu) = \frac{\dot{Q}(\nu)}{\dot{Q}(\infty) - \dot{Q}(\nu)} D_B$$

$D(\nu)$ exhibits LRT to SLRT crossover

$$D(\nu) \quad = \quad \left[\overline{\left(\frac{w_n}{w_\beta + w_n}\right)}\right] \left[\overline{\left(\frac{1}{w_\beta + w_n}\right)}\right]^{-1}$$

$$D_{[\text{LRT}]} \quad = \quad \overline{g_n}\, \nu \qquad \text{[weak driving]}$$

$$D_{[\text{SLRT}]} \quad = \quad [\overline{1/g_n}]^{-1} \nu \qquad \text{[strong driving]}$$

Expressions above assume n.n. transitions only.

Summary of main results

1. Due to the sparsity of the perturbation matrix, the NESS is of glassy nature [1]
2. An extension of the Fluctuation-Dissipation phenomenology has been proposed [1]
3. A log-wide distribution of couplings leads to Sinai-type physics [2]
4. In the Sinai regime, the fluctuations of the current reflect the log-wide distribution of the couplings [3]