Semi-linear response and RMT for the heating rate of cold atoms in vibrating traps

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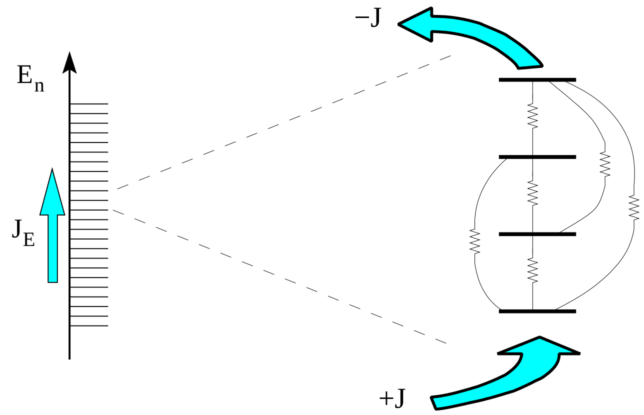

References:

A. Stotland, R. Budoyo, T. Peer, T. Kottos and D. Cohen, JPA (FTC) (2008)

A. Stotland, D. Cohen and N. Davidson, EPL (2009)

A. Stotland, T. Kottos, and D. Cohen, in preparation

http://physics.bgu.ac.il/∼stotland

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Diffusion and Energy absorption

Driven chaotic system with Hamiltonian $\mathcal{H}(X(t))$

X = some control parameter

$\dot{X}$ = rate of the (noisy) driving

$\rightsquigarrow$ diffusion in energy space:

$$\boldsymbol{D} = \boldsymbol{G}_{\text{diffusion}} \ \overline{\dot{X}^2}$$

$\rightsquigarrow$ energy absorption:

$$\dot{\boldsymbol{E}} = \boldsymbol{G}_{\text{absorption}} \ \overline{\dot{X}^2}$$

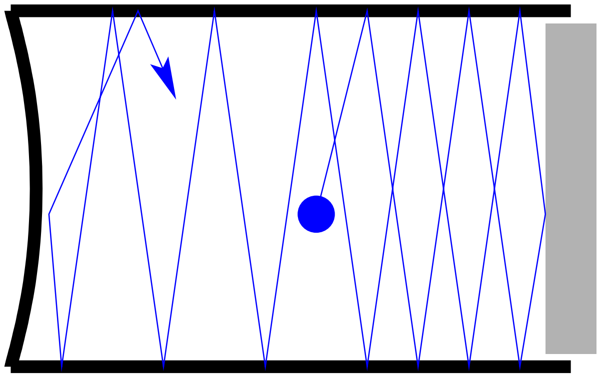
There is a dissipation-diffusion relation.
In the canonical case $\dot{E} = D/T$.

Below we use for G scaled units.

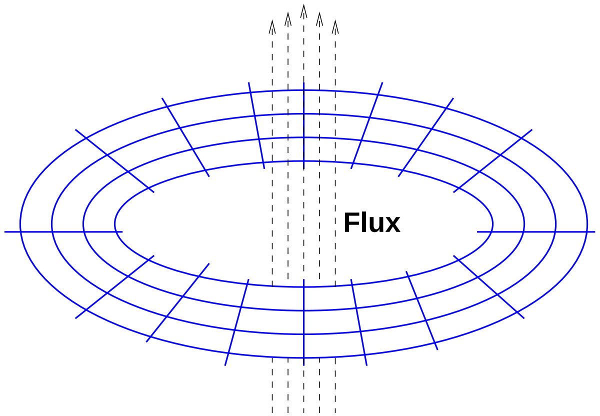
[Ott, Brown, Grebogi, Wilkinson, Jarzynski, Cohen]

Models

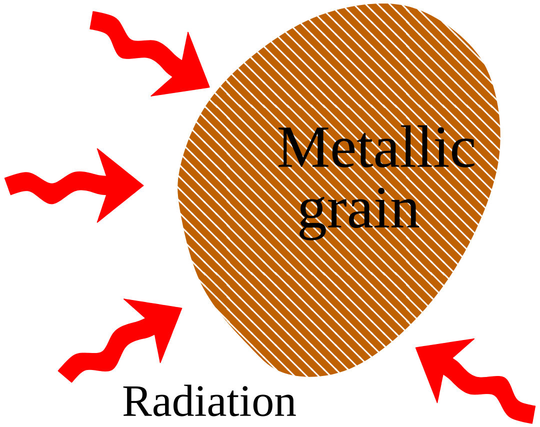
$$\mathcal{H} = \{E_n\} - X(t)\{V_{nm}\}$$



2-D box with a vibrating wall.



Disordered ring driven by EMF.



Radiation absorbtion by metallic grains.

[1] D. Cohen, T. Kottos and H. Schanz, JPA (2006)

[2] S. Bandopadhyay, Y. Etzioni and D. Cohen, EPL (2006)

[3] M. Wilkinson, B. Mehlig and D. Cohen, EPL (2006)

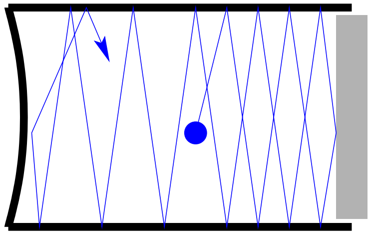
[4] D. Cohen, PRB (2007)

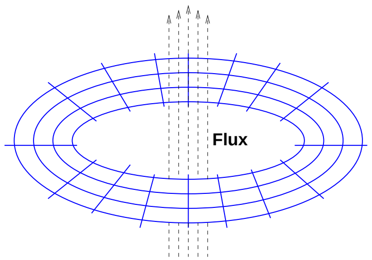
[5] A. Stotland, R. Budoyo, T. Peer, T. Kottos and D. Cohen, JPA(FTC) (2008)

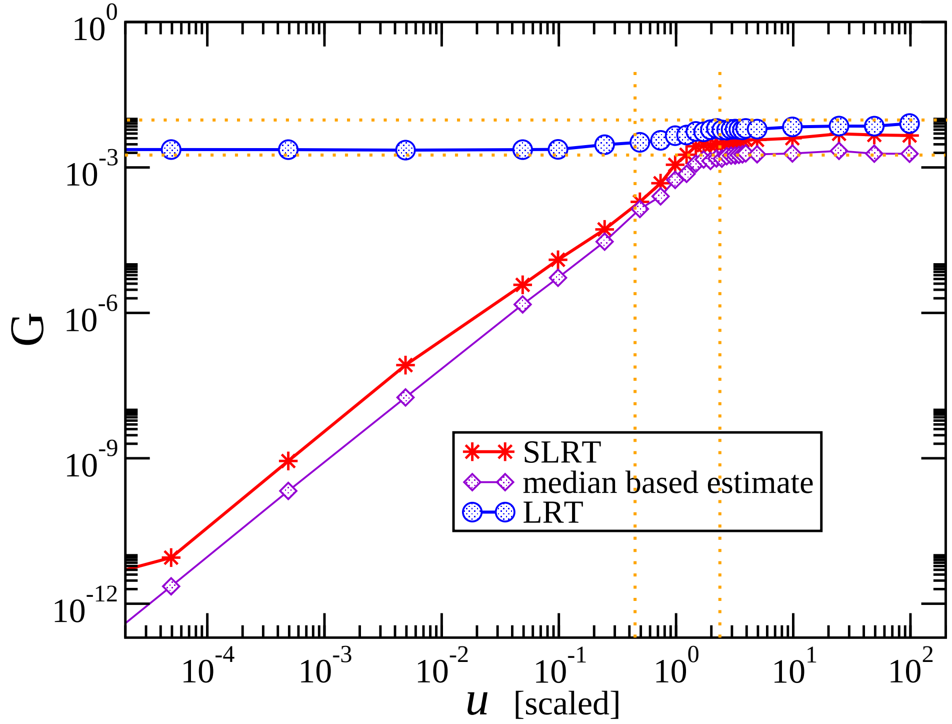
[6] A. Stotland, D. Cohen and N. Davidson, EPL (2009)

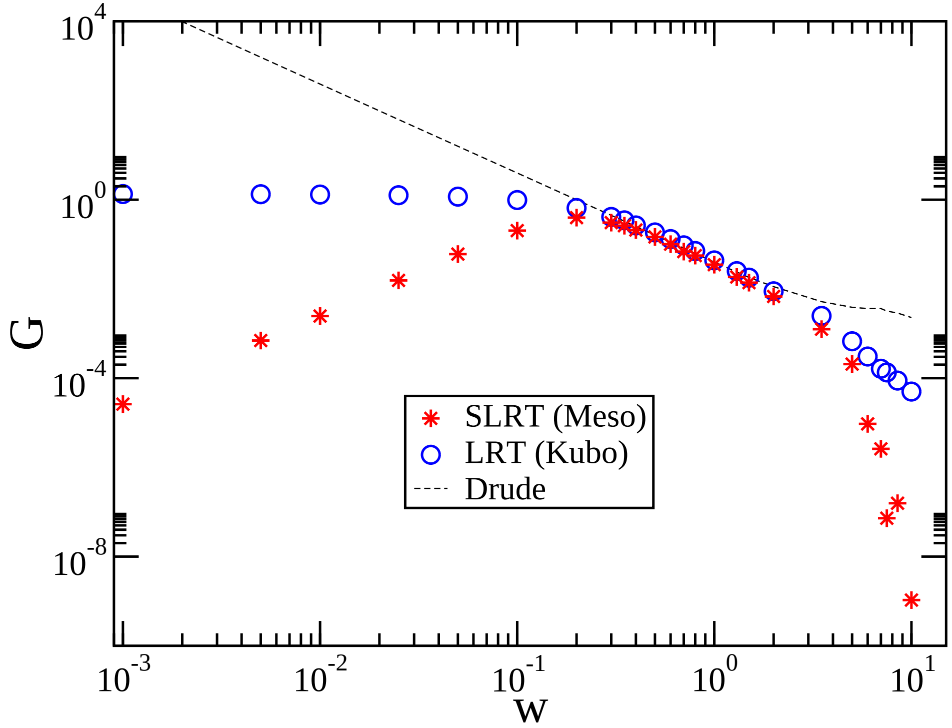
[7] A. Stotland, T. Kottos and D. Cohen, in preparation

Some results









u - strength of the deformation

W - strength of the disorder

The resistor network calculation

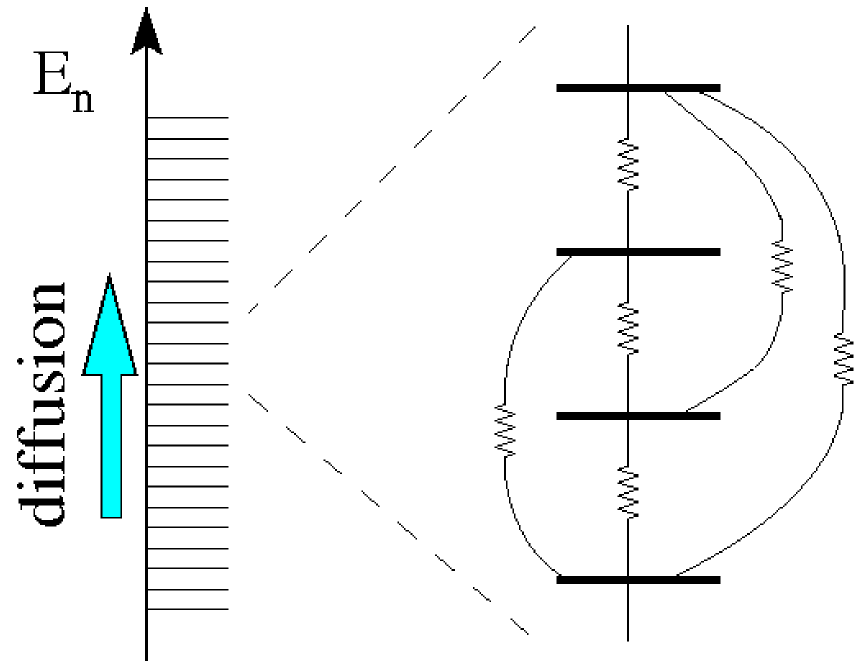
$$\mathcal{H} = \{E_n\} - X(t)\{V_{nm}\}$$

$$\boldsymbol{G} \;=\; \pi\varrho_{\text{E}}\ \langle\langle|V_{mn}|^2\rangle\rangle_{\text{SLRT}}$$

$$g_{\text{SLRT}} \;\equiv\; \frac{\langle\langle|V_{mn}|^2\rangle\rangle_{\text{SLRT}}}{\langle\langle|V_{mn}|^2\rangle\rangle_{\text{algebraic}}}$$

$$\mathrm{g}_{nm} = 2\varrho_{\text{F}}^{-3}\ \frac{|V_{nm}|^2}{(E_n-E_m)^2}\ \tilde{S}(E_n-E_m)$$

$$\langle\langle|V_{mn}|^2\rangle\rangle_{\text{SLRT}} \equiv \text{inverse resistivity}$$



$$\langle\langle|V_{mn}|^2\rangle\rangle_{\text{harmonic}} \ll \langle\langle|V_{mn}|^2\rangle\rangle_{\text{SLRT}} \ll \langle\langle|V_{mn}|^2\rangle\rangle_{\text{algebraic}}$$

SLRT *vs.* LRT

$$\mathcal{H}(X(t)) \approx \mathcal{H}_0 + f(t)V$$
$$\mathcal{H}_0 = \mathcal{H}(X_0)$$
$$f(t) = X(t) - X_0$$
$$V = \frac{\partial \mathcal{H}}{\partial X}$$
$$\tilde{C}(\omega) = \mathrm{FT}\,\langle V(t)V(0)\rangle$$
$$\tilde{S}(\omega) = \mathrm{FT}\,\langle \dot{f}(t)\dot{f}(0)\rangle$$

Kubo formula:

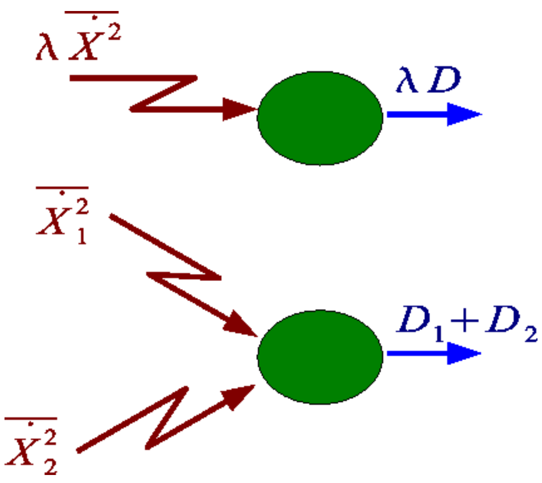
$$\boldsymbol{D} = \int \tilde{C}(\omega)\tilde{S}(\omega)d\omega$$

SLRT example:

$$\boldsymbol{D} = \left[\int R(\omega)\left[\tilde{S}(\omega)\right]^{-1} d\omega\right]^{-1}$$

Linear response implies

$$\tilde{S}(\omega) \mapsto \lambda\tilde{S}(\omega) \Longrightarrow \boldsymbol{D} \mapsto \lambda\boldsymbol{D}$$
$$\tilde{S}(\omega) \mapsto \sum_i \tilde{S}_i(\omega) \Longrightarrow \boldsymbol{D} \mapsto \sum_i \boldsymbol{D}_i$$



The model

A particle in a 2-D box with a vibrating wall.

Deforming potential: smooth Gaussian / s-scatterer

The Hamiltonian in the $\boldsymbol{n} = (n_x, n_y)$ basis:

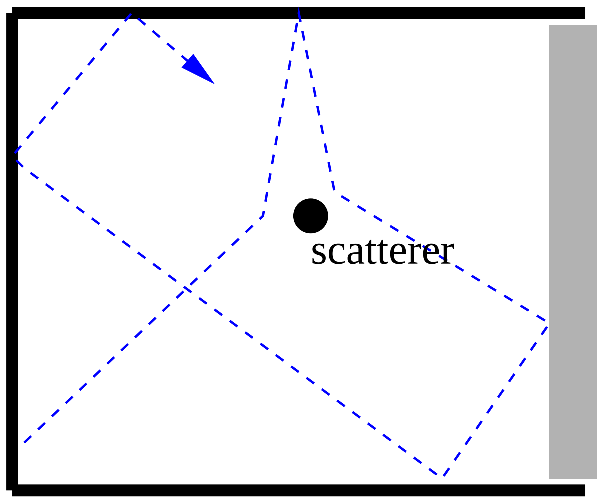
$$\mathcal{H} = \mathrm{diag}\{E_{\boldsymbol{n}}\} + u\{U_{\boldsymbol{nm}}\} + f(t)\{V_{\boldsymbol{nm}}\}$$

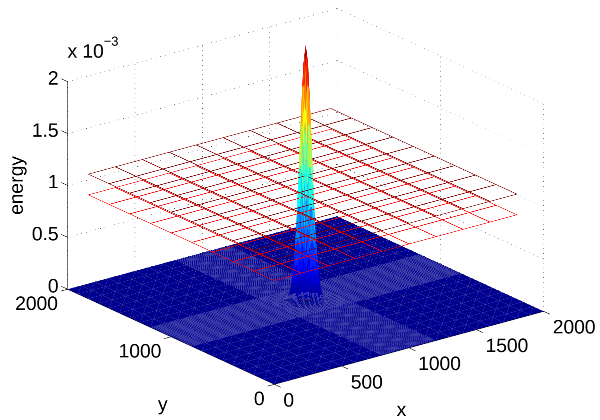
The matrix elements for the wall displacement:

$$V_{\boldsymbol{nm}} \quad = \quad -\delta_{n_y,m_y} \times \frac{\pi^2}{\mathsf{M} L_x^3} n_x m_x \qquad \text{[sparse]}$$

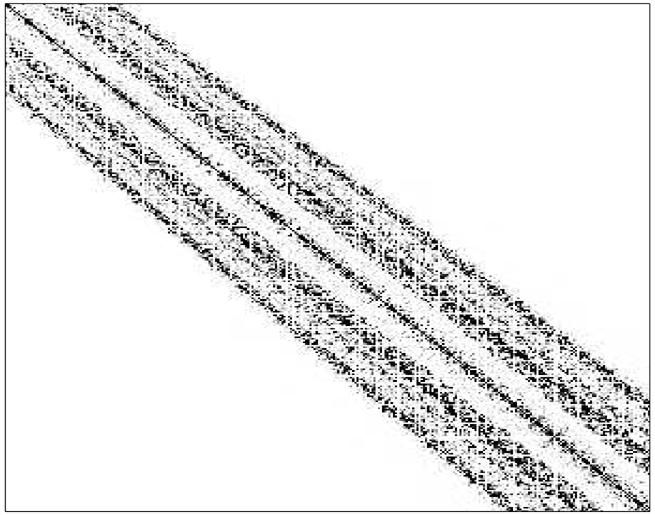
The Hamiltonian in the E_n basis:

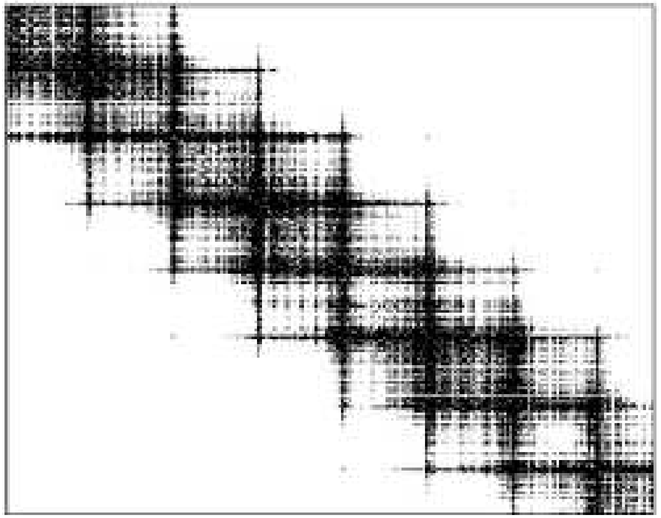
$$\mathcal{H} = \mathrm{diag}\{E_n\} + f(t)\{V_{nm}\}$$

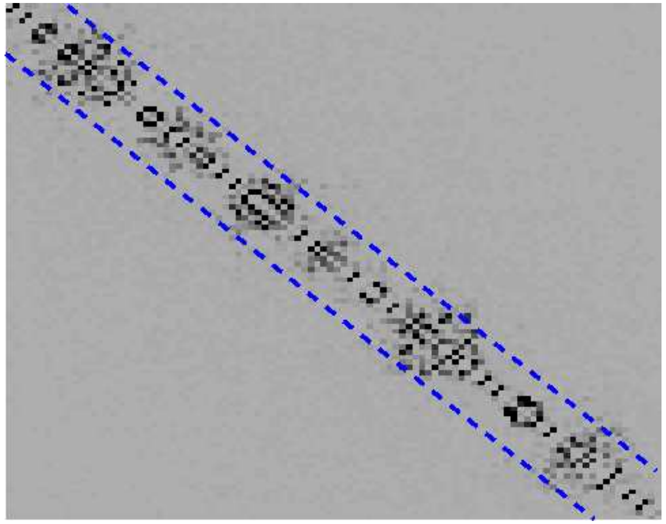




Matrices - Gallery





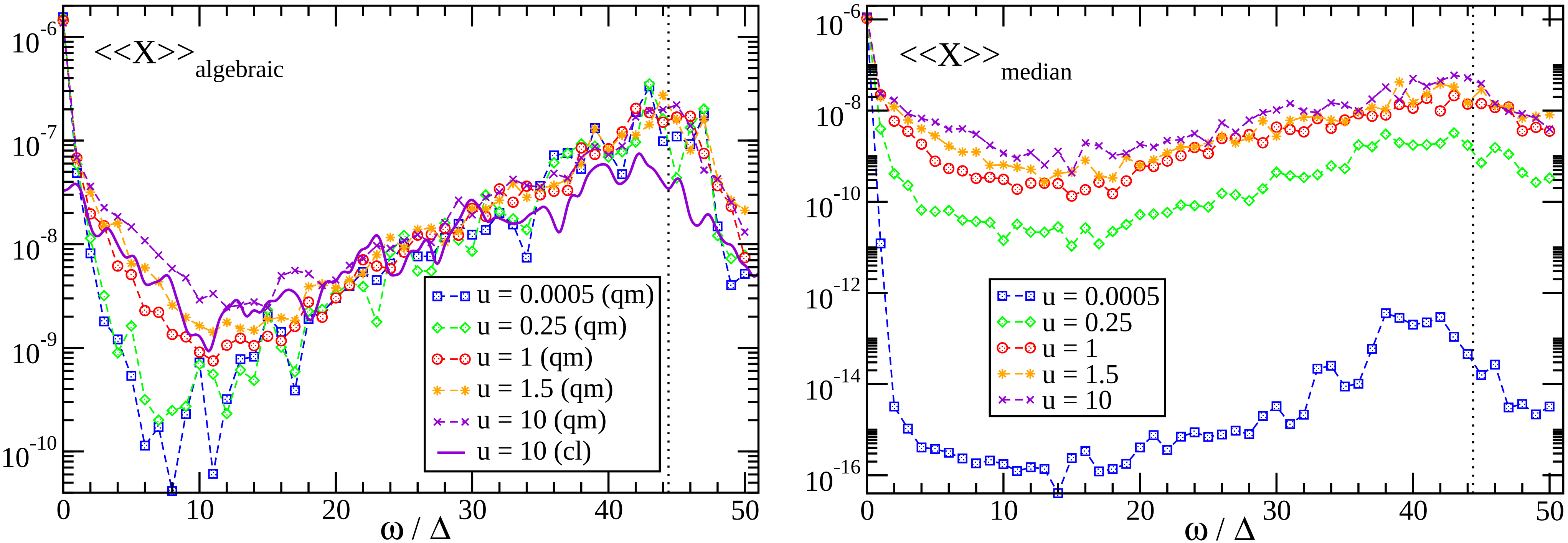


$$\tilde{C}(\omega) \equiv \langle\langle |V_{nm}|^2 \rangle\rangle_{\text{algebraic}} = \left\langle \sum_n |V_{nn_0}|^2 \; \delta(\omega - (E_n - E_{n_0})) \right\rangle_{\text{algebraic}}$$

Sparsity and texture are not reflected by $\tilde{C}(\omega)$!

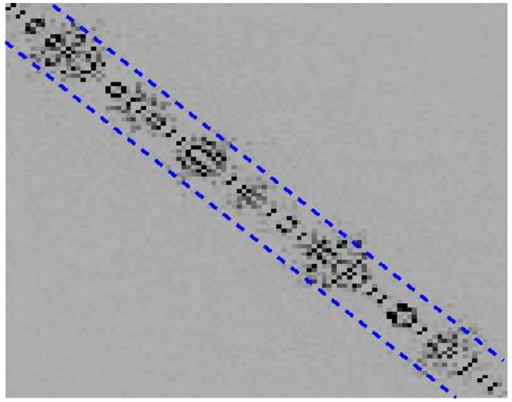
$$\boldsymbol{G} \; = \; \pi \varrho_{\text{E}} \; \langle\langle |V_{mn}|^2 \rangle\rangle$$

Spectral functions



$$\langle\langle|V_{nm}|^2\rangle\rangle_{\text{algebraic/median}} = \left\langle \sum_n |V_{nn_0}|^2 \ \delta(\omega - (E_n - E_{n_0})) \right\rangle_{\text{algebraic/median}}$$

$\{|V_{nm}|^2\}$ as a random matrix $\{X\}$

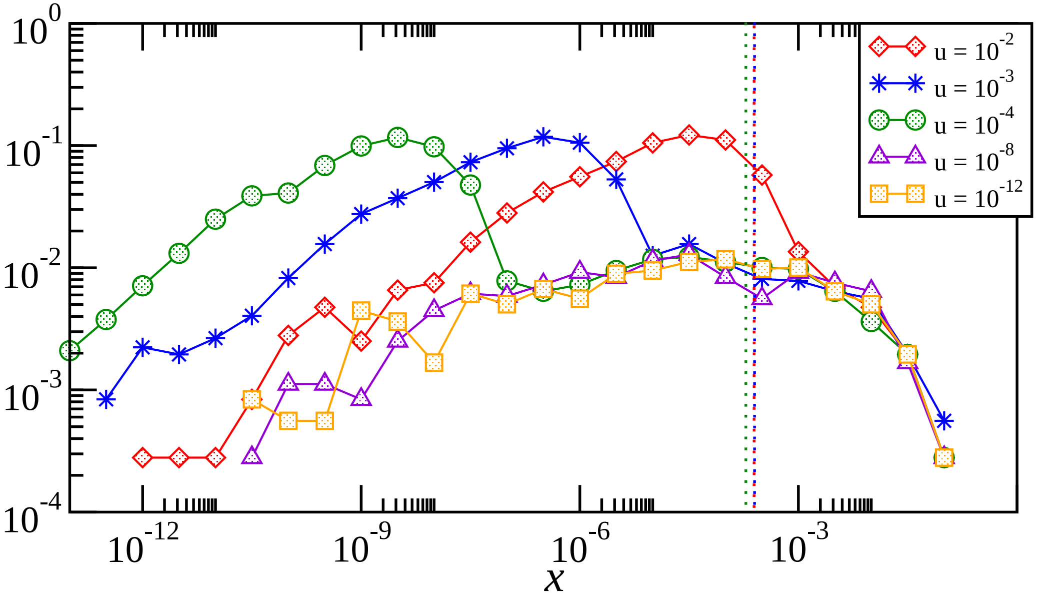


- bandwidth
- texture
- sparsity

$$q \equiv \frac{\langle\langle x \rangle\rangle_g}{\langle\langle x \rangle\rangle_a}$$

$$p \equiv \text{Prob}\Big(X > \langle X \rangle\Big)$$

Histogram of X :



$X \sim$ LogNormal

Averages:

Algebraic: $\langle\langle x \rangle\rangle_a = \langle x \rangle$

Harmonic: $\langle\langle x \rangle\rangle_h = [\langle 1/x \rangle]^{-1}$

Geometric: $\langle\langle x \rangle\rangle_g = \exp[\langle \log x \rangle]$

For "log - wide" distributions:

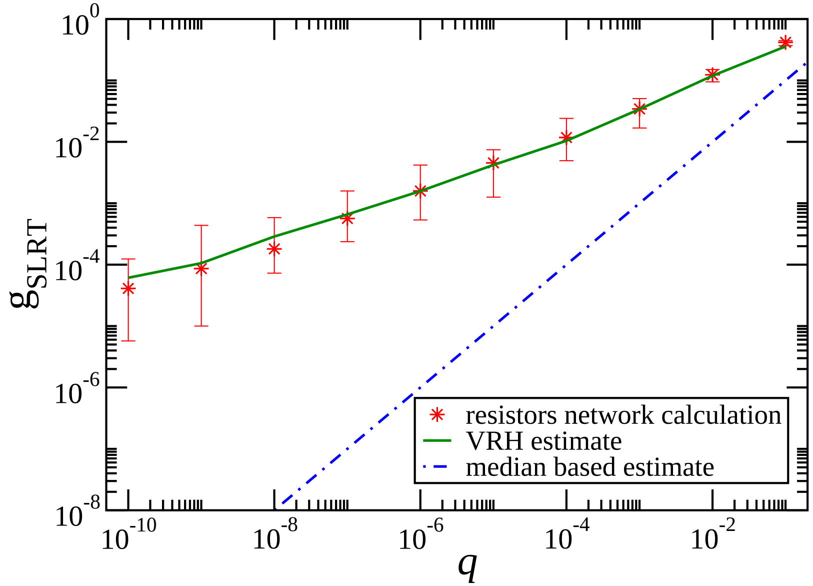
$$\langle\langle x \rangle\rangle_h \ll \langle\langle x \rangle\rangle_g \ll \langle\langle x \rangle\rangle_a$$

The RMT modeling - Results

- Log-normal RMT modeling

For the rectangular $\tilde{S}(\omega)$ of width ω_c

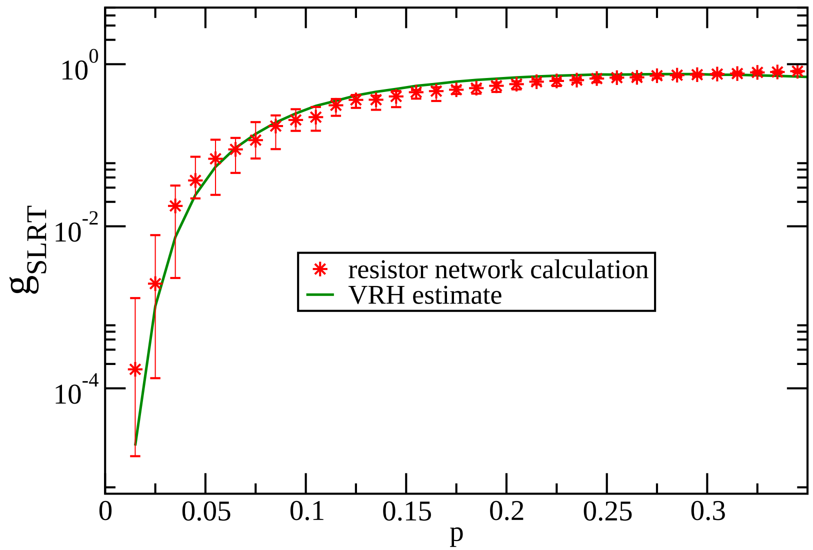
$$g_{\mathrm{SLRT}} \approx q \exp\left[2\sqrt{-\ln q \, \ln(\omega_c/\Delta)}\right]$$



- Log-box RMT modeling

For the exponential $\tilde{S}(\omega)$ of width ω_c

$$g_{\mathrm{SLRT}} \approx \frac{1}{p} \exp\left[-2\sqrt{\frac{\Delta}{p\,\omega_c}}\right]$$



Conclusions

(*) Wigner ($\sim$ 1955):
The perturbation is represented by a random matrix whose elements are taken from a Gaussian distribution.

Not always...

1. No “strong quantum chaos“ $\Longrightarrow$ log-normal distribution.
2. The heating process $\sim$ a percolation problem.
3. Resistors network calculation to get G_{SLRT}.
4. Generalization of the VRH estimate
5. SLRT is essential whenever the distribution of matrix elements is wide (“sparsity”) or if the matrix has “texture”.

Appendix: Generalized VRH

A typical matrix element for connected transitions:

$$\left(\frac{\omega}{\Delta}\right) \text{Prob}\Big(x > x_\omega\Big) \;\sim\; 1$$

Generalized VRH estimate:

$$G_{\text{SLRT}} \quad = \quad \int x_\omega \tilde{S}(\omega) d\omega$$

In the standard case (strong disorder):

$$\begin{aligned} \text{Prob}(x) \;&=\; \text{log-box} \\ x_\omega \;&\approx\; v_{\text{F}}^2 \;\exp\left(\frac{\Delta_l}{|\omega|}\right) \end{aligned}$$

$$\begin{aligned} \tilde{S}(\omega) \;&\propto\; \exp\left(-\frac{|\omega|}{T}\right) \\ G_{\text{SLRT}} \;&\propto\; \int \exp\left(-\frac{|\omega|}{T}\right) \exp\left(\frac{\Delta_l}{|\omega|}\right) d\omega \end{aligned}$$