Quantum decay into a non-flat continuum

Itamar Sela, Ben-Gurion University

Collaborations:

Doron Cohen (BGU)

James Aisenberg (Wesleyan)

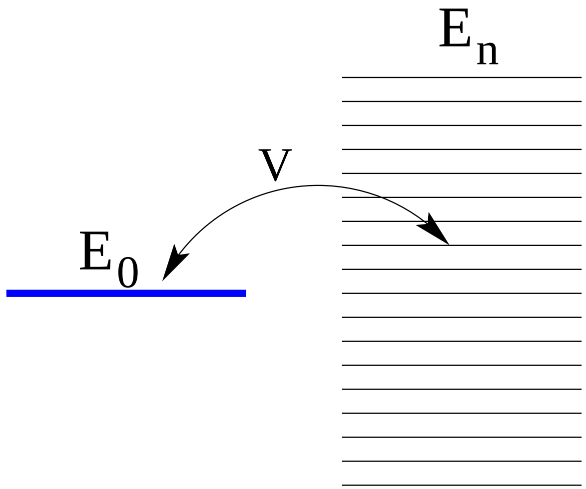
Tsampikos Kottos (Wesleyan)

Alex Elgart (Virginia Tech)

$BSF, $DIP

Quantum decay

$$\mathcal{H} = \left(\begin{array}{c|ccccc} E_0 & & & & & \\ \hline & E_1 & & & & \\ & & E_2 & & & \\ & & & \cdot & & \\ & & & & E_n & \\ & & & & & \cdot \end{array}\right) + V$$



Survival probability

$$P(t) \;\; \equiv \;\; \left| \langle 0 | \mathrm{e}^{-i\mathcal{H}t} | 0 \rangle \right|^2$$

Flat continuum [s=1]

$$P(t) \;\; = \;\; \mathrm{e}^{-t/t_0}$$

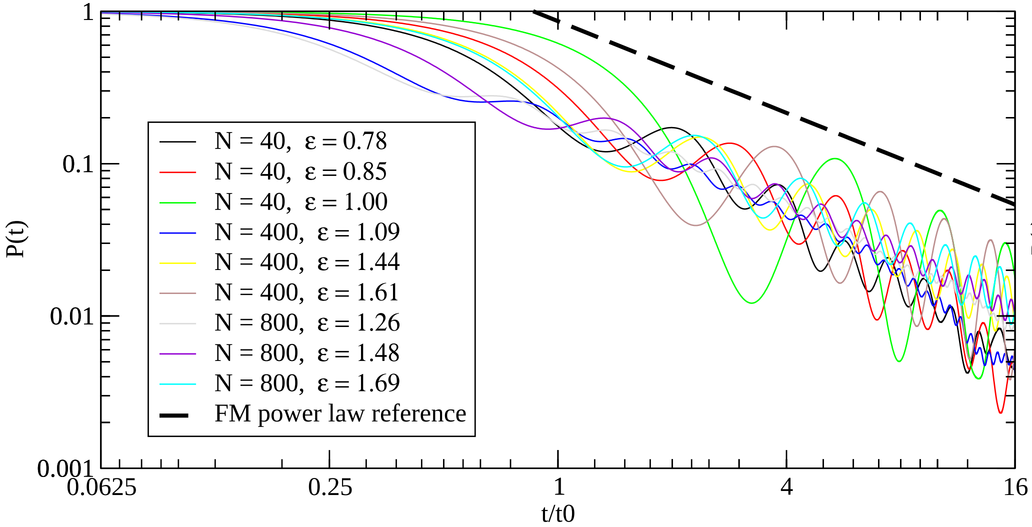
$$|V_{n,0}|^2 \propto |E_n - E_0|^{s-1}$$

FM = Friedrichs model

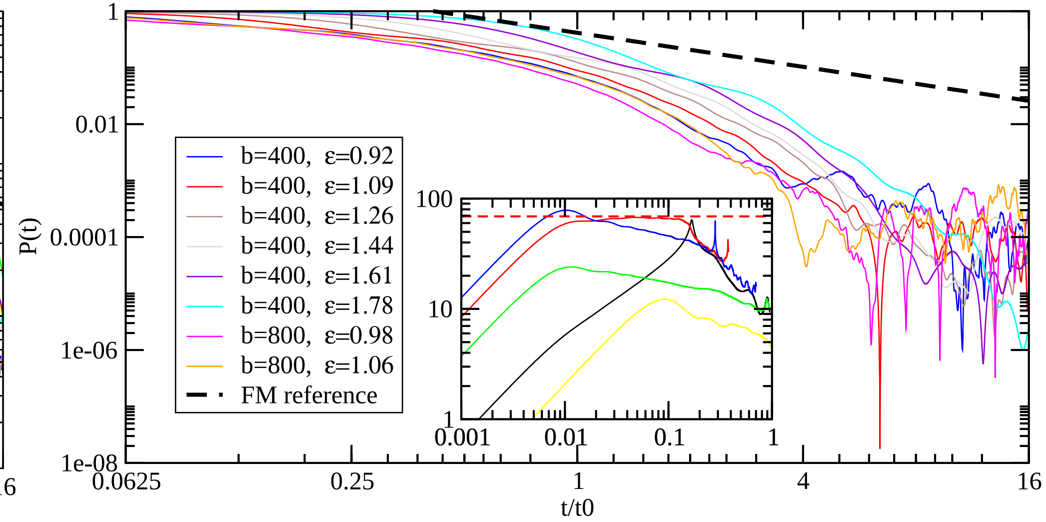
WM = Wigner model

Main results

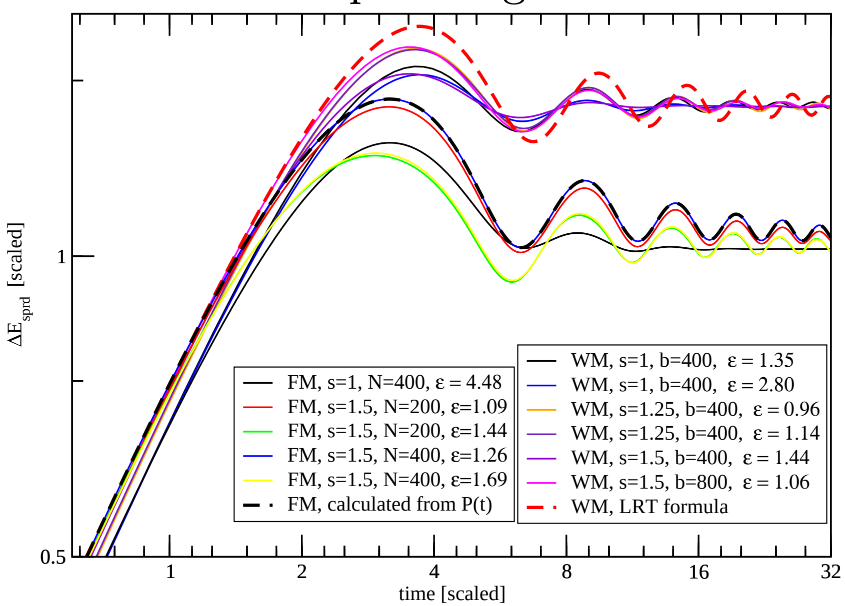
FM



WM



Spreading



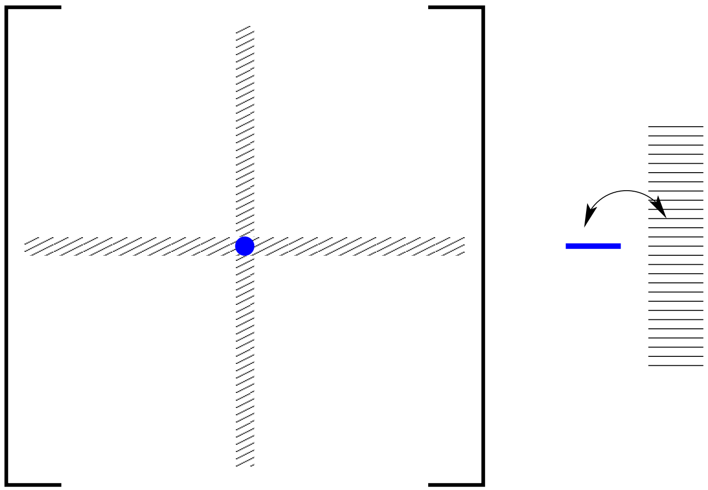
$$P(t) \approx \quad \exp[-(t/t_0)^{2-s}]$$

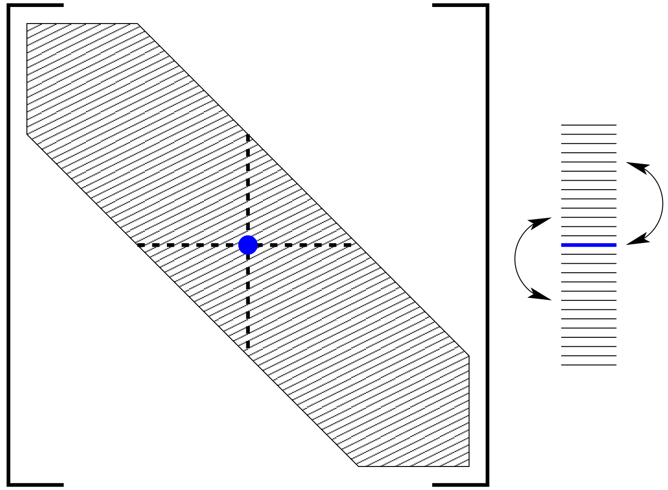
$$P(t)\Big|_{\mathrm{FM}} = \left|\frac{2\sin((s-1)\pi)}{(2-s)\pi\ (t/t_0)^{2-s}}\right|^2$$

$$\Delta E_{\mathrm{sprd}}(t)\Big|_{\mathrm{WM}} = \left[2\Big(C(0) - C(t)\Big)\right]^{1/2}$$

$$\Delta E_{\mathrm{sprd}}(t)\Big|_{\mathrm{FM}} = \left[(1+c^2(t))C(0) - \dot{c}(t)^2 + 2c(t)\ddot{c}(t)\right]^{1/2}$$

Modeling

$V_{\mathrm{FM}} =$ 

$V_{\mathrm{WM}} =$ 

$$|V_{n,0}|^2 \propto |E_n - E_0|^{s-1} \qquad\qquad |V_{n,m}|^2 \propto |E_n - E_m|^{s-1}$$

The spectral function

$$\tilde{C}(\omega) \;=\; \sum_{n\neq 0} |V_{n,0}|^2 2\pi\delta(\omega - (E_n - E_0)) \;=\; 2\pi\epsilon^2|\omega|^{s-1}\mathrm{e}^{-|\omega|/\omega_c}$$

Regime of interest $0 < s < 2$

$s = 1 =$ Flat continuum (Ohmic case)

Time scales (dimensional analysis)

$$
\begin{aligned}
t_{\mathrm{H}} &= 2\pi\varrho && \text{Heisenberg time} \\
t_c &= 2\pi/\omega_c && \text{transient time} \\
t_0 &= \left(\frac{2\pi\epsilon^2}{\mathbf{\Gamma}(3-s)\sin(s\pi/2)}\right)^{-1/(2-s)} && \text{generalized Wigner time}
\end{aligned}
$$

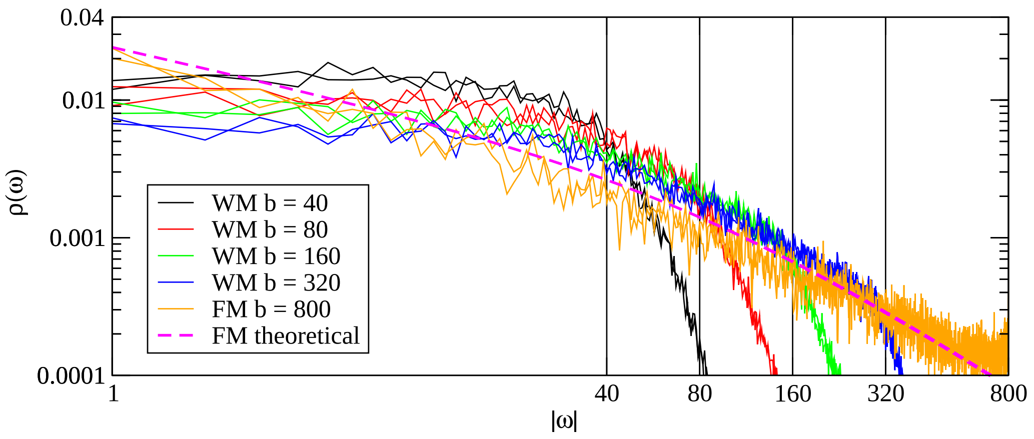
Strategy

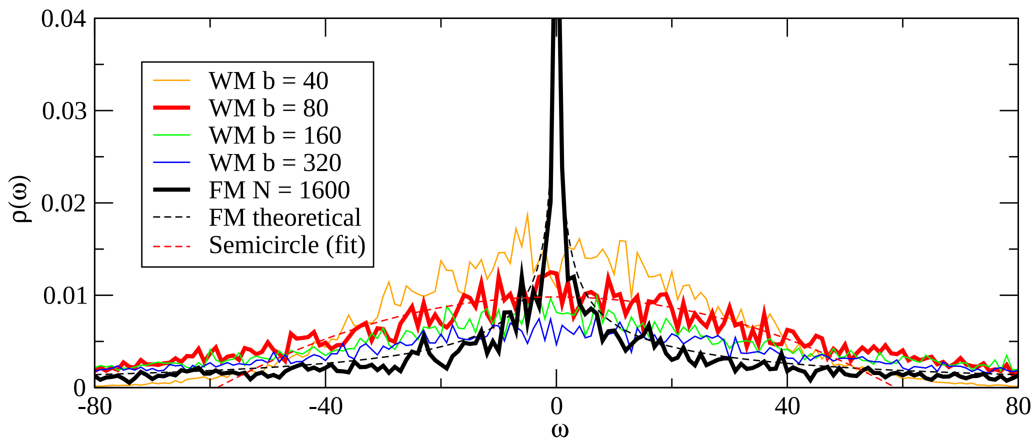
$$
P(t) \equiv \left|\langle 0|\mathrm{e}^{-i\mathcal{H}t}|0\rangle\right|^2 = \left|\mathrm{FT}\left[2\pi\rho(\omega)\right]\right|^2
$$

Local Density of States (LDoS)

$$
\rho(\omega) = \sum_n |\langle n|E_0\rangle|^2 \delta(\omega - (E_n - E_0))
$$

LDoS - FM





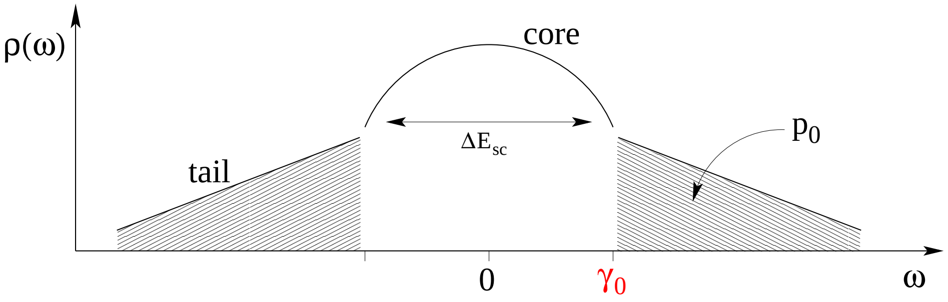
$$
\begin{aligned}
\rho(\omega) &= \sum_n |\langle n|E_0\rangle|^2 \delta(\omega - (E_n - E_0)) \\
&= \frac{1}{\pi} \frac{\Gamma(\omega)/2}{(\omega - \Delta(\omega))^2 + (\Gamma(\omega)/2)^2} \\
&= \frac{1}{\pi} \left\{ \frac{\Gamma(\omega)/2}{\Delta(\omega)^2 + (\Gamma(\omega)/2)^2}, \quad \frac{\Gamma(\omega)/2}{\omega^2} \right\} \\
&= \left\{ \left(\frac{\sin(s\pi/2)}{\pi\epsilon}\right)^2 \frac{1}{|\omega|^{s-1}}, \quad \frac{\epsilon^2}{|\omega|^{3-s}} \right\}
\end{aligned}
$$

$$
\Gamma(\omega) = \tilde{C}(\omega) = 2\pi\epsilon^2|\omega|^{s-1}
$$

$$
\begin{aligned}
\Delta(\omega) &= \int_{-\infty}^{+\infty} \frac{\tilde{C}(\omega')}{\omega - \omega'} \mathrm{d}\omega' \\
&= \epsilon^2 \pi \cot\left(\frac{s\pi}{2}\right) |\omega|^{s-1} \mathrm{sgn}(\omega)
\end{aligned}
$$

LDoS - WM

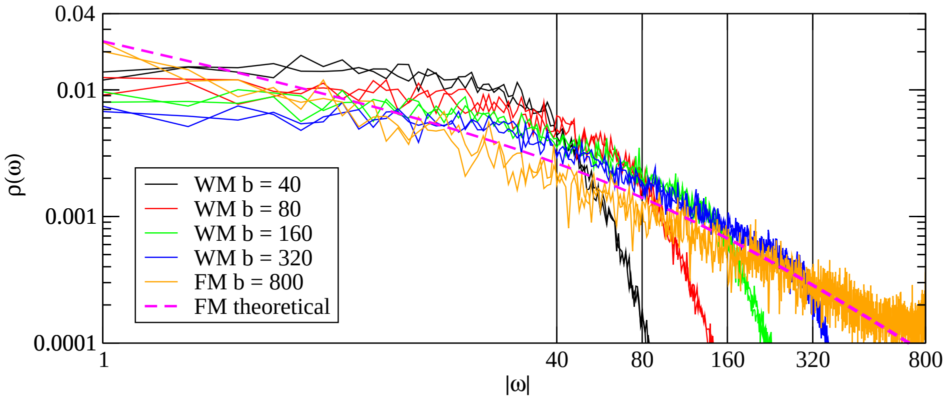
$$\rho(\omega) \approx \left\{ \text{Semi-circle}, \quad \frac{\epsilon^2}{|\omega|^{3-s}} \right\}$$

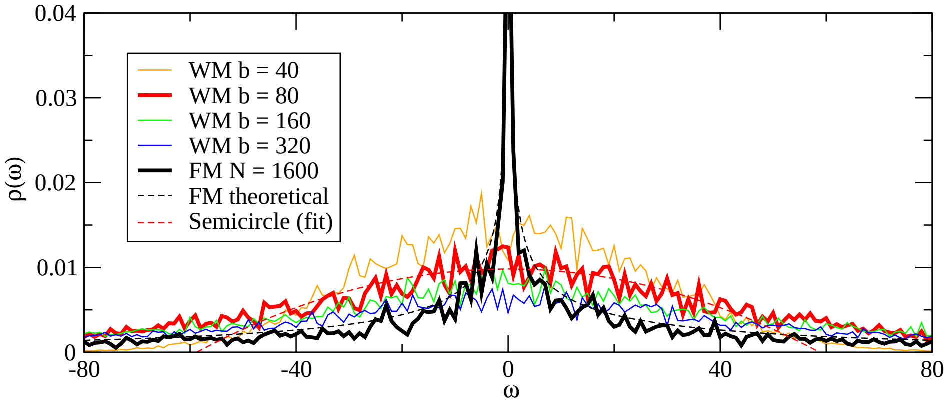


$$p_0 = \sum \left| \frac{Vn0}{E_n - E_0} \right|^2 = \int_{\gamma_0}^{\infty} \frac{\tilde{C}(\omega)}{\omega^2} \frac{d\omega}{2\pi} \sim \mathcal{O}(1)$$

$$\Delta E_{\mathrm{sc}} = \left[\int_0^{\gamma_0} \tilde{C}(\omega) d\omega \right]^{1/2} \sim \gamma_0$$

$$\Rightarrow \quad \gamma_0 \sim \epsilon^{2/(2-s)} \quad \Rightarrow \quad t_0$$





E. Wigner, Ann. Math 62 548 (1955); 65 203 (1957)
M. Feingold, Europhysics Letters 17, 97 (1992)
D. Cohen, E.J. Heller, Phys. Rev. Lett. 84, 2841 (2000)
D. Cohen and T. Kottos, Phys. Rev. E 63, 36203 (2001)

Fourier Transform

$$\rho(\omega)\big|_{\mathrm{WM}} \approx \left\{ \text{Semi-circle}, \quad \frac{\epsilon^2}{|\omega|^{3-s}} \right\} \sim \frac{1}{|\omega|^{1+\alpha}} \sim \quad \alpha \text{ stable Levy distribution}$$

$$\rho(\omega)\big|_{\mathrm{FM}} = \left\{ \left(\frac{\sin(s\pi/2)}{\pi\epsilon}\right)^2 \frac{1}{|\omega|^{s-1}}, \quad \frac{\epsilon^2}{|\omega|^{3-s}} \right\}$$

$$\mathrm{FT}\left[\mathrm{e}^{-A|t|^{\alpha}}\right] \sim \mathrm{FT}\left[-A|t|^{\alpha}\right] \sim \frac{1}{|\omega|^{1+\alpha}} \qquad \alpha = 2-s$$

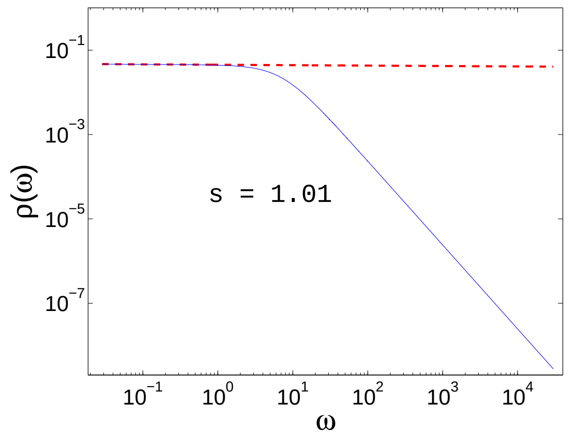
$$\mathrm{FT}\left[|\omega|^{\beta-1}\right] = \left[\frac{1}{\pi}\Gamma(\beta)\cos\left(\beta\frac{\pi}{2}\right)\right]\frac{1}{|t|^{\beta}} \qquad \beta = 2-s > 0$$

Exponential-like $\exp\left[-At^{\alpha}\right]$ is associated with $1/|\omega|^{1+\alpha}$ tails.

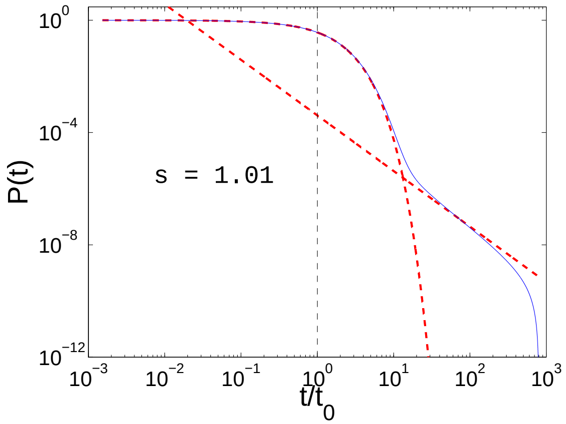
Singular LDoS core $1/|\omega|^{1-\beta}$ is associated with $1/t^{\beta}$ for long times.

Fourier Transform

$$\rho(\omega)\Big|_{\rm FM} = \left\{ \left(\frac{\sin(s\pi/2)}{\pi\epsilon}\right)^2 \frac{1}{|\omega|^{s-1}}, \quad \frac{\epsilon^2}{|\omega|^{3-s}} \right\}$$



$$P(t)\Big|_{\rm FM} = \left\{ {\rm e}^{-(t/t_0)^{2-s}}, \quad \left| \frac{2\sin((s-1)\pi)}{(2-s)\pi\ (t/t_0)^{2-s}} \right|^2 \right\}$$

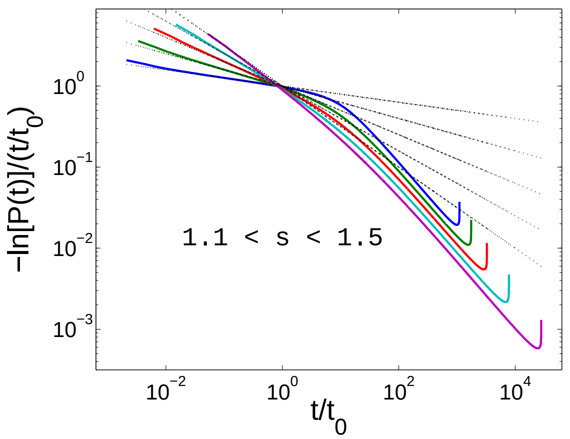


Generalized Wigner time

$$t_0 \quad = \quad \left(\frac{2\pi\epsilon^2}{\mathbf{\Gamma}(3-s)\sin(s\pi/2)} \right)^{-1/(2-s)}$$

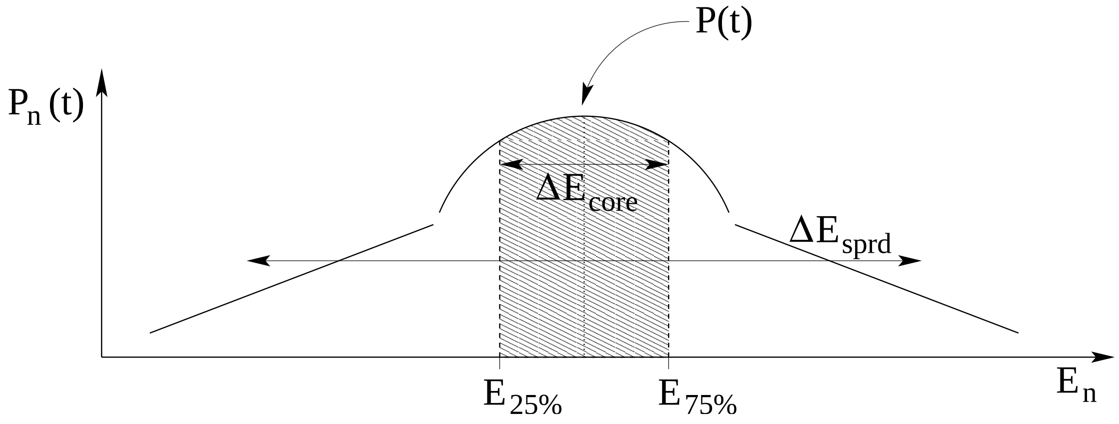
Cross-over at

$$t_0' = \left| 2\ln\left(\frac{2\sin((s-1)\pi)}{(2-s))\pi} \right) \right|^{1/(2-s)} t_0$$

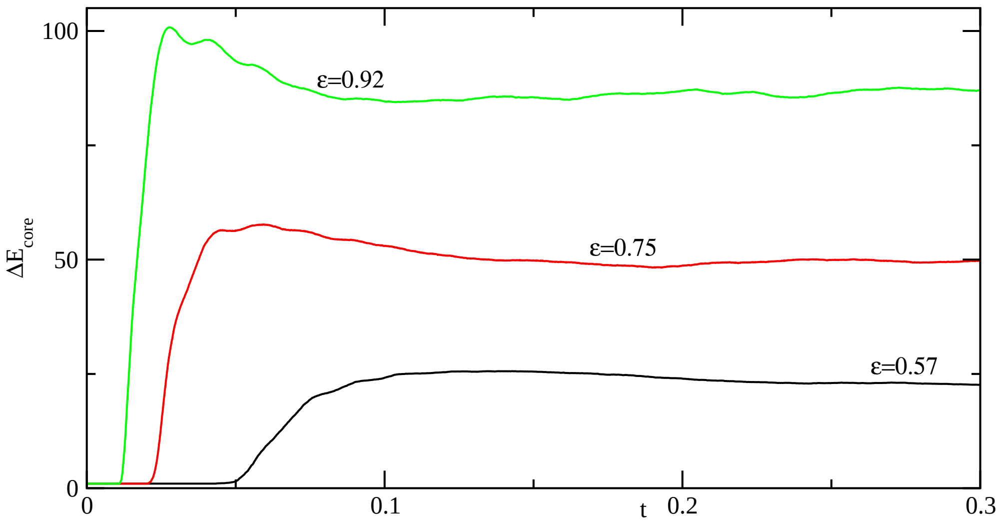


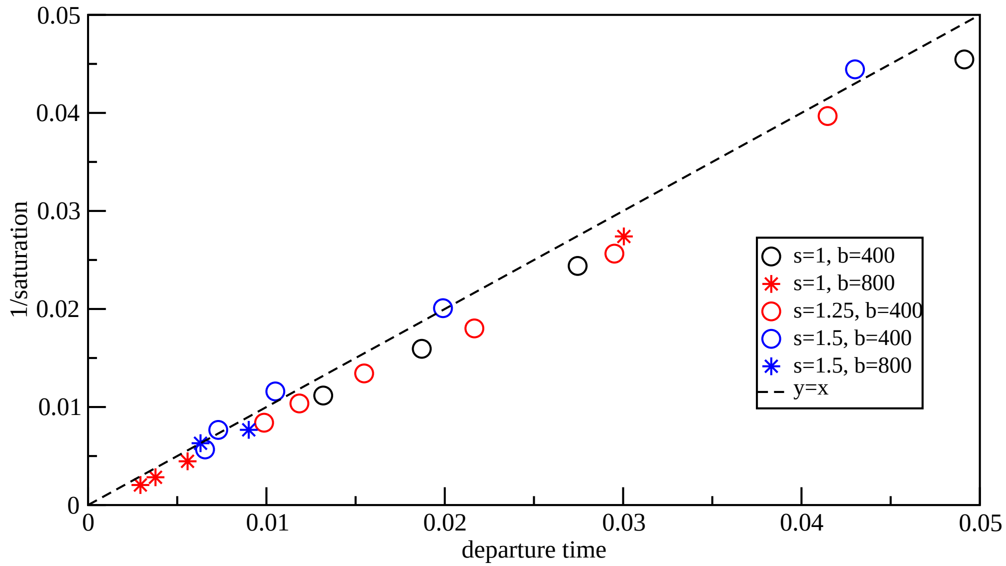
Departure time

$$\Delta E_{\mathrm{core}}(t) = E_{75\%} - E_{25\%}$$



$P(t)$ and ΔE_{core} are dominated by the core.
ΔE_{sprd} is dominated by the tails.





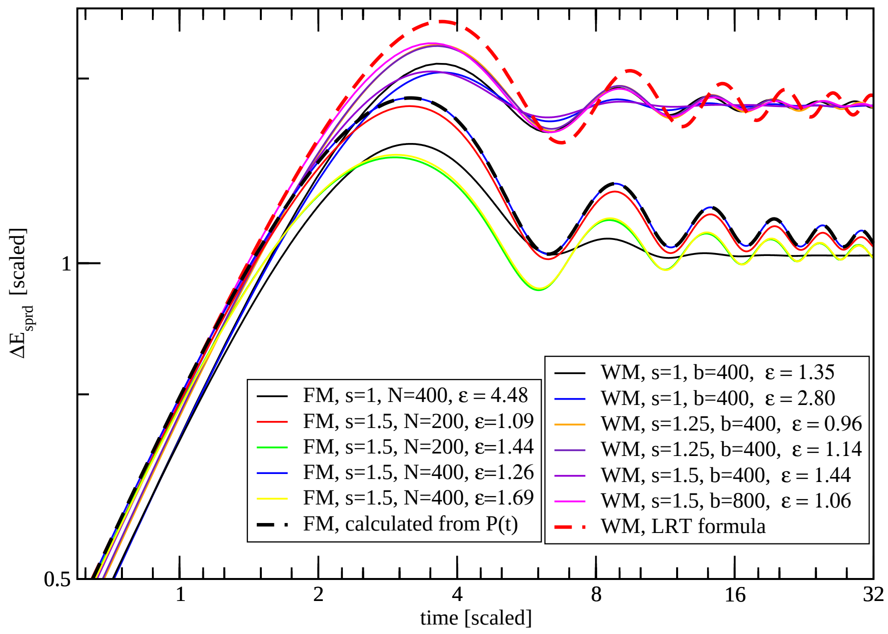
Rise at $t \sim t_0$, Saturation value $\sim \gamma_0$

One paramter scaling

Spreading

$$P(t) \quad = \quad P_t(n{=}0) \quad = \quad |c(t)|^2$$

$$\Delta E_{\text{sprd}}(t) = \left[\sum_n (E_n - E_0)^2 \, P_t(n)\right]^{1/2}$$



$$\Delta E_{\text{sprd}}(t)\big|_{\text{WM}} = \left[2\Big(C(0) - C(t)\Big)\right]^{1/2} \sim \left[2C(0)\right]^{1/2}$$

D. Cohen, F.M. Izrailev, T. Kottos, Phys. Rev. Lett. 84, 2052 (2000)

$$\Delta E_{\text{sprd}}(t)\big|_{\text{FM}} = \left[(1{+}c^2(t))C(0) - \dot{c}(t)^2 + 2c(t)\ddot{c}(t)\right]^{1/2} \sim \left[(1{+}P(t))C(0)\right]^{1/2}$$

Summary

- The “integrable“ FM and the ”chaotic” WM have the same spectral properties but different dynamics.

- The LDoS has a core-tail structure. The non-perturbative core is model-specific while the FOPT tail is universal.

- The stretched-exponential decay is characterized by a universal generalized Wigner time scale t_0.

- In the FM there is a cross-over at time t'_0 to a power-law decay.